Paper

Modified State Predictive Control Aiming at Improving Robust Stability*

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Finite spectrum assignment, also known as state predictive control, is an effective control method for systems with time delay in the input. This paper considers introducing some modification on the control law of state predictive control, where the modification can be interpreted, roughly speaking, as suitably taking account of the (intentionally introduced) deviation of the past input from what is desired in the sense of the conventional state predictive control. The motivation for introducing such modification lies in an attempt to modify the dynamics of the controller while maintaining a feature of the conventional state predictive control to a certain extent. In particular, we aim at improving robust stability for non-parametric uncertainties of the plant. We first derive the characteristic equation of the modified state predictive control systems, and give a necessary and sufficient condition for stability. We then derive an explicit representation of the complementary sensitivity function associated with the robust stability analysis problem for multiplicative uncertainties. Finally, we demonstrate through a numerical example that modified state predictive control can indeed be useful for improving robust stability if the modification is introduced suitably.

1. Introduction

The Smith method [1] and finite spectrum assignment, also known as state predictive control [2–4], are well known as an effective control method for systems with input delay. The latter method is applicable to unstable systems, and its key idea is to virtually remove the effect of input delay from the closed-loop system by predicting the future state $x(t+h)$, where t is the current time and h denotes the input delay. This paper considers modified state predictive control, whose main idea, roughly speaking, is to take account of the (intentionally introduced) deviation of the past input from what is desired in the sense of the conventional state predictive control. We further study its effectiveness in improving robust stability against non-parametric uncertainties.

This paper is organized as follows. As a preliminary, we first review the conventional state predictive control as well as an early conference paper on modified state predictive control by the present authors [5] so that the motivation for the present study can be manifested. We then generalize the control law in this earlier study and derive the characteristic equation of the resulting closed-loop system, which immediately leads to the necessary and sufficient condition for stability. We are ultimately interested in the effect that our modified treatment can possibly offer on robust stability against the multiplicative uncertainties of the plant. Hence, we derive an explicit representation of the complementary sensitivity function of the modified state predictive control system. Finally, we demonstrate through a numerical example that state predictive control with appropriate multiple modification terms can indeed improve robust stability compared with the conventional state predictive control.

An earlier version of this paper extending the treatment in [5] was presented in the conference paper [6], but the derivations of the characteristic equation and the complementary sensitivity function of the closed-loop system, together with the comparison of the simulation results, were not given. The present paper also considers a different improved numerical example as well as non-parametric uncertainties in the simulations so that the effectiveness of modified state predictive control in enhancing robust stability against such uncertainties can be demonstrated also in the

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time domain.

In this paper, $\mathbf{R}$ and $\mathbf{N}$ denote the set of real numbers and that of positive integers, respectively. The symbol $|\cdot|$ denotes the determinant of a matrix.

2. Conventional State Predictive Control and a Basic Idea of Modified State Predictive Control

This section first reviews the conventional state predictive control method [2–4]. To motivate the present study, an attempt of modified state predictive control in the earlier study [5] is then reviewed. The latter corresponds to a special case of the more general treatment in this paper, but starting with this simple case would make it easier to intuitively understand the idea behind modifying the control law in the conventional state predictive control.

2.1 State Predictive Control

Consider the continuous-time plant

$$\dot{x}(t) = Ax(t) + Bv(t), \quad v(t) = u(t-h), \tag{1}$$

where $x(t) \in \mathbf{R}^n$, $v(t) \in \mathbf{R}^m$, $A \in \mathbf{R}^{n\times n}$ and $B \in \mathbf{R}^{n\times m}$, and h denotes the input delay and $u(t)$ denotes the plant input at time t. We assume that $x(0)$ and $v(\tau) = u(\tau - h)$ $(0 \le \tau < h)$ are given as the initial condition at the initial time $t = 0$. Let F be a stabilizing state feedback gain for the stabilizable pair (A,B). A basic idea of the conventional state predictive control is to virtually apply this gain as

$$u(t) = Fx(t+h) \tag{2}$$

so that the input delay can be canceled and the closed-loop system would behave virtually as

$$\dot{x}(t) = (A+BF)x(t). \tag{3}$$

Obviously, however, (2) cannot be implemented directly as a control law because $x(t+h)$ is a future state. Yet, the state equation (1) admits the solution for $x(t+\theta)$ $(\theta \in [0,h])$ with the (present) state $x(t)$ and $v(\tau)$, $\tau \in [t,t+h)$ given by

$$x(t+\theta) = e^{A\theta}x(t) + \int_t^{t+\theta} e^{A(t+\theta-\tau)}Bv(\tau)d\tau, \tag{4}$$

so that the above issue is readily circumvented by considering

$$u(t) = F\Big\{e^{Ah}x(t) + \int_t^{t+h} e^{A(t+h-\tau)}Bu(\tau-h)d\tau\Big\}. \tag{5}$$

