Modified LSC Theory of Tearing Instability

研究代表者：清水　徹（愛媛大学　宇宙進化研究センター）
Tohru Shimizu, (RCSCE, Ehime University, Japan)
shimizu@cosmos.ehime-u.ac.jp

研究目的 (Research Objective):

This paper summarizes the author's studies of the linear theory of tearing instability called modified LSC theory, which started from Shimizu's KDK ResearchReport2017 and AAPPS-DPP2018, and then, was continued about 7 years. Those studies already have been or are scheduled to be published, totally in 4 full papers [1-4]. Unfortunately, the first paper [1] has been rejected 7 times between 2018-2024. For the reason, the publication of subsequent papers is also delayed. This paper may help you to read the first paper and subsequent three papers [2,3,4]. Author expects you to improve and extend this study, moreover.

1. Introduction:

The modified LSC theory is based on the original LSC theory, which was introduced by Loureiro,et.al. (PoP2007). In fact, the linear perturbation equations solved in the modified LSC were taken from that of original LSC. Hence, most notations in the modified LSC are based on the Loureiro's definitions, where Φ and Ψ are respectively perturbed potential functions of flow and magnetic fields. The prime is the derivative for the direction normal to the current sheet, where $f(\xi)$ is the equilibrium function of magnetic field Bxo. Then, the most important target is knowing the linear growth rate of the tearing instability and the critical condition, beyond which the current sheet is destabilized. This paper shows some topics important to understand about linear theory of tearing instability in my viewpoints.

2. Equilibrium must be rigorous:

Every perturbation theory must start from rigorous equilibrium. Fig.1 shows an image to explain that. Evidently, if the equilibrium is not rigorous, such theories are meaningless or have a delicate problem. At this point, FKR theory (Fruth,et.al.,PhFl1963) has a delicate problem, where a null-flow equilibrium field was employed. The null-flow equilibrium field is rigorous only in ideal-MHD limit but not in resistive-MHD. Because, the tearing instability does not occur in ideal-MHD. Against the problem, FKR focused on when the resistivity is sufficiently close to zero. Meanwhile, if the current sheet thickness is infinity, the null-flow equilibrium field can be rigorous in resistive-MHD. In that case, even with large resistivity, it is rigorous. However, in such a special case, the tearing instability is suggested to be stable [1]. Finally, employing non-zero-flow equilibrium field shown in Fig.2, LSC theory completely removed the problem. Hence, in resistive-MHD, the original and modified LSC theories can exactly study whether the current sheet is destabilized or not.

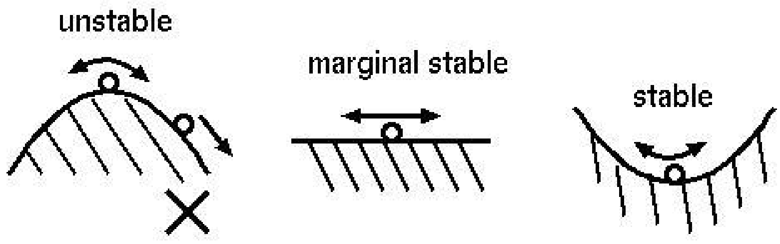

Fig.1: Rigorous equilibrium is needed to study perturbation theories. One of the circles in the left figure is not in the equilibrium.

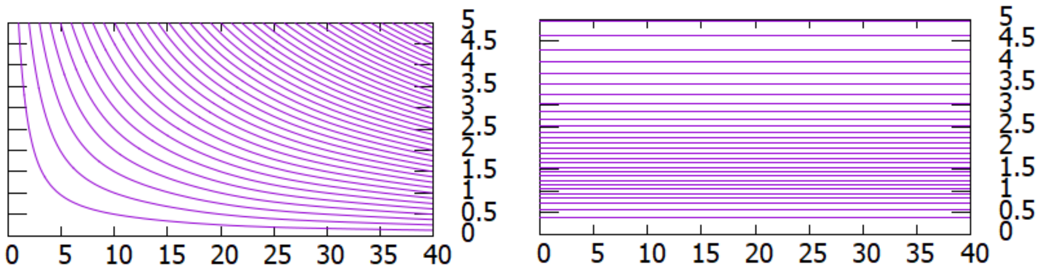

Fig.2: Rigorous equilibrium employed in LSC theory is established by magnetic annihilation, where magnetic reconnection does not occur. Left is the 2D flow potential field Φ (stream function). Right is 1D magnetic field Bxo. The current sheet is along the horizontal axis, and symmetric boundary is assumed at the origin.

