

# THERMOMECHANICAL NOISE OF ARRAYED CAPACITIVE ACCELEROMETERS WITH 300-NM-GAP SENSING ELECTRODES

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## ABSTRACT

Thermomechanical noise of arrayed capacitive accelerometers with sub-micrometer sensing electrodes was evaluated. The unit accelerometer of the array was 80- $\mu\text{m}$  square, designed as a proportional scale-down of a conventional single-axis accelerometer. Since the size effect shows the capacitance sensitivity per unit volume increases by proportional downsizing, a 10-by-10 array of the one-tenth sized unit accelerometer would have the same sensitivity of a single accelerometer of same occupied area. However, the thermomechanical noise needs to be controlled and reduced by vacuum encapsulation because size reduction causes noise increase. By measuring the electrical impedance at the resonant frequency, the damping coefficient was estimated using electrical equivalent circuit modeling. The estimated thermomechanical noise was reduced below  $3 \mu\text{g}/\sqrt{\text{Hz}}$  by encapsulating at 100 Pa, which is low enough for instrumentation applications.

## KEYWORDS

Capacitive accelerometer array, size effect, thermomechanical noise, squeezed film damping, equivalent circuit

## INTRODUCTION

Performance improvement of MEMS capacitive accelerometers are being demanded strongly for expanding their applications for instrumentation, such as seismic, geophysical, and infrastructure monitoring measurements [1]. To realize compact, low power and reliable system, better acceleration resolution up to  $1 \mu\text{g}$  without increasing device size has been requested. However, a conventional way for improving performance is increase of their size, which makes total system large, expensive and unreliable. Our previous report pointed out that an array of proportionally down-scaled accelerometers increases sensitivity (Table 1) because the sensitivity per unit volume is inversely proportional to the representative size

of the device structure. The concept was verified by demonstrating 10-by-10 array of accelerometers of 80- $\mu\text{m}$  square [2]. However, the size effect of the squeezed film damping of parallel plate electrodes tells us that the thermomechanical noise per unit volume also increases inversely proportional to the gap width of the device. In this work, the thermomechanical noise of the arrayed accelerometer having sub-micrometer gap was evaluated through electrical impedance measurement and electrical equivalent circuit analysis.

## PRINCIPLE

The capacitance sensitivity (capacitance change per unit acceleration)  $\Delta C$  of an in-plane capacitive accelerometer of the differential parallel plate type, shown in Fig. 1, is described as;

$$\Delta C = 2\varepsilon_0 \frac{hl}{d^2} \frac{m}{k}, \quad (1)$$

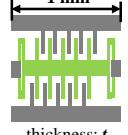

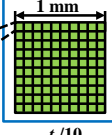
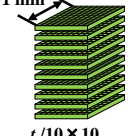
where  $\varepsilon_0$  is the dielectric constant,  $h$  and  $d$  are the height and gap of capacitance, respectively,  $l$  is the total length of one side of differential capacitances,  $m$  is the mass weight and  $k$  is the spring constant of suspending beam. Since  $m$  and  $k$  are proportional to cubic and linear of the representing size of the device structure, respectively, the capacitance sensitivity per unit volume will be inversely proportional to the size, when its dimensions are proportionally changed along all axis directions. This concept means that the total sensitivity of a thousand accelerometers is ten times of a single accelerometer of the same volume (Table 1).

However, the thermomechanical noise that is one of the dominant factors of the noise floor of capacitive accelerometers becomes an issue. The thermomechanical noise from the ambient air (Brownian noise) is;

$$ND_{tm} = \frac{\sqrt{4ck_B T}}{m}, \quad (2)$$

where  $c$  is the damping coefficient,  $k_B$  is Boltzmann constant,  $T$  is ambient temperature. The damping coefficient of the present device is dominated by the

Table 1: Capacitive accelerometer performance by arraying downscaled device structures.