This is a control law involving the prediction of $x(t+h)$ in terms of the past plant input $u(\tau)$, $\tau \in [t-h,t)$ and is implementable as long as the state is accessible. This control method is called state predictive control, and the characteristic equation of the resulting closed-loop system [4,7] is given by

$$|sI - A - BF| = 0. \tag{6}$$

2.2 Output Feedback Case

When the state $x(t)$ is not accessible, we can introduce an observer in a mostly usual fashion. Suppose that $y(t) = Cx(t)$ is available, where $C \in \mathbf{R}^{l\times n}$, and (C,A) is detectable. Then, a full-order observer in the context of state predictive control is given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t-h) + L(C\hat{x}(t) - y(t)), \tag{7}$$

where $\hat{x}(t) \in \mathbf{R}^n$ is the estimate of $x(t)$, and $L \in \mathbf{R}^{n\times l}$ is an observer gain such that $A+LC$ is Hurwitz (i.e., stable in the continuous-time sense). If $x(t)$ is replaced by $\hat{x}(t)$ in (5), then the characteristic equation of the closed-loop system [4] is given by

$$|sI - A - BF| \cdot |sI - A - LC| = 0. \tag{8}$$

This obviously leads to the necessary and sufficient condition for closed-loop stability and implies that the so-called separation principle holds also in state predictive control.

2.3 Modified State Predictive Control with a Single Modification Term

This section reviews the modified state predictive control method studied in [5] under the state-feedback setting. The idea of this method corresponds to virtually applying a modified form of (2) given by

$$u(t) = Fx(t+h) + M_0(u(t-h) - Fx(t)), \tag{9}$$

where the coefficient matrix $M_0 \in \mathbf{R}^{m\times m}$ for the modification term is Schur (i.e., stable in the discrete-time sense) for the reason stated shortly. The actual control law is readily given by

$$u(t) = F\left(e^{Ah}x(t) + \int_t^{t+h} e^{A(t+h-\tau)}Bv(\tau)d\tau\right) + M_0(u(t-h) - Fx(t)) \tag{10}$$

where the first term is nothing but the control law of the conventional state predictive control. The second term could be interpreted as reflecting (through the matrix M_0) the deviation of the past input $u(t-h)$ from what is considered to be desirable in the sense of (2).

(Remark 1) Such a deviation arises actually by the use of the modification term itself; the standpoint of introducing such a term intentionally is that it would be useful if the coefficient matrix M_0 is chosen appropriately. After introducing more general treatment on the modification of the control law, the present paper aims at confirming such usefulness from a perspective different from the one taken in [5]; unlike the parametric uncertainties in the delay h and the steady-state gain of the plant, the present paper is interested in analyzing robustness against non-parametric multiplicative uncertainties.

The characteristic equation of the closed-loop system given by (1) and (10) is

$$|sI-A-BF|\cdot|I-M_0e^{-hs}|=0. \tag{11}$$

Now, the set of the roots of $\left|I-M_0e^{-sh}\right|=0$ is given by

$$\{(1/h)\log\nu:|\nu I-M_0|=0,\ \nu\neq 0\} \tag{12}$$

where log is the (infinitely-multivalued) complex logarithm function. Hence, we see that the closed-loop system is stable if and only if $A+BF$ is Hurwitz and M_0 is Schur.

Obviously, introducing a modification term generally leads to the presence of new closed-loop poles that were absent in the conventional state predictive control (unless $M_0\neq 0$ is nilpotent). Roughly speaking, this could be interpreted as some of the invisible closed-loop poles at $-\infty$ (corresponding to the zero eigenvalues of $M_0=0$ for the conventional case; consider the limit $\nu\to 0$ in (12)) being shifted to the right, and one might thus argue that the use of the modification term is pointless. Yet, the basic idea of introducing the modification term actually lies exactly in exploiting its ability in giving some freedom in the locations of the closed-loop poles and thus in the frequency-domain characteristics of the closed-loop system. The idea could be restated as follows: with the difference in the closed-loop poles in mind, the conventional state predictive control might correspond to high-gain feedback whereas the modification term could possibly lead to reducing the controller gain and thus improving some performance of the closed-loop system while maintaining a feature of state predictive control to a certain extent. This standpoint for the use of the modification term motivates us to consider exploiting more freedom, i.e., introducing more general modification terms to discuss further possible improvement of the closed-loop performance.