3. <u>An interpretation of Δ'-index</u>:

The delta-prime index (Δ'-index) was introduced in FKR, and then, is traditionally employed in many linear theories of tearing instability, such as the original LSC. The index is defined at a discontinuity of perturbed magnetic field Ψ at the neutral sheet ($\xi=0$), i.e., $\Delta'=(\Psi'(\xi=+0)-\Psi'(\xi=-0))/\Psi(\xi=0)$. When the index is positive, tearing instability occurs, i.e., the current sheet is unstable. Meanwhile, modified LSC refers to $\Psi''(\xi=0)$, instead of the index.

Fig.3 shows how the discontinuity appears in the instability. When the instability occurs, the current density $\Psi''=-J_z$ increases. It means the magnetic field $\Psi'=B_{x1}$ piles up around the neutral sheet, against the magnetic diffusion. It results in $\Psi''(0)>0$, and hence, Ψ has a local maximum point separated from the neutral sheet, i.e., $\xi=0$, because of the magnetic field convection.

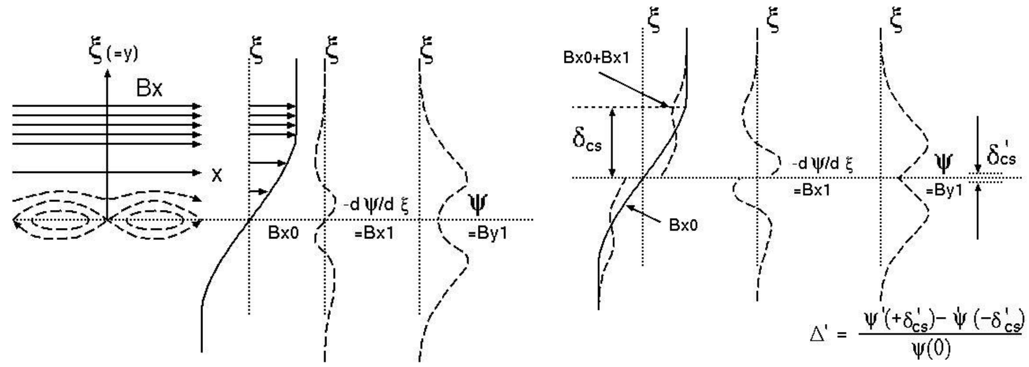

Fig.3: The most-left is the tearing instability in 2D. The middle is the equilibrium field Bxo, perturbed field Ψ'=Bx1, and Bxo+Bx1, assumed in modified LSC. The most-right is Bxo, Ψ'=Bx1, and Bxo+Bx1 in FKR and original LSC. Note that δcs'<<δcs.

As shown in Fig.3 (middle and right), the difference between the original and modifed LSCs is whether the differential discontinuity of Ψ exists at ξ=0, or not. The former assumes the discontinuity and solves outside of the discontinuity as ideal-MHD. Meanwhile, the latter does not assume the discontinuity and seamlessly solves resistive-MHD through the inside and outside of the sheet.

4. Introduction of upstream open boundary condition:

Another difference between the original and modified LSCs, is the introduction of the upstream open boundary, which is close to what is often employed in numerical simulations. There are some types of the open boundary condition. The basic type is Φ=Ψ=0 at ξ=ξc, which results in zero-crossing solution of Φ and Ψ [1,2]. The other types are Φ=Ψ=Φ'=0 [2,4] or Φ=Ψ=Ψ'=0 [3], which results in zero-contact solution.

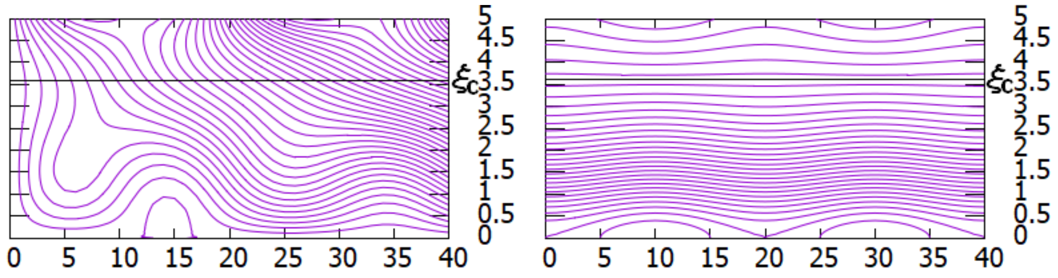

Fig.4 : Equilibrium + perturbed fields numerically obtained in modified LSC. The left is flow potential field Φo+aΦ1 and the right is the magnetic field Ψo+aΨ1, where adjustable parameter "a" must be small but, for the visuality, is extremely emphasized. For the reason, the flow field (left figure) is extremely distorted. The upetream open boundary is assumed at ξc=3.6. A plasmoid chain of magnetic islands appears along the neutral sheet, i.e., horizontal axis of the right figure.

