Summary of thesis: Topology and Strong correlation effect of Hidden symmetry breaking superconductor *Kosuke Nogaki*

Odd-parity superconductivity serves as a foundational platform for realizing topological superconductivity, wherein Majorana fermions emerge at the edges or defects of systems. Majorana fermions are recognized as key candidates for fault-tolerant quantum computing. Consequently, the pursuit and realization of topological superconductivity have been central topics in condensed matter physics. Among the various potential candidates, spin-triplet superconductors are particularly promising due to their odd parity of inversion symmetry. However, naturally occurring spin-triplet superconductors are exceedingly rare, leading to a demand for alternative principles to achieve odd-parity superconductivity. In 2012, Yoshida et al. introduced a groundbreaking principle, suggesting that sublattice degrees of freedom could enable the formation of odd-parity superconductivity in spin-singlet superconductors. About a decade after this proposal, the discovery of CeRh₂As₂ by Khim *et al.* in 2021, which exhibits a multiple superconducting phase diagram, has attracted considerable attention. This discovery has spurred numerous experimental studies aimed at elucidating the characteristics of CeRh₂As₂, unveiling several mysteries. In light of this context, this thesis focuses on the topological aspects and strong correlation effects in CeRh₂As₂, as well as field-induced superconductivity, which is intrinsically linked to the sublattice degrees of freedom.

First, we establish the presence of topological crystalline superconductivity in the high-field phase of CeRh₂As₂. By clarifying the algebraic relationships of the space group, we demonstrate that a one-dimensional odd-parity superconducting state can be defined within a restricted Hilbert space. This space is conceptualized as a glide-symmetry-preserved one-dimensional subspace embedded in the full three-dimensional space. Within this framework, we define the Zak phase in this space and express it through a Fermi-surface formula. These formulas are instrumental in indicating the topological number through the shape of the Fermi surface. Conducting electronic structure calculations, we predict the existence of topological crystalline superconductivity. Furthermore, the validity of our formulas is corroborated by analyzing a simple tight-binding model, applicable across all odd-parity irreducible representations.

Second, we delve into the strong correlation effects in $CeRh_2As_2$. While the phase diagram of this material can be interpreted using Yoshida *et al.*'s original proposal, discrepancies arise concerning the value of the parity transition field. To address this, we developed a bilayer Rashba-Hubbard model, which we analyzed using the fluctuation-exchange approximation. This model highlights the interplay between staggered Rashba spin-orbit coupling and magnetic fields, leading us to discover two-dimensional XY-type magnetic fluctuations. These fluctuations are consistent with the nuclear-magnetic resonance measurements reported by Kitagawa et al. Further, by solving the linearized Eliashberg equation, we observed a parity transition characterized by a dominant d-wave and a subdominant p-wave gap function, both contributing to superconductivity. Our phase diagram, for a wide range of spin-orbit coupling strengths, indicates a significant enhancement of parity transition fields due to antiferromagnetic fluctuations. Notably, these antiferromagnetic quantum critical fluctuations confer robustness to the system against external magnetic fields.

Third, we introduce a novel mechanism for field-induced superconductivity. Historically, field-induced superconductivity has been observed in Chevrel-phase materials and organic superconductors, typically attributed the Jaccarino-Peter effect. Additionally, in to uranium-based superconductors, this phenomenon is linked to ferromagnetic quantum critical fluctuations. Recently, the discovery of a field-induced phase transition within the superconducting state of CeRh₂As₂ has shed light on the role of sublattice degrees of freedom in strongly correlated superconductors. Notably, certain uranium-based superconductors and the locally noncentrosymmetric cerium-based superconductor CeSb₂, along with magic-angle twisted trilayer graphene, also exhibit field-induced superconductivity. In our study, we demonstrate how the degeneracy of multipole fluctuations can be lifted, creating an unconventional channel for inter-sublattice Cooper pairing. The application of a magnetic field, which symmetry, disrupts time-reversal facilitates the emergence of unconventional Cooper pairing, driven by these degeneracy-lifted multipole fluctuations. Our calculated phase diagrams indicate the formation of a field-induced odd-parity superconducting state across a wide spectrum of spin-orbit coupling strengths.