3. Modified State Predictive Control

This paper considers more general modification of the control law of the conventional state predictive control under the output-feedback setting. More precisely, we introduce multiple modification terms into the control law (5) of the conventional state predictive control and replace $x(t)$ with the state estimate $\hat{x}(t)$. As mentioned above, the single modification term in the earlier study [5] was based on the deviation of the past control input $u(t-h)$ from that given by (2) with t shifted to the past by h. The key idea in the present paper is to generalize the modification term by considering similar deviations for other values of the shift in the interval $(0,h]$.

3.1 Modified State Predictive Control and Stability

First, suppose for the moment that the state is accessible. Now, we take N $(\in\mathbf{N})$ values of shift by which we consider shifting t in (2) to the past. These values are denoted by $\mu_i h$ with $0<\mu_0<\mu_1<\ldots<\mu_{N-1}\leq 1$. This implies that we consider $u(t-\mu_i h)-Fx(t-\mu_i h+h)$ in modifying the control law (5) so that we can virtually apply the control input given by

$$u(t)=Fx(t+h)+\sum_{i=0}^{N-1}M_i\{u(t-\mu_i h)-Fx(t-\mu_i h+h)\} \tag{13}$$

where $M_i\in\mathbf{R}^{m\times m}$ $(i=0,\ldots,N-1)$ are the coefficient matrices satisfying the condition given later. However, not only the first term but also the second term is not implementable directly because $x(t-\mu_i h+h)$ at the present time t is a future state for each $i=0,\ldots,N-2$ as well as $i=N-1$ (unless $\mu_{N-1}=1$). This issue can also be circumvented by the use of the prediction formula (4), with $v(\tau)$ replaced by $u(\tau-h)$, for $x(t-\mu_i h+h)$. This readily leads to a generalized version of the control law (10) corresponding to an implementable form of (13), which is precisely the control law of modified state predictive control.

(Remark 2) By suitably splitting the interval for the finite-interval integral in (10), the introduction of different finite-interval integrals in (4) with $\theta=\mu_i h$ $(i=0,\ldots,N-1)$ leads to virtually no additional computations in terms of the controller implementation.

As shown in the appendix, the characteristic equation of the closed-loop system is given by

$$|sI-A-BF|\cdot|I-\sum_{i=0}^{N-1}M_ie^{-s\mu_i h}|=0. \tag{14}$$

This implies that the modified state predictive control system is stable if and only if the roots of this characteristic equation all lie in the open left half plane.

3.2 Roots of the Characteristic Equation

When $\mu_i=(i+1)/N$ $(i=0,\ldots,N-1)$, the only nontrivial part in solving the characteristic equation is

$$\left|I-\sum_{i=0}^{N-1}M_ie^{-s(\frac{i+1}{N})h}\right|=0, \tag{15}$$

for which we have the following theorem (the proof is easy and thus is omitted).

[**Theorem 1**] The set of the roots of (15) is given by

$$\{(N/h)\log\nu:|\nu I-M|=0,\ \nu\neq 0\}, \tag{16}$$

where

$$M=\begin{bmatrix} 0 & I_m & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & & I_m \\ M_{N-1} & M_{N-2} & \cdots & M_0 \end{bmatrix}. \tag{17}$$

It readily follows for the above special case that the modified state predictive control system is stable if and only if $A+BF$ is Hurwitz and M is Schur. The closed-loop poles for the case when μ_i $(i=0,\ldots,N-1)$ have mutually rational ratios can readily be obtained through the results for this special case (see [6] for details).

3.3 Characteristic Equation for the Output Feedback Case

It is not hard to show that when the state is not accessible, the observer (7) may be introduced without essentially affecting the above arguments; for $x(t+\theta)$ in (13) with $\theta=h$ and $\theta=-\mu_i h+h$, it is replaced by $\hat{\hat{x}}(t+\theta|t)$, which is defined as the right hand side of (4) with $x(t)$ replaced by $\hat{x}(t)$. Then, we can readily show that the characteristic equation of the closed-loop system (leading to an obvious stability condition) is given by

$$|sI-A-BF|\cdot\left|I-\sum_{i=0}^{N-1}M_i e^{-s\mu_i h}\right|\cdot|sI-A-LC|=0. \tag{18}$$

4. Complementary Sensitivity Function and Robust Stability of Modified State Predictive Control Systems

By introducing modified state predictive control, we are ultimately interested in its possible ability in improving robust stability against non-parametric multiplicative uncertainties, compared with the conventional state predictive control.