Fig.4 shows an image of the combination of equilibrium and perturbed fields in the zero-contact solution, where the perturbed field is extremely emphasized for the visualization. The upstream side $\xi > \xi c$ of the upstream open boundary (ξc=3.6) is physically meaningless. At this point, the physical interpretation of the outside is similar to that of MHD simulations. The open boundary condition is hard to observe in Fig.4, but exactly satisfied there.

5. Initial value problem vs eigen value problem in numerical studies:

In those 4 papers [1-4], the perturbation equations were numerically solved as an initial value problem (IVP). Meanwhile, there are many studies where an eigen value problem (EVP) is solved, instead of IVP. Many peoples expect there is no difference in the selection, but those 4 papers show there are some differences. Since the numerical studies are always affected by numerical errors, the simple IVP will have advantages. In particular, the behavior of solutions at $\xi c=+\infty$ will be sensitive for the numerical errors. Regardless of IVP and EVP, any approximation or prediction is required to numerically explore the behaviors at $\xi c=+\infty$. In a viewpoint of numerical errors, solving IVP is close to particle simulations (PS), rather than fluid simulations (FS). Because, FSs always have dissipative errors while PSs can be neutral for numerical errors. The dissipative errors will distort the numerical results.

6. Introduction of viscosity and the non-uniformity:

The first paper [1] suggested that tearing instability is not perfectly stabilized by any resistivity, i.e., in resistive-MHD. In other words, resistivity can slow the growth of the instability but even infinite resistivity cannot stop it. To stop it, the introduction of viscosity is suggested. The second paper [2] studied the viscosity effect and also the non-uniformity. In contrast to the first paper, the equilibrium field $f(\xi)$ in the subsequent three papers [2,3,4] was modified to more rigorously keep the equilibrium. In fact, $f(\xi)$ in the first paper is assumed to be constant outside of the current sheet, which breaks the differential continuity of $f(\xi)$ at the outer edge of the sheet, i.e., ξ=1.307. The $f(\xi)$ in the subsequent three papers [2-4] is rigorous for the introductions of viscosity and hyper-resistivity.

In the second paper [2], it was also shown that the non-uniformity of viscosity can enhances the growth of the instability. It suggests that the uniform resistivity and viscosity do not effectively work to attain the fast growth, i.e., fast magnetic reconnection. Another remarkable result is the derivation of the critical condition, beyond which the instability stops. It was also shown that the critical condition consists of three dimensionless parameters, i.e., Lundquist number S, magnetic Prandtl number Pm, and the ratio $\xi c/1.307$, which is the ratio of the current sheet thickness $\xi o=1.307$ and the distance between the open boundary point ξc and the neutral sheet $\xi=0$. The critical condition may be applicable for substorms and solar flares observed in space plasma observations. However, it is still unclear what corresponds to the upstream open boundary in real space plasmas. Since the real current sheet will be always maintained in the 3D plasma convections of magnetosphere and flux tubes, any 3D closed equilibrium field may be considered, instead of the 2D equilibrium field with upstream open boundary.

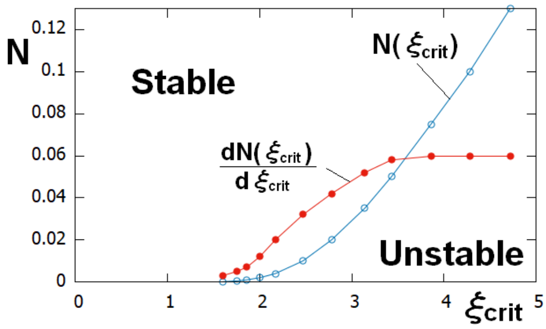

Fig.5: The critical condition of tearing instability. N is viscosity and ξ crit is the location of the critical upstream open boundary point.

Fig.5 shows the critical condition obtained in the modified-LSC with uniform viscosity N [2]. The horizontal axis ξ crit is related to the intensity of the uniform resistivity. Above the curve of N is unstable, and below is stable. Large ξ crit means when the current sheet is thin. For example, imagine when the viscosity N in plasma is constant. If the sheet is thinner than a value for ξ crit, the sheet is unstable for the tearing mode.

7. <u>Hyper-resistivity</u>:

The third paper [3] studied when tearing instability is caused by hyper-resistivity, where resistivity and hyper-resistivity mixedly work. Before exploring the perturbation theory, the rigorous equilibrium was numerically obtained, which is determined by two inflow Lundquist numbers Si and S_{Hi}, respectively, for resistivity and hyper-resistivity. Then, the perturbed solutions were numerically obtained on the basis of the equilibrium. Hence, depending on the ratio of resistivity and hyper-resistivity, the equilibrium magnetic field was modified from when resistivity only works, where the equilibrium flow field was fixed at that of the second paper [2]. Eventually, it was shown that hyper-resistivity steadily enhances the growth of instability, rather than resistivity.

