Summary of thesis:

Bulk-boundary correspondence in non-Hermitian point-gap topological phases

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In recent years, there has been significant interest in both theoretical exploration and experimental investigation of topological phases of matter in non-Hermitian systems. Unlike Hermitian systems, non-Hermitian systems exhibit energy spectra with complex values. This leads to the emergence of two distinct gap structures: a line gap, which can be viewed as a natural extension of a gap in Hermitian systems, and a point gap, which is an inherent feature of non-Hermitian systems. These various gap structures give rise to a richer diversity of topological phases, namely, line-gap and point-gap topological phases [1,2].

The bulk-boundary correspondence (BBC), which signifies a one-to-one correspondence between the bulk topological invariant defined under periodic boundary conditions (PBCs) and the number of gapless boundary states appeared under open boundary conditions (OBCs), is a characteristic boundary phenomenon in Hermitian topological phases and serves as a manifestation of nontrivial topology. However, it is known that non-Hermiticity can obscure and disrupt the BBC in non-Hermitian system. This is due to the difference between the bulk spectrum under OBCs and that under PBCs, a phenomenon referred to as the non-Hermitian skin effects (NHSEs) [3]. Previous research has indeed demonstrated that the BBC can be restored for line-gap topological phases when the bulk spectra under OBCs are appropriately treated [3,4]. In other words, the physical consequences of nontrivial line-gap topology align with the conventional BBC. On the other hand, the situation regarding nontrivial point-gap topology remains uncertain and necessitates further exploration: In one-dimensional systems, it has been proven that a nontrivial one-dimensional winding number or $\mathbb{Z}_2$ index in PBCs result in the NHSEs [5,6], indicating unconventional boundary physics. Conversely, in three-dimensional systems, there is a suggestion that topological surface states originate from a nontrivial three-dimensional winding number under PBCs [7], implying the presence of the conventional BBC.

In this thesis, we unveil the boundary phenomena associated with point-gap topological phases and establish the BBC for point-gap topology. Furthermore, as a practical application of our theory, we propose a simple and universal platform of point-gap topological phases based on topological materials. This thesis is organized as follows.

In Chapter 1, we provide a brief overview of fundamental concepts related to topological phases. This includes discussions on symmetry, gap structure, topological classification, and the BBC, spanning both Hermitian topological phases and non-Hermitian systems.

In Chapter 2, we clarify the boundary phenomena in point-gap topological phases and prove the BBC for point-gap topological phases in a conventional sense, namely, a one-to-one correspondence between topologically protected surface states and a nontrivial point-gap topological invariant [8]. We first start from elucidating the formulation of the BBC in three-dimensional class A, using the three-dimensional winding number in OBCs. Then, we generalize it to point-gap topological phases with symmetry. A crucial consequence we reveal in this chapter is that possible point-gap topological phases can differ from PBCs and OBCs, and nontrivial point-gap topology in OBCs manifests the BBC [8].

In Chapter 3, we propose a systematic and straightforward platform of point-gap topological phases [9]. We here focus on special classes of symmetry known as $\mathrm{AZ}^\dagger$ symmetry classes, which naturally arises in effective non-Hermitian Hamiltonians of materials. By coupling topological insulators or superconductors to the environment, we implement point-gap topological phases on their surfaces. Our approach is applicable to any topological materials in Hermitian systems and pave the way to explore boundary physics in point-gap topological phases.

References

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