The associated complementary sensitivity function is well known to be quite important in the theoretical treatment of the robust stability problem. This section is mainly interested in explicitly describing the complementary sensitivity function of the modified state predictive control systems, so that robustness analysis can be carried out for numerical examples.

We consider the output feedback case with the full-order observer (7), and derive an explicit formula for the associated complementary sensitivity function $T(s)=(I+G(s)K(s))^{-1}G(s)K(s)$, where $G(s)=C(sI-A)^{-1}Be^{-sh}$ denotes the transfer matrix of the plant and $K(s)$ denotes that of the modified state predictive controller consisting of the state feedback gain, the full-order observer and the state prediction mechanisms relevant to (13). First, an explicit representation of $K(s)$ can be obtained by applying the Laplace transformation to (7), as well as (13) modified for the output feedback case as stated in Subsection **3.3**, and computing the transfer matrix from $-Y(s)$ to $U(s)$, where $Y(s)$ and $U(s)$ denote the Laplace transforms of y and u, respectively. This procedure is rather tedious but straightforward, in principle, which leads to

$$K(s)=-K_{\rm d}(s)^{-1}K_{\rm n}(s) \tag{19}$$

where

$$\begin{aligned}K_{\rm d}(s)&=I-\sum_{i=0}^{N-1}M_i e^{-s\mu_i h}\\ &\quad-\Bigg(F(I-e^{Ah}e^{-sh})-\\ &\quad\sum_{i=0}^{N-1}e^{-s\mu_i h}M_iF(I-e^{A(1-\mu_i)h}e^{-s(1-\mu_i)h})\Bigg)\\ &\quad\times(sI-A)^{-1}B\\ &\quad-e^{-sh}\left(Fe^{Ah}-\sum_{i=0}^{N-1}M_iFe^{A(1-\mu_i)h}\right)\\ &\quad\times(sI-A-LC)^{-1}B\end{aligned} \tag{20}$$

$$\begin{aligned}K_{\rm n}(s)&=-\left(F-\sum_{i=0}^{N-1}M_iFe^{-A\mu_i h}\right)e^{Ah}\\ &\qquad\times(sI-A-LC)^{-1}L\end{aligned} \tag{21}$$

Eq. (19) together with the matrix inversion lemma leads to the following representation of the complementary sensitivity function:

$$T(s)=-G(s)\{K_{\rm d}(s)-K_{\rm n}(s)G(s)\}^{-1}K_{\rm n}(s). \tag{22}$$

The details of the derivation of these transfer matrices are given in the appendix.

5. Robustness Comparison with a Numerical Example

This section studies the effectiveness of modified state predictive control through a numerical example by observing how the modified treatment can indeed contribute to adjusting the frequency response of the complementary sensitivity function and thus the robust stability radius.

5.1 Adjusting Complementary Sensitivity Function of Modified State Predictive Control Systems

This subsection gives a numerical example showing that the modified treatment can contribute to reducing, over some (high) frequency ranges, the gains of the complementary sensitivity function.

Consider the system given by

$$A=\begin{bmatrix}0&1&0\\0&0&1\\-4&-6&-4\end{bmatrix},\ B=\begin{bmatrix}0\\0\\1\end{bmatrix},\ C=\begin{bmatrix}2&4&3\end{bmatrix} \tag{23}$$

and the delay $h=1$. Let the state feedback gain F be such that $A+BF$ has the eigenvalues -4, -2 and

-1. Suppose that the state is not accessible and take the full-order observer gain L such that $A+LC$ has the same eigenvalues as the above. In the following, we fix F and L, and consider different modification terms to examine the effect of adding the modification terms.

(Remark 3) We aim at confirming that adding the modification terms to the conventional state predictive control law could be useful. More importantly, our main focus lies on the choice of the associated tuning parameters N, μ_i $(i=0,\ldots,N-1)$ and M_i $(i=0,\ldots,N-1)$ for the modification terms, rather than the underlying F and L. In this sense, we intend to make the example as simple as possible with respect to F and L, and thus the observer poles are taken identical with the regulator poles. One might argue that it is a practical guideline to choose the observer poles faster than the regulator poles, but it is the standpoint of the authors that such a guideline need not be taken care of in this example.

We first consider the conventional state predictive control (i.e., without any modification terms), for which the complementary sensitivity function $T(s)$ is denoted by $T_0(s)$. We next consider the case of a single modification term (i.e., $N=1$) with $\mu_0=1$ and $M_0=0.5$, taking account of the stability condition $|M_0|<1$. The corresponding $T(s)$ is denoted by $T_1(s)$. We then consider the use of multiple modification terms with $N=2$ and $N=3$. The following two cases are considered:

- $\mu_0=1/5$, $\mu_1=1$, $M_0=0.56$, $M_1=0.1$.
- $\mu_0=1/8$, $\mu_1=1/4$, $\mu_2=1$, $M_0=0.17$, $M_1=0.7$, $M_2=-0.07$.