Conventional	1/10 scale down	10×10 2D array	10×10×10 3D array
			
thickness: $t$	$t/10$	$t/10$	$t/10 \times 10$
Volume = 1	1/1000	1/10	1
Sensitivity = 1	1/100	1	10
Thermomechanical noise = 1	$100\sqrt{10}$	$10\sqrt{10}$	10

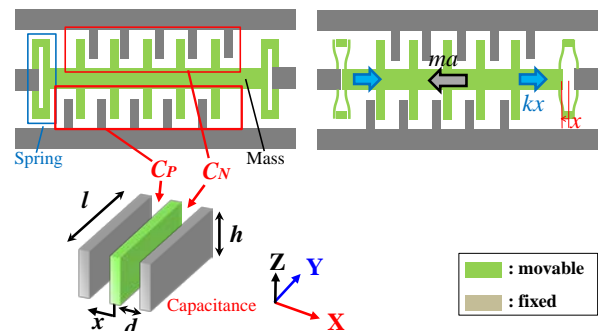


Figure 1: Basic structure of in-plane capacitive accelerometer.

squeezed film damping at the capacitance gaps, described as;

$$c_{sq} = \frac{96\mu lh^3}{d^3\pi^4}, \quad (3)$$

where  $\mu$  is the viscosity of the air. On the proportional down scaling, the squeezed film damping decreases proportional to the size. Therefore, the thermomechanical noise increases the 2.5th power of the size of unit accelerometer. Then, the noise per unit volume in arrayed accelerometers increases inversely proportional to the dimensions, since the noise is reduced by averaging effect on arraying. The size effect means that the noise increases by 10 times if we employ a three dimensional array, which cancels out the sensitivity improvement.

To solve this issue, vacuum encapsulation is employed. When the ambient pressure decreases to the Knudsen-flow regime, the viscosity of the air becomes low. At the relatively low Knudsen number ( $K_n < 1000$ ) the effective damping coefficient  $\mu_{eff}$  was used to express the change and the coefficient has been derived from the measured data and fitted to equations. In this research, we use the following equation by Li [3], since we observed good correlation to our measurements;

$$\mu_{eff} = \mu / (1 + 6.8636K_n^{0.9906}). \quad (4)$$

To get the same noise level,  $\mu_{eff}$  should be one hundredth of  $\mu$ . The ambient pressure is required to be around 1 kPa.

## EXPERIMENTAL

### Design and Fabrication

Since the conventional MEMS capacitive accelerometer has about 1 mm square in-plane size and several tens of micrometers in its thickness, the unit accelerometer of 80- $\mu\text{m}$  square with the thickness of 5  $\mu\text{m}$  was designed in our previous research [2]. The specification of the device is listed in Table 2.

Table 2: Specification of accelerometer array

In-plane dimensions (unit)	80 $\mu\text{m}$ $\times$ 80 $\mu\text{m}$
Thickness	5 $\mu\text{m}$
Max. beam length	25 $\mu\text{m}$
Min. width	0.3 $\mu\text{m}$
Min. gap	0.3 $\mu\text{m}$
Sensitivity (unit: design)	0.0105 fF/g
Number of sensors	100
Measured sensitivity (total)	0.99 fF/g

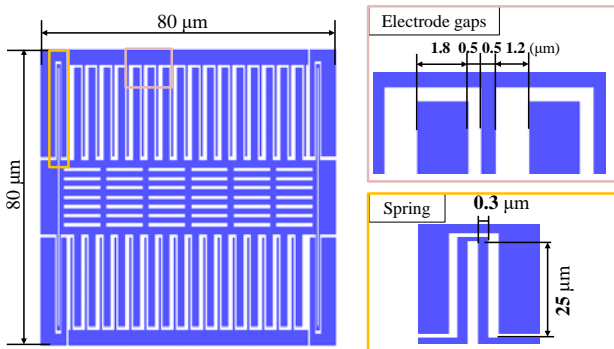


Figure 2: Fabricated 10 $\times$ 10 array of 80  $\mu\text{m}$  capacitive accelerometers. Whole device foot print is 1mm square, which is comparable to existing capacitive accelerometer.

On designing the structure of the unit-accelerometer, the width and length of the capacitance electrodes were optimized to avoid stiction during its operation. The optimized design was shown in Fig. 2. The thicker movable electrodes contribute as the mass weight.

Standard device fabrication process was used with a silicon-on-insulator (SOI) wafer of 5- $\mu\text{m}$ -thick device layer. Electron beam lithography was employed to pattern 300-nm wide beams with high accuracy. HF vapor etching with a custom made setup was used for sacrificial oxide removal.