8. Improvement of WKB approximation:

Fig.6 shows the image of the growth of plasmoid chain formed in plasmoid instability [1,5]. In Fig.6(a), plasmoids are generated around the origin, and then, move to downstream, i.e., the right of the figure. Since the movement of the plasmoids is accelerated in the sheet, the wave length lcs is gradually extended, as it moves to downstream. Fig.6(c) shows the movements of some X-points formed in the plasmoid chain, which is accelerated, as time proceeds. For this reason, in original and modified LSC theories, the wave number k of the plasmoid chain changes in time. However, the original LSC solved when k does not change in time. That is the zeroth-order WKB approximation. Meanwhile, modified LSC solves the first-order WKB.

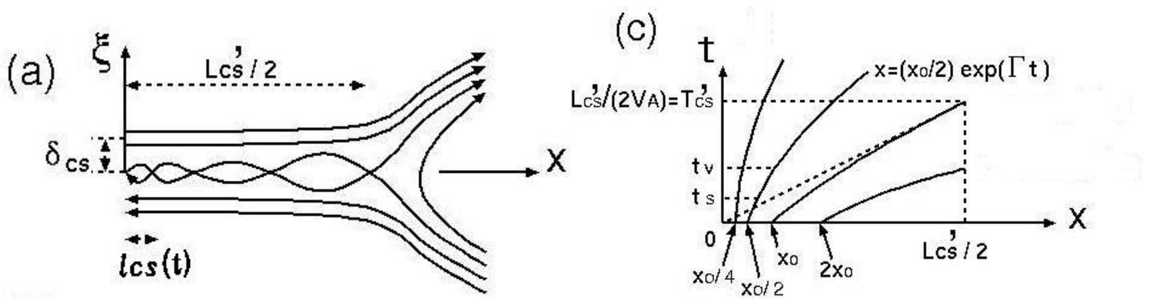

Fig.6: The growth of plasmoid chain in Plasmoid instability.

In fact, the fourth paper [4] studied the first-order WKB approximation in the modified LSC. Originally, the perturbed equations have been introduced by Loureiro, et.al., (PoP2007). As reported in KDK Research Report 2022, most of our studies have been already completed, and now, the numerical error check is being extensively made. The improvement of the WKB approximation tends to suppress the growth of the instability. Hence, the current sheet is more stable than that of the zeroth-order WKB case.

9. Intermediate shock problem:

The problems of tearing instability and magnetic reconnection process are essentially related to the intermediate shock problem (e.g., T.Hada, AAPPS -DPP2020). It suggests that it is important to consider outside of the current sheet in the viscous-resistive-MHD. In other words, uniform viscosity is important to stabilize the current sheet. At this point, in non-viscous case [1], current sheet is always unstable, i.e., cannot exist. Rather, since current sheet is commonly observable in real space plasmas, that must be often stable.

10. Conclusoins:

Plasmoid instability may be interpreted as an instability driven by plasmoids, i.e., through a kind of feedback process of the plasmoid movements. If so, the concept is essentially the same as that of the spontaneous fast reconnection model introduced by Ugai (JPP1977, PhFl1986, etc.). However, the spontaneous model is driven by anomalous (i.e., extreme non-uniform) resistivity, resulting in steady state Petsheck model. The plasmoid instability is driven by uniform resistivity, resulting in non-steady state Sweet-Parker model, which is a turbulent model. To attain the fast magnetic reconnection required for substorms and solar flares, the non-uniformity of resistivity and viscosity will be effective. Plasmoid instability established by uniform resistivity and viscosity may be a candidate for the fast magnetic reconnection but those author's studies [1-4] suggest that the present numerical studies of the plasmoid instability must be carefully rechecked [5]. Historically, the science is always developed by reducing errors, which includes numerical errors in numerical studies and instrumental errors in observations and so on. The MHD scenario introduced by Alfven may have any problem for the application to real plasmas, and also, the perturbation equation introduced by Loureiro may have any problem. However, reducing the numerical errors, it is firstly important to try to exactly explore the mathematical characteristics included in the equations.

公表状況（Publications and Presentations）：

1. Tohru Shimizu and K.Kondoh, A New Approach of Linear Theory of Tearing Instability in Uniform Resistivity, physics.plasma-ph, http://arxiv.org/abs/2209.00149
2. Tohru Shimizu, Linear Theory of Visco-Resistive Tearing Instability, (submitted to arxiv).
3. Tohru Shimizu and K. Fujimoto, Tearing Instability by Hyper-resistivity, (submitted).
4. Tohru Shimizu, a paper for WKB approximation, (being prepared to submit).
5. T.Shimizu, K.Kondoh, and S.Zenitani, Numerical MHD study for plasmoid instability in uniform resistivity, Phys. Plasmas 24, 112117 (2017).