These parameters are determined in a trial and error fashion, but both cases satisfy the stability condition. The corresponding complementary sensitivity functions are denoted by $T_2(s)$ and $T_3(s)$, respectively. For reference, the responses of y and u corresponding to the parameters of $T_0(s)$ and $T_3(s)$ (with $x(0)=[0\ \ 0\ \ 1]^T$, $u(t)=0$ $(t\in[-h,0))$ and $\hat{x}(0)=0$) are shown in Fig. 1. The comparison of the responses in this figure suggests that the state predictive controller (without any modification terms) corresponds to high-gain feedback, while the modification terms introduced in this example (for the case of $N=3$) successfully reduce the controller gain, as we have described in Section **2.3** (for the case of $N=1$) as the underlying aim that is hopefully attained by modified state predictive control. Indeed, the gain plots of the controllers (denoted by K_N) corresponding to T_N $(N=0,\cdots,3)$ are given in Fig. 2, by which we can confirm that the reduction of the controller gains have successfully been achieved in this example by the modification term(s), particularly in a high-frequency range.

To see how this gain reduction is reflected on the closed-loop performance, the gain plots of the frequency responses of $T_N(s)$ $(N=0,\ldots,3)$ are shown in Fig. 3. We could observe from these figures that appropriately introduced modification terms would indeed contribute to achieving some sort of mild control

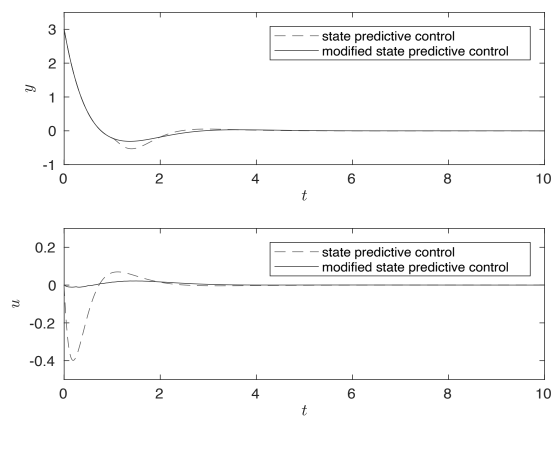


Fig. 1 Comparison of the simulation results ($N=0$ and $N=3$)

