Long-term Research Visit
2022L-01

Disaster Prevention Research Institute
Kyoto University

Earthquake source modeling (source time function, seismic moment tensor, slip, rupture velocity, risetime) by inversion of only direct waves

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1. Introduction

Earthquake source mechanisms are of primary importance in monitoring local, regional, and global seismicity, as they reflect the stress structure operating in the study area and can help map its tectonic structure. On a global scale, the sources of information are large and moderate earthquakes that occur in active tectonic regions and are recorded by seismic stations around the world. On a regional and local scale, sources of information can be seismic events that occur in a separate territory and are registered by seismic stations that are part of the networks of a specific region. For this case, we can cite the seismically active region of Transcarpathia, which is located in the western part of Ukraine. earthquakes with a small magnitude (up to M4) occur here, but the nature of these events requires additional research, as so-called repeated earthquakes occur quite often in this region. So, in particular, in 2015, about 400 events with a magnitude from 0.7 to 3.0 took place near the city of Mukachevo over two weeks, which were not felt, but whose nature requires additional research. Therefore, the use of various methods to determine the parameters of the earthquake focus is an important seismological problem for this region. Determining the focal mechanisms for regions with low seismicity using standard methods is an often unsolved problem. Using the method to construct focal mechanisms based on the first arrivals of P waves is impossible due to the insufficient number of seismic stations that registered the event. This project shows the method of determining the seismic tensor by inversion only direct waves using a limited number of stations. This technique is based on the Thomson-Haskell matrix method (Thomson, 1950; Haskell, 1951), for which straight waves are extracted from the full wave field. Thus, this research project presents a moment tensor inversion method for only direct P- and S-waves, which are relatively less sensitive to modeling trajectory effects than reflected and transformed waves.

Thus, based on forward modeling, a numerical technique is developed for the inversion of observed waveforms for the components of the moment tensor, obtained by generalized inversion. Addressing the problem of unavoidable inaccuracy of seismic waves modelling, PI proposes to invert only the direct P- and S-waves instead of the full wavefield. An advantage of inverting only the direct waves consists in their much lesser distortion, if compared to reflected and converted waves, by inaccurate modelling of velocity structure of the Earth.

2. Methodology

The method presented here enables obtaining of focal mechanism solution by inversion of waveforms recorded at only limited number of seismic stations. The inversion scheme consists of two steps. First (forward modeling), propagation of seismic waves in vertically inhomogeneous media is considered and a version of matrix method for calculation of synthetic seismograms on the upper surface of the horizontally layered isotropic medium is developed. The point source is located inside a layer and is represented by seismic moment tensor. The displacements on the upper surface are presented in matrix form in frequency and wave number domain, separately for far-field and near-field (Malytskyy & Kozlovskyy, 2014). Subsequently, only the far-field displacements are considered and the wave-field from only direct P- and S-waves is isolated with application of eigenvector analysis reducing the problem to system of linear equations (Malytskyy, 2016). Subsequently (inverse modeling), spectra of the moment rate tensor components are calculated using a solution of generalized inversion and transformed to time domain by applying the inverse Fourier transform.

Forward modelling. Assuming that the point source, represented by symmetric moment rate tensor $\mathbf{M}(t)$, is located within a stack of solid, isotropic, homogeneous, perfectly elastic layers with horizontal and perfectly contacting interfaces, the layers characterized by thickness, density, and velocities of P- and S-waves, the following expressions have been obtained by Malytskyy (2010, 2016) in cylindrical coordinates for the displacements $u_z^{(0)}(t,r,\varphi)$, $u_r^{(0)}(t,r,\varphi)$ and $u_\varphi^{(0)}(t,r,\varphi)$ on the upper surface of the half-space at z=0:

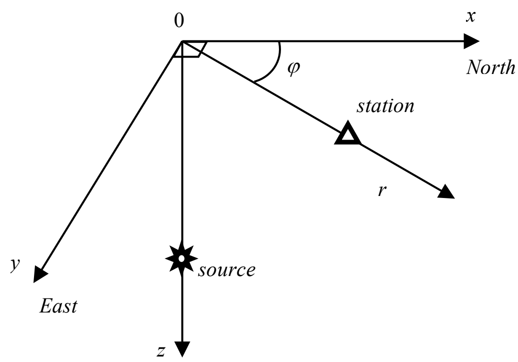

Fig. 1. Cylindrical and Cartesian coordinates of the source and the station.