### Thermomechanical Noise Evaluation

Since the voltage output noise of the capacitive accelerometers was dominated by the readout circuit used in our experiments, it is difficult to evaluate thermomechanical noise directly. Instead, the damping factor of the device structure was evaluated from the impedance spectra around its resonant frequency. The bias voltage was applied on the parallel plate capacitance and the movable electrodes were connected to the mass to transduce the force and to sense the displacement. The current that flowed on the capacitance was measured. A lumped model of the accelerometer, when the ac and dc bias voltage at the one set of the capacitance, is shown in Fig. 3a. In order to analyze the mechanical and electrical parameters, equivalent circuit analysis was performed, where the mechanical components were converted to corresponding electrical components. In this work, the parallel plate capacitance was modelled with the circuit shown in Fig. 3b which has been developed previously [4].

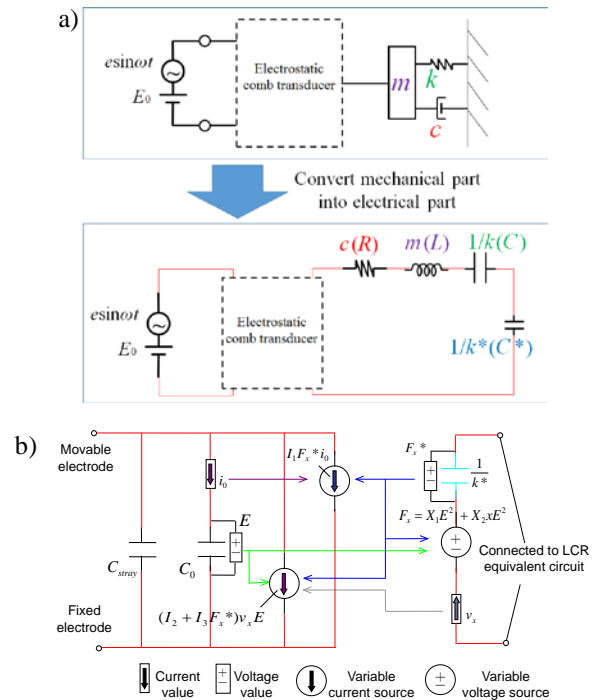


Figure 3: Equivalent circuit modeling of accelerometer. a) Modeled measurement setup and its equivalent circuit. b) Circuit model of electrostatic parallel plate transducer.

The model expresses perfectly the theoretical equations of the capacitance so that it can accommodate any dc bias voltage. It means that once the parameters of the circuit component are fixed, the model is valid for any electrical or mechanical biasing conditions.

The accelerometer mounted on a ceramic package was placed in a vacuum chamber evacuated using a rotary pump. The impedance spectrum was measured using an RF impedance analyzer (4294A, Keysight).

## RESULTS AND DISCUSSION

The fabricated device is shown in Fig. 4. The performance of the device as a single-axis accelerometer was measured and reported in [2]. The capacitance sensitivity of the arrayed accelerometer was measured as 0.99 fF/g, which is as same as that of a single accelerometer having the same device area. This demonstrates the concept of performance improvement by arrayed scale-down devices.

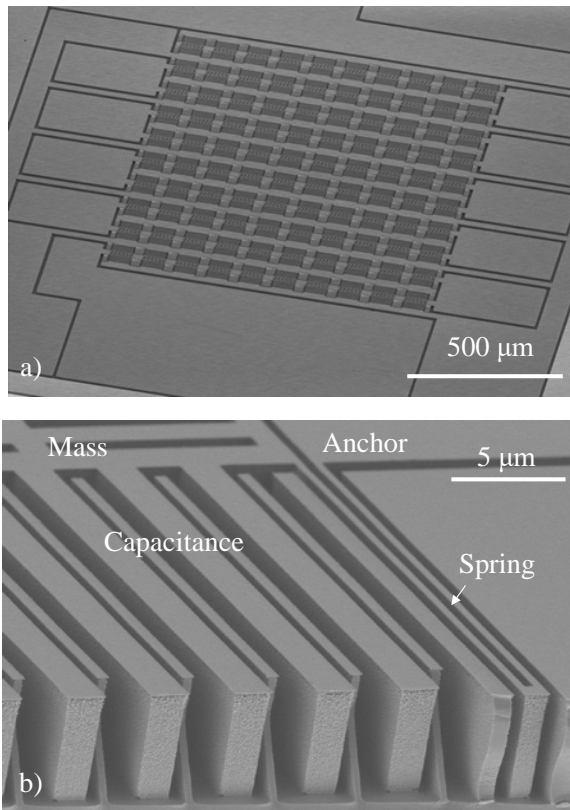


Figure 4: Fabricated  $10 \times 10$  array of  $80 \mu\text{m}$  capacitive accelerometers. Whole device foot print is  $1\text{mm}$  square, which is comparable to existing capacitive accelerometer.