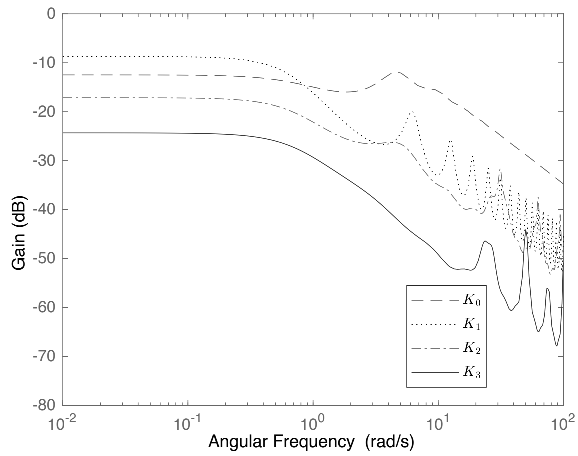


Fig. 2 Frequency responses of the controllers

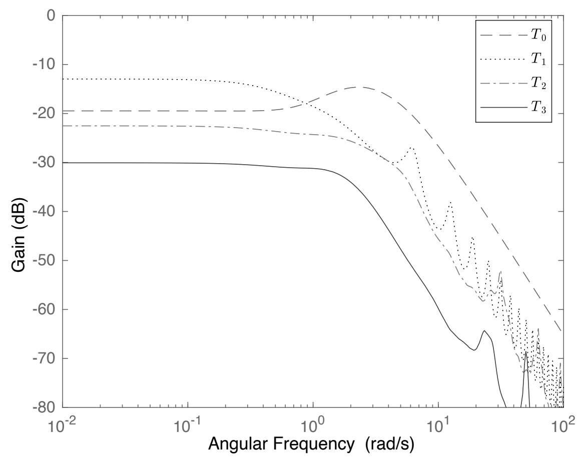
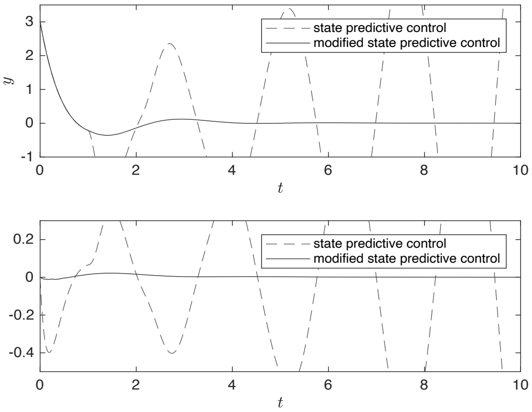
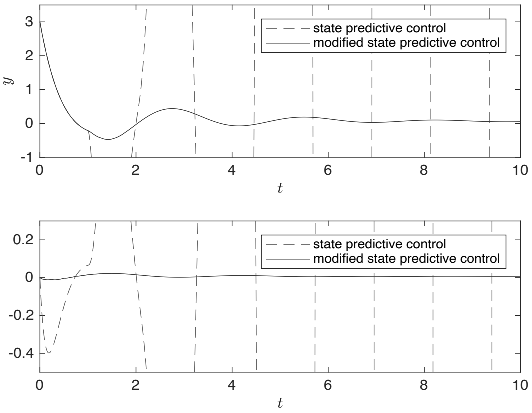


Fig. 3 Frequency responses of the complementary sensitivity functions



(a) Gain is multiplied by 8



(b) Gain is multiplied by 25

Fig. 4 Comparison of the responses under the gain perturbations

with improved control performance, which we further confirm in the following.

In particular, we see from Fig. 3 that the gains of $T_2(s)$ and $T_3(s)$ are always much smaller than the gain of $T_0(s)$, and this tendency is (mostly) more prominent in a high frequency range. Since the complementary sensitivity function is relevant to the robust stability condition with respect to multiplicative uncertainties, which typically increase as the angular frequency increases, this feature is generally considered to contribute to improving robust stability. Furthermore, the H_∞ norm of $T_3(s)$ is nearly 1/10 times smaller than that of $T_0(s)$. Thus, we can expect that, e.g., the gain margin is much larger for the case of $N=3$ than $N=0$ (the conventional state-predictive control). Thus, it is suggested that modified state predictive control can be useful in improving robust stability if the modification terms are determined appropriately. For reference, Fig. 4 shows that the conventional state-predictive control system becomes unstable if the plant gain increases by the factor of 8 but that the modified state-predictive control system remains stable even if it increases by the factor of 25.

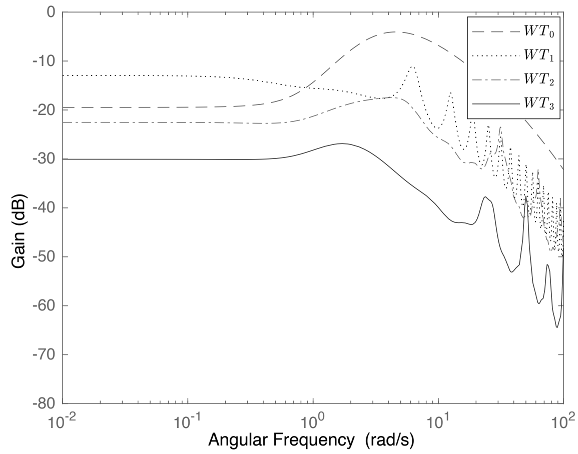


Fig. 5 Frequency responses of the weighted complementary sensitivity functions

Table 1 The comparison of the robust stability radii

	$T_0(s)$	$T_1(s)$	$T_2(s)$	$T_3(s)$
robust stability radius	1.60	3.55	7.50	22.1

5.2 Improvement of the Robust Stability Radius

We next consider confirming the effectiveness of appropriately introduced modification terms in a more quantitative manner. More specifically, the qualitative effectiveness suggested in the preceding subsection is reinforced by further computing numerically the robust stability radius, which is defined as the supremum of $\delta > 0$ satisfying the following condition: the control system with the plant G replaced by $(I+\Delta W)G$ remains stable for whatever stable transfer function $\Delta(s)$ whose H_∞ norm is bounded by δ. In this example, we assume that $W(s)$ determining the frequency-dependent bound on the multiplicative uncertainties on the output side is given by

$$W(s) = \frac{50(s+1)}{s+50}. \tag{24}$$

The gain plots for the frequency responses of $WT_N(s)$ $(N=0,\ldots,3)$ are shown in Fig. 5, by which the associated robust stability radii are as shown in Table 1 (which are given as the reciprocals of the H_∞ norms of the above transfer functions; see [8,9]; or [10] for similar arguments). Even though the modification terms corresponding to T_2 and T_3 were chosen only in a trial and error fashion (without so much difficulties), they succeed in drastically improving the robust stability radius by a factor of nearly 5 and 14, respectively, compared with that for the conventional state predictive control.