$$\begin{pmatrix} u_z^{(0)} \\ u_r^{(0)} \end{pmatrix} = \sum_{i=1}^{3} \int_0^\infty k^2 \mathbf{I}_i L^{-1}\left[m_i \mathbf{g}_i\right] dk \,, \qquad u_\varphi^{(0)} = \sum_{i=5}^{6} \int_0^\infty k^2 J_i L^{-1}\left[m_i g_{\varphi i}\right] dk \,, \tag{1}$$

where

$$m_1 = M_{xz}\cos\varphi + M_{yz}\sin\varphi \,, \quad m_2 = M_{zz} \,, \quad m_3 = \cos^2\varphi \cdot M_{xx} + \sin^2\varphi \cdot M_{yy} + \sin 2\varphi \cdot M_{xy} \,,$$
$$m_4 = -\cos 2\varphi \cdot M_{xx} + \cos 2\varphi \cdot M_{yy} - 2\sin 2\varphi \cdot M_{xy} \,, \quad m_5 = M_{yz}\cos\varphi - M_{xz}\sin\varphi \,, \tag{2}$$
$$m_6 = \sin 2\varphi \cdot M_{xx} - \sin 2\varphi \cdot M_{yy} - 2\cos 2\varphi \cdot M_{xy} \,,$$

M_{xx}, M_{xy},…, M_{zz} are the Cartesian components of the moment rate tensor **M** in the frequency domain representing the source located at r=0, axis x pointing North and y East, φ is the station's azimuth (Fig.1), k is the horizontal wave number, functions $\mathbf{g}_i = (g_{zi}, g_{ri})^T$, and $g_{\varphi i}$ contain propagation effects between the source and the receiver, $\mathbf{I}_1 = \begin{pmatrix} J_1 & 0 \\ 0 & J_0 \end{pmatrix}$, $\mathbf{I}_2 = \begin{pmatrix} J_0 & 0 \\ 0 & J_1 \end{pmatrix}$, $\mathbf{I}_3 = \mathbf{I}_2$, $J_5 = J_0$, $J_6 = J_1$ are the Bessel functions of argument kr, and L^{-1} is the inverse Laplace transform, from frequency to time domain.

Further, only the far-field displacements are considered and the wave-field from only direct P- and S-waves is isolated with application of eigenvector analysis reducing the problem to a system of linear equations (Malytskyy, 2016). Eq. (1) is then expressed in matrix form for only the direct P- and S-waves on the upper surface of the half-space in frequency and wave number domain (ω, k) (Malytskyy, 2010):

$$\mathbf{U}^{(0)} = \mathbf{K} \cdot \boldsymbol{M} \tag{3}$$

where vector $\mathbf{U}^{(0)} = \left(U_x^{(0)P}, U_x^{(0)S}, U_y^{(0)P}, U_y^{(0)S}, U_z^{(0)P}, U_z^{(0)S}\right)^T$ contains the six Cartesian displacement components of direct P- and S-waves, vector $\boldsymbol{M} = \left(M_{xz}, M_{yz}, M_{zz}, M_{xx}, M_{yy}, M_{xy}\right)^T$ consists of the six independent Cartesian components of moment rate tensor **M**, matrix **K** accounting for path effects and transformations between the Cartesian and cylindrical coordinates:

$$\mathbf{K} = \begin{pmatrix} K_{11}^P & K_{12}^P & K_{13}^P & K_{14}^P & K_{15}^P & K_{16}^P \\ K_{21}^S & K_{22}^S & K_{23}^S & K_{24}^S & K_{25}^S & K_{26}^S \\ K_{31}^P & K_{32}^P & K_{33}^P & K_{34}^P & K_{35}^P & K_{36}^P \\ K_{41}^S & K_{42}^S & K_{43}^S & K_{44}^S & K_{45}^S & K_{46}^S \\ K_{51}^P & K_{52}^P & K_{53}^P & K_{54}^P & K_{55}^P & K_{56}^P \\ K_{61}^S & K_{62}^S & K_{63}^S & K_{64}^S & K_{65}^S & K_{66}^S \end{pmatrix}.$$

Inversion modeling. Now, that a direct relation between the moment rate tensor and the displacements on the free surface of the half-space is defined by eq. (3), the moment tensor can be determined from the displacements by inverting the relation.