The impedance spectra of the accelerometer array at  $35\text{mVac}$  and  $500\text{mVdc}$  bias and at the pressure range from 100 to  $0.1\text{ kPa}$  were measured. At the atmospheric pressure, the impedance has no peak. Below  $1\text{ kPa}$ , the peak corresponding to the resonance was observed. The spectrum was fitted to the equivalent circuit model. At  $1\text{ kPa}$  (Fig. 5), the damping coefficient was estimated as  $300\text{ nNs}$ , which agreed with the analysis by Li [3]. However, the peak shape at  $100\text{ Pa}$  was too broad to fit the extracted parameter (Fig. 6). This would be caused by variation in

the resonant frequency (mainly from the beam spring constant) due to fabrication non-uniformity. The variation became apparent at low pressure since the peak width was smaller than the variation. A good fitting was obtained by assuming that the resonant frequency has the normal distribution and its standard deviation is  $0.8\text{ kHz}$ . The fitted damping coefficient was  $25\text{ nNs}$ , which agreed well with the estimated value using Li's analysis (Fig.6).

The measured damping coefficients were used to estimate the thermomechanical noise of the arrayed accelerometer as  $8.0$  and  $2.6 \mu\text{g}/\sqrt{\text{Hz}}$  at  $1\text{ kPa}$  and  $100\text{ Pa}$  of ambient pressures, respectively.

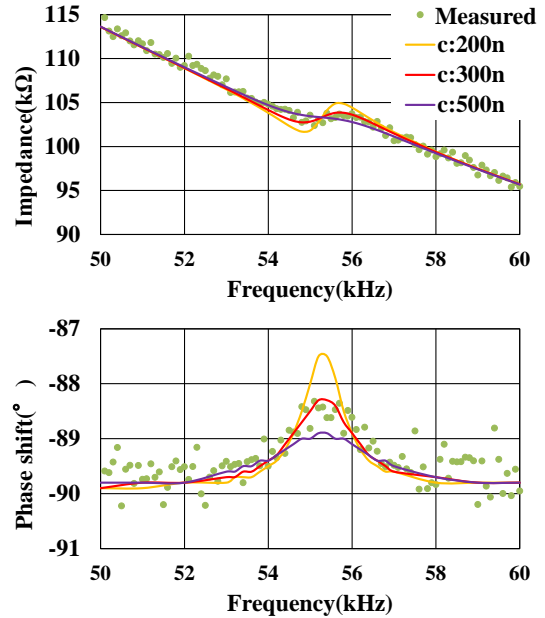


Figure 5: Impedance spectra at  $1\text{ kPa}$  with  $35\text{ mVac}$  and  $0.5\text{ Vdc}$  bias. Fitting curves show different damping factors.

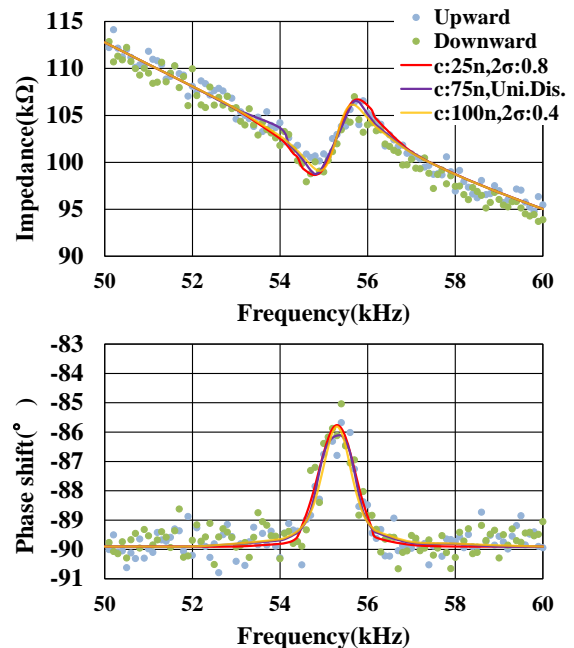


Figure 6: Impedance spectra at  $100\text{ Pa}$  with  $35\text{ mVac}$  and  $0.5\text{ Vdc}$  bias. Fitting curves show assumed different estimated deviations of resonant frequencies.