With the above analysis taken into account, we finally consider the case where the plant is subject to non-parametric uncertainties as well as the parametric uncertainties in the delay h and the gain. Let us denote by G_0 the system given by (23). Then, all the

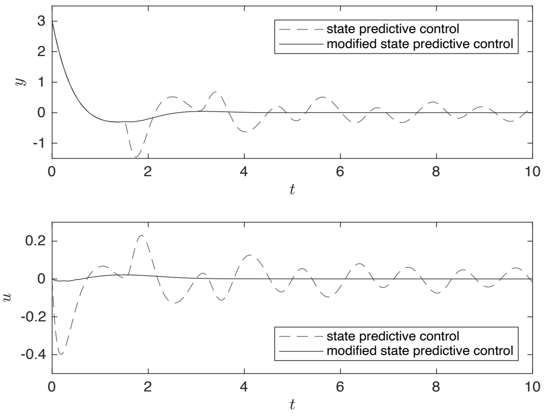


Fig. 6 Responses under non-parametric uncertainties

preceding simulations correspond to the case where both the nominal plant and the actual plant with the input delay removed are given by this G_0 (unless the gain perturbations are considered). Instead, we consider the situation where the actual plant with the input delay removed is given by $3G_0G_1/2$, where G_1 denotes a subsystem placed at the input side of G_0. In particular, we suppose that the transfer function of G_1 is given by $10(s+1)/(s+10)$ (whose gain increases as the angular frequency increases). Noting that the steady-state gain of G_1 is 1, considering G_0 as the nominal plant (as in the preceding treatment) corresponds to the situation where this subsystem G_1 was ignored in the modeling process, e.g., for simplifying the nominal plant or the poor knowledge on G_1. On the other hand, the factor of 3/2 introduced in the above corresponds to the existence of 50% increase in the gain compared with the nominal plant. Furthermore, we suppose that the input delay of the actual plant is 50% times larger than that of the nominal plant (which is $h=1$), i.e., the actual input delay is 1.5 (although the controller obviously works assuming the nominal plant delay $h=1$ and with only the associated memory of the plant input). We note that we can confirm the Δ corresponding to this actual plant (and $W(s)$ given in (24)) has the H_∞ norm slightly less than 1, where this value is smaller than the value for $N=0$ in Table 1 and is much smaller than that for $N=3$. Hence, the closed-loop system is ensured to remain stable for $N=0$ and $N=3$. The simulation results for this situation are given in Fig. 6, where the initial state for G_1 is assumed to be 0, and all the other situation remain the same as that for Fig. 1 so that the comparison of the responses in these two figures could be meaningful; in particular, the initial state of G_0 are taken the same, and $u(t)=0$ $(t\in[-1.5h,-h))$ is further assumed. The simulation results in Fig. 6 obviously demonstrate the improvement of robustness achieved by the introduction of adequate modification terms.

All the above observations imply that introducing appropriate modification terms actually contributes to enhancing robust stability of the closed-loop system.

6. Conclusions

This paper first considered modifying the control law of state predictive control. Next, the characteristic equation of the closed-loop systems was derived and a necessary and sufficient condition was given for its stability. The complementary sensitivity function of the closed-loop system was further derived so that robust stability for multiplicative uncertainties can be analyzed. Finally, a numerical example was provided demonstrating that introducing appropriate modification terms can contribute to improving robust stability and the robust stability radius compared with those for the conventional state predictive control.

As opposed to such a promising side of introducing the modification terms, it is quite important to further study a systematic procedure for somehow optimizing the modification terms, unlike the trial and error fashion employed in the example. Unfortunately, however, this is beyond the scope of the present paper, and further investigation on the issue remains our very important future studies.

Before closing the paper, we remark that the arguments in this paper up to the introduction of multiple modification terms and the derivation of the characteristic equation can be regarded as a sort of counterpart of the relevant study for discrete-time systems with input delay [11]. In this sense, optimization of the modification terms is also an interesting topic for discrete-time systems and it might be a simpler problem to start with. On the other hand, such arguments in discrete-time might further have some relation with the discretization treatment of the finite-interval integral in the control law of the (modified) state predictive control. It is known that the discretization treatment in the state predictive control law could destabilize the closed-loop system [12,7], and it would also be an interesting topic to study how the introduction of the modified terms in the control law could affect this issue in the continuous-time case.