A least-squares solution to the over-determined system of eq. (3) for **M** can be obtained by generalized inversion (Aki & Richards, 2002):

$$\mathbf{M} = (\widetilde{\mathbf{K}}^* \cdot \mathbf{K})^{-1} \widetilde{\mathbf{K}}^* \mathbf{U}^{(0)} \tag{4}$$

where the wave denotes complex conjugation and transposition, and -1 inversion.

Thus, since all the six independent components of moment tensor **M** contribute to the waveforms $\mathbf{U}^{(0)}$ in the over-determined system of Eq. 3 the inversion scheme of Eq. 4 should enable, at least theoretically, to obtain a unique solution for each of them. Within the limitations of current source presentation and path effects modeling, the solution is exact and convergence is reached after a single iteration. The inverse problem in this case consists in determining the parameters of the point source under condition that the source location and origin time is known, as well as distribution of velocities of seismic waves between the source and the station.

3. Results

In this section, we present the inversion results for the earthquake that occurred in Japan on May 11, 2021 (06:08:44.03 UTC, 34.836°N, 135.616°E, Mw3.3, depth 12.49 km). On Fig. 1, the records of this earthquake are shown. The seismic moment tensor of the earthquake, shown in Fig. 2, was determined by specialists of the Japanese National Research Institute for Earth Science and Disaster Resilience (NIED: National Research Institute for Earth Science and Disaster Resilience) by inverting the whole observed waveforms from three broadband stations.

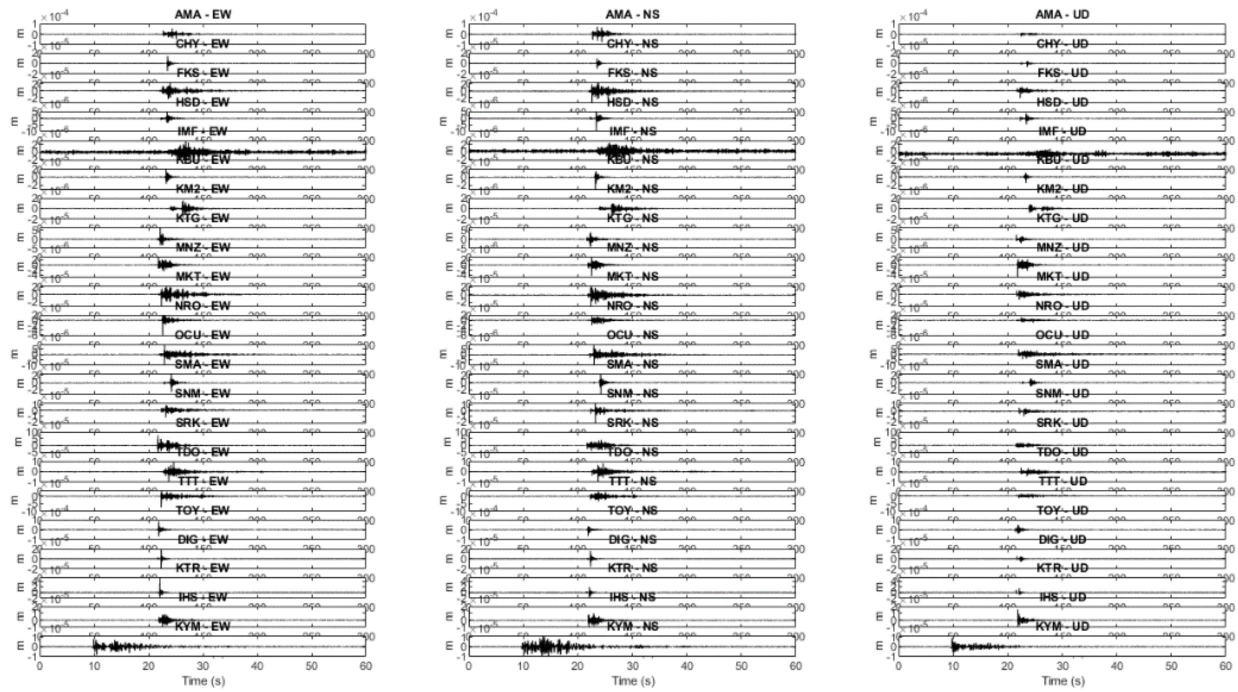

Fig.1. Records (in displacements) of the Kyoto-Osaka earthquake of May 11, 2021 (data from CEORKA, http://www.ceorka.org/).