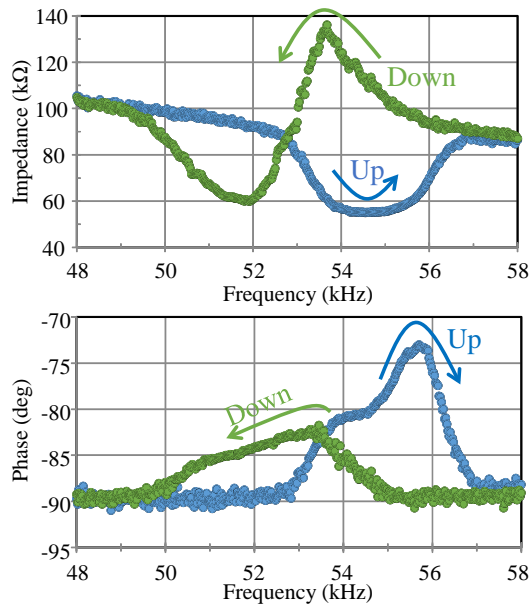


Figure 7: Impedance spectrum at 100 Pa with 140 mVac and 1 Vdc bias.

The impedance spectra at 70 mVac and 1 Vdc bias shows hysteresis, as shown in Fig. 7. On the upward sweep, plotted with blue dots, the impedance decreased from 53 kHz and the decrease was kept to 56 kHz. On the downward sweep, plotted with green dots, the impedance increased from 56 kHz and turned to a decrease at 53 kHz and the decrease was kept to 50 kHz. Assuming that the change in the impedance was caused by the resonant vibration, the frequency width was too large compared to the width at lower bias in Fig. 6. The reason might be the nonlinear vibration due to large static or dynamic displacement.

To investigate the response, the static capacitance (sum of the stray and base capacitances) is subtracted from Fig.7 and the rest of the impedance was plotted in Fig. 8. The response indicated that each resonator would have soft-spring effect. However, it is difficult to explain the reason of larger bandwidth at the resonance. Larger dc biases caused offset of the position of electrode and the offset might cause larger scatterings of the resonant frequency. The positional offset also causes lowering the resonant frequency in Fig.7 in comparison with Fig.6. Additional analysis with nonlinear effect should be conducted.

## CONCLUSION

The thermomechanical noise of the arrayed capacitive accelerometer with sub-micrometer gap was evaluated through impedance spectrum analysis using electrical equivalent circuit. The impedance spectrum around the resonant frequency was widened because of the fabrication variation in each device structure. In addition, the nonlinear response was also observed at lower pressure. It is proved that the thermomechanical noise is low enough for the application to instrumentation, when it is encapsulated at 100 Pa vacuum levels.

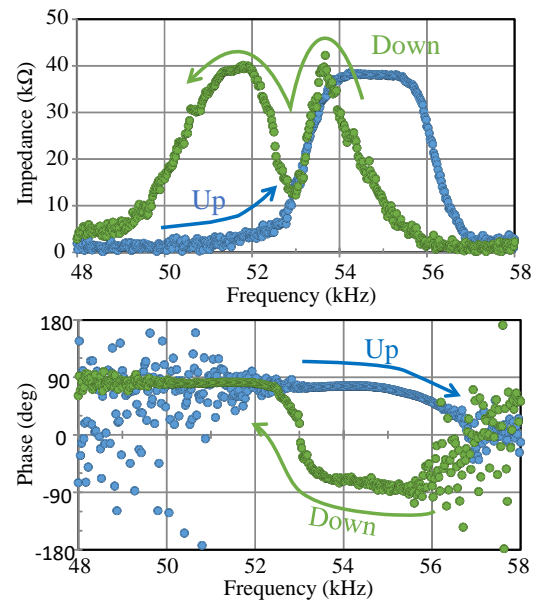


Figure 8: Impedance of arrayed accelerometer by subtracting stray capacitance.

## ACKNOWLEDGEMENTS

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