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Appendix

Appendix 1. Derivation of the Characteristic Equation (14)

We assume the initial condition (stated below (1)) is zero, and note that the Laplace transform of the second term in the right hand side of (4) rewritten as

$$\int_0^{t+\theta} e^{A(t+\theta-\tau)}Bv(\tau)d\tau - e^{A\theta}\int_0^{t} e^{A(t-\tau)}Bv(\tau)d\tau \tag{A1}$$

is given by

$$\begin{aligned}&(e^{s\theta}I-e^{A\theta})(sI-A)^{-1}e^{-sh}BU(s)\\ &\quad = e^{-s(h-\theta)}Z^{\theta}(s)BU(s)\end{aligned} \tag{A2}$$

where $Z^{\theta}(s) := (I-e^{(A-sI)\theta})(sI-A)^{-1} = \int_0^{\theta} e^{(A-sI)t}dt$. The Laplace transforms of (1) and (13) with (4) taken into account are given respectively by

$$sX(s) = AX(s) + e^{-sh}BU(s) \tag{A3}$$

$$\begin{aligned}U(s) &= F(e^{Ah}X(s) + Z(s)BU(s))\\ &\quad + \sum_{i=0}^{N-1} M_i e^{-s\mu_i h}(I - FZ_i(s)B)U(s)\\ &\quad - \sum_{i=0}^{N-1} M_i F e^{A(1-\mu_i)h}X(s),\end{aligned} \tag{A4}$$

where $Z(s) := Z^h(s)$ and $Z_i(s) := Z^{(1-\mu_i)h}(s)$. Hence, by defining

$$\Phi = \begin{bmatrix}\Phi_{11} & \Phi_{12}\\ \Phi_{21} & \Phi_{22}\end{bmatrix} \tag{A5}$$

so that (A3) and (A4) can be rearranged as $\Phi[X^T\ U^T]^T = 0$, where

$$\begin{aligned}\Phi_{11} &= sI - A\\ \Phi_{12} &= -e^{-sh}B\\ \Phi_{21} &= -\left(Fe^{Ah} - \sum_{i=0}^{N-1} M_i F e^{A(1-\mu_i)h}\right)\\ \Phi_{22} &= I - FZ(s)B - \sum_{i=0}^{N-1} M_i e^{-s\mu_i h}(I - FZ_i(s)B),\end{aligned} \tag{A6}$$

it readily follows that the characteristic equation of the closed-loop system is given by

$$|\Phi| = 0. \tag{A7}$$

Pre- and post-multiplying Φ by the nonsingular matrices

$$\begin{aligned}T_1 &= \begin{bmatrix} I & 0\\ -e^{sh}FZ(s) & I\end{bmatrix}\\ T_2 &= \begin{bmatrix} I & 0\\ -e^{sh}FZ(s) + \sum_{i=0}^{N-1} M_i F e^{s(1-\mu_i)h}Z_i(s) & I\end{bmatrix}\end{aligned} \tag{A8}$$

respectively yields

$$T_1\Phi T_2 = \begin{bmatrix} sI - A - BF & -e^{-sh}B\\ 0 & I - \sum_{i=0}^{N-1} M_i e^{-s\mu_i h}\end{bmatrix}. \tag{A9}$$

This completes the derivation of the characteristic equation (14).

Appendix 2. Derivation of the Controller Transfer Matrix (19) and the Complementary Sensitivity Function (22)

First, the Laplace transform of the full-order observer (7) is given by

$$\hat{X}(s) = (sI - A - LC)^{-1}(e^{-sh}BU(s) - LY(s)). \tag{A10}$$

On the other hand, we have (A4) with $X(s)$ replaced by $\hat{X}(s)$ under the output feedback setting. Substituting (A10) to this modified equation on $U(s)$ and rearranging the result, we have $K_{\rm d}(s)U(s) = K_{\rm n}(s)Y(s)$, where $K_{\rm d}(s)$ and $K_{\rm n}(s)$ are given by (20) and (21), respectively. Hence, the transfer matrix of the modified state predictive controller treated in the negative feedback form is given by (19). Substituting this transfer matrix to the expression of the complementary sensitivity function $T(s)$, we have

$$\begin{aligned}T &= (I + GK)^{-1}GK\\ &= -(I - GK_{\rm d}^{-1}K_{\rm n})^{-1}GK_{\rm d}^{-1}K_{\rm n}\\ &= -G(I - K_d^{-1}K_nG)^{-1}K_d^{-1}K_n\\ &= -G(K_d - K_nG)^{-1}K_n.\end{aligned} \tag{A11}$$

This completes the derivation of (22).

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