

数理解析研究所講究録 2247

RIMS 共同研究 (公開型)

Nonlinear and Random Waves

京都大学数理解析研究所

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数理解析研究所講究録は、京都大学数理解析研究所の共同利用研究集会および共同研究の記録として1964年に刊行が開始されました。当研究所が全国共同利用研究所として発足した翌年のことでしたが、以来半世紀、毎年数十巻を刊行し、2016年には第2000巻が刊行されるに至りました。第1巻から第2000巻までに収録された論文数は29,265編、総頁数は342,960頁という膨大なものであり、最先端の数学・数理科学分野の研究状況を伝えるのみならず、我が国の数学・数理科学の発展の歴史を留める文献として、他に類例を見ない論文集となっています。

講究録の内容は当研究所のウェブサイトおよび京都大学の学術情報リポジトリにおいても公開され、年間の総アクセス数は1,380,032回（2017年度）を数えるなど、多数の方にご利用いただいています。

講究録の使用言語は論文著者の判断に任されていますが、結果的に日本語が多用されていることが特徴の一つとなっています。その結果、講究録は、数学・数理科学の広い領域における最先端の専門知識に母国語でアクセスできるものとして、近年の英語化の流れの中で、重要な文献となりつつあります。

当研究所の共同利用事業に参加し講究録の論文を執筆していただいた多数の方々に対し、講究録を大きく成長させていただいたことを深く感謝いたしますとともに、これからも、当研究所の国際共同利用・共同研究拠点(*)としての活動にご参加いただき、講究録の発展にご協力いただけますよう心よりお願い申し上げます。

*数理解析研究所は2018年11月13日、共同利用・共同研究拠点の認定が廃止され、新しく国際共同利用・共同研究拠点に認定されました。

講究録

Kôkyûroku

RIMS Kôkyûroku was started in 1964 as the proceedings of symposia, colloquia and workshops supported by RIMS, the Research Institute for Mathematical Sciences, Kyoto University. It was the next year of the establishment of RIMS as one of the Nationwide Cooperative Research Centers. For half a century since then, several dozen volumes have been issued each year, and the 2,000th volume was issued in 2016. The volumes of Kôkyûroku from the 1st through the 2,000th, containing enormous 29,265 articles and 342,960 pages, not only deliver the latest research activities in mathematics and mathematical sciences but also constitute valuable and incomparable collections of articles that pass down history of progress of mathematics and mathematical science in Japan.

Articles in Kôkyûroku are available on the websites of RIMS and Kyoto University Research Information Repository. They are very frequently accessed on the internet, with a total of as many as 1,380,032 accesses in 2017.

The authors choose the languages to write articles, and many are written in Japanese, which is one of the characteristics of Kôkyûroku. As a result, Kôkyûroku is regarded as a significant and important literature which allows easy access to the latest specialized knowledge in the large fields of mathematics and mathematical sciences written in native language for Japanese readers, while more and more research papers are being written in English in recent years.

We are deeply grateful to many of those who have participated in cooperative research activities of RIMS and greatly developed Kôkyûroku. We heartily ask for your continuous participation in research activities at RIMS as an International Joint Usage/Research Center(*) and your warm support and cooperation for the fruitful development of Kôkyûroku.

* RIMS was certified as an International Joint Usage/Research Center on Nov. 13, 2018.

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Nonlinear and Random Waves

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This is a report of research done at the Research Institute for Mathematical Sciences,
an International Joint Usage/Research Center located in Kyoto University.
The papers contained herein are in final form and will not be submitted for publication elsewhere.

RIMS workshop “Nonlinear and Random Waves”

Preface

This publication is the proceedings of the hybrid format workshop entitled “Nonlinear and Random Waves”, which was held during October 3-5, 2022, at the Research Institute for Mathematical Sciences (RIMS), Kyoto University, Japan.

The main scientific subject of the workshop was nonlinear singular stochastic dispersive equations. Studies of nonlinear wave propagation in a random environment, or forced by a stochastic perturbation are of great importance in engineering and physics: nonlinear optics, condensed matter physics, fluid mechanics, turbulence analysis. Since Martin Hairer’s contribution in the field of stochastic PDEs, singular stochastic parabolic equations were very competitive research subjects these last five years, but applications of Hairer’s theory are limited for nonlinear singular dispersive equations due to the lack of smoothing properties, although the wave equations can be accessible somehow by case. On the other hand, Bourgain’s almost-everywhere approach by the use of Gibbs measure was followed by numerous developments, and simultaneously the study of the propagation of randomness under Hamiltonian flows, like wave equations, attracts now many researchers in the world. And both topics are closely related.

The purpose of this workshop was to broaden such arguments for random nonlinear dispersive equations from different and various aspects, for example, the propagation of randomness under nonlinear dispersive equations, asymptotic properties like large deviation principle and scaling limits, qualitative properties of nonlinear dispersive-type equations, scattering and stability of special solutions.

The content of this proceedings consists of short notes by Anne-Sophie de Suzzoni, Aurélien Deya, Hirotsu Nagoji, Itsuko Hashimoto, Laurent Thomann, Mamoru Okamoto, Masaya Maeda, Minami Watanabe and Yukimi Goto. Those notes are related to the topics of their talks in the workshop and cast a spotlight on recent developpements in nonlinear and random wave problems : The note by A-S. de Suzzoni is concerned with the propagation of chaos in wave turbulence modeled by the incompressible Euler equation on the torus with size L , and explains in which sense a solution corresponding to a random initial data with independent Gaussian Fourier coefficients sees its Fourier coefficients at a later fixed time remain independent for sufficiently large L . The note by A. Deya summarizes the well-posedness issues for the linear wave equation driven by a multiplicative space-time fractional noise, proposing two approaches to give sense to the stochastic product: Skorohod approach and pathwise approach. H. Nagoji offers in his note a new result as an application of the I -method for the global existence of solutions for two-dimensional nonlinear wave equations driven by subordinate cylindrical noises, following the local existence result given in his talk. I. Hashimoto explains her result on the asymptotic stability of the stationary solution on the outflow problem described by the compressible Navier-Stokes equation in the exterior domain of a ball. L. Thomann gives an overview of his results on the almost sure global existence and the almost sure scattering of the solution of the one-dimensional nonlinear Schrödinger equation in the whole space with random initial data. The new idea is the use of the absolute continuity between the linear flow measure and the nonlinear flow measure, originated from the quasi-invariance of the initial measure in the Hamiltonian structure, and the monotonicity of the Gibbs type measure. This method seems more applicable to various situations than Bourgain’s original arguments since it does not make use of the ‘invariance’ of the measure. M. Okamoto, in his note, focuses on the phenomenon of phase transition in terms of non normalizability/normalizability for Φ_3^3 -measure and Gibbs measure under Hartree type interactions. M. Maeda’s note, as a brief résumé of the works by Kowalczyk, Martel, Muñoz, Van den Bosch, introduces a virial method to prove the decay estimate of the solution of Schrödinger equations, and develop some ideas on how to remove the eigenvalues which are an obstacle for such decay properties. M. Watanabe considers the classification problem in the nonlinear Schrödinger equation, in case of double power nonlinearities. Due to the double nonlinearity, the estimate of the distance between the stationary solution and the general evolution seems not immediate by the method used in Duyckaerts and Merle for the case of pure power nonlinearity, but in place the application of the one pass Lemma by Nakanishi

and Schlag gets over this problem. Y. Goto's note addresses a partial resolution of Solovej's conjecture for the approximation of Born-Oppenheimer potential energy in the case of Kohn-Sham model.

As another objective of the workshop was the promotion of women, the speakers were equally matched in gender. This "parity" structure between men and women allowed equitable discussions, and interactions with a large part of the researchers concerned with the thematic; this would certainly not have been possible in a workshop with female-only audience. We believe that the concern about diversity of speakers should be taken into account in all conferences, as our workshop shows that such a concern does not impact the quality of the meeting. On the other hand, we had in addition an interesting opportunity which was called Cambridge INI-RIMS networking events. The idea was to share by zoom one conference on each side: from our workshop and from the organized semester at Newton Institute. In the present case, we had the opportunity to share a conference with the dispersive hydrodynamics program organized by Gennady El and Mark Hoefer; this opened us up to some different aspects of the analysis of dispersive hydrodynamic systems that was very much appreciated. We are grateful to Koji Ohkitani who proposed us this nice idea.

We would like to thank all the speakers for their cooperation. We also would like to thank the participants for the success of the workshop. The workshop clearly allowed to initiate discussions and collaborations among the participants. Last but not least, we greatly acknowledge the support from RIMS.

Organizers of the workshop "Nonlinear and Random Waves":

Anne de Bouard, Centre de mathématiques appliquées, Ecole Polytechnique, France

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Nonlinear and Random Waves
RIMS 共同研究 (公開型) 報告集

2022 年 10 月 3 日～10 月 5 日
研究代表者 福泉 麗佳 (Reika Fukuizumi)
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