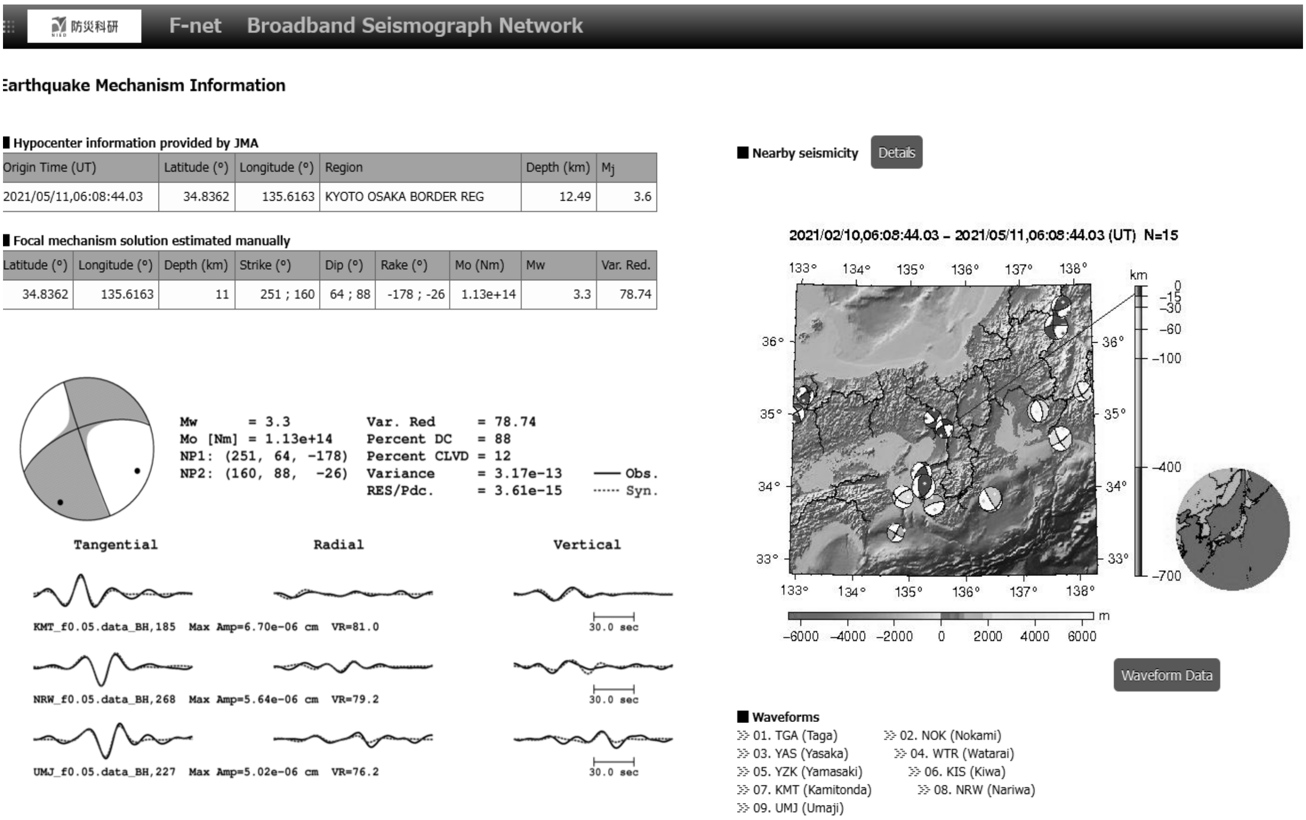

Fig. 2. Focal mechanism of the Kyoto-Osaka earthquake obtained by the seismic moment tensor inversion by NIED F-net (https://www.fnet.bosai.go.jp/event/tdmt.php?_id=20210511060700&LANG=en).

The seismic moment tensor was determined using our method by inverting the waveforms corresponding only to direct waves recorded at four stations: DIG, NRO, OCU, and MNZ and eleven stations of the local broadband seismological observation network CEORKA (The Committee of Earthquake Observation and Research in the Kansai Area): DIG, NRO, OCU, MNZ, TOY, HSD, KM2, KTR, IHS, SMA, and TDO (Fig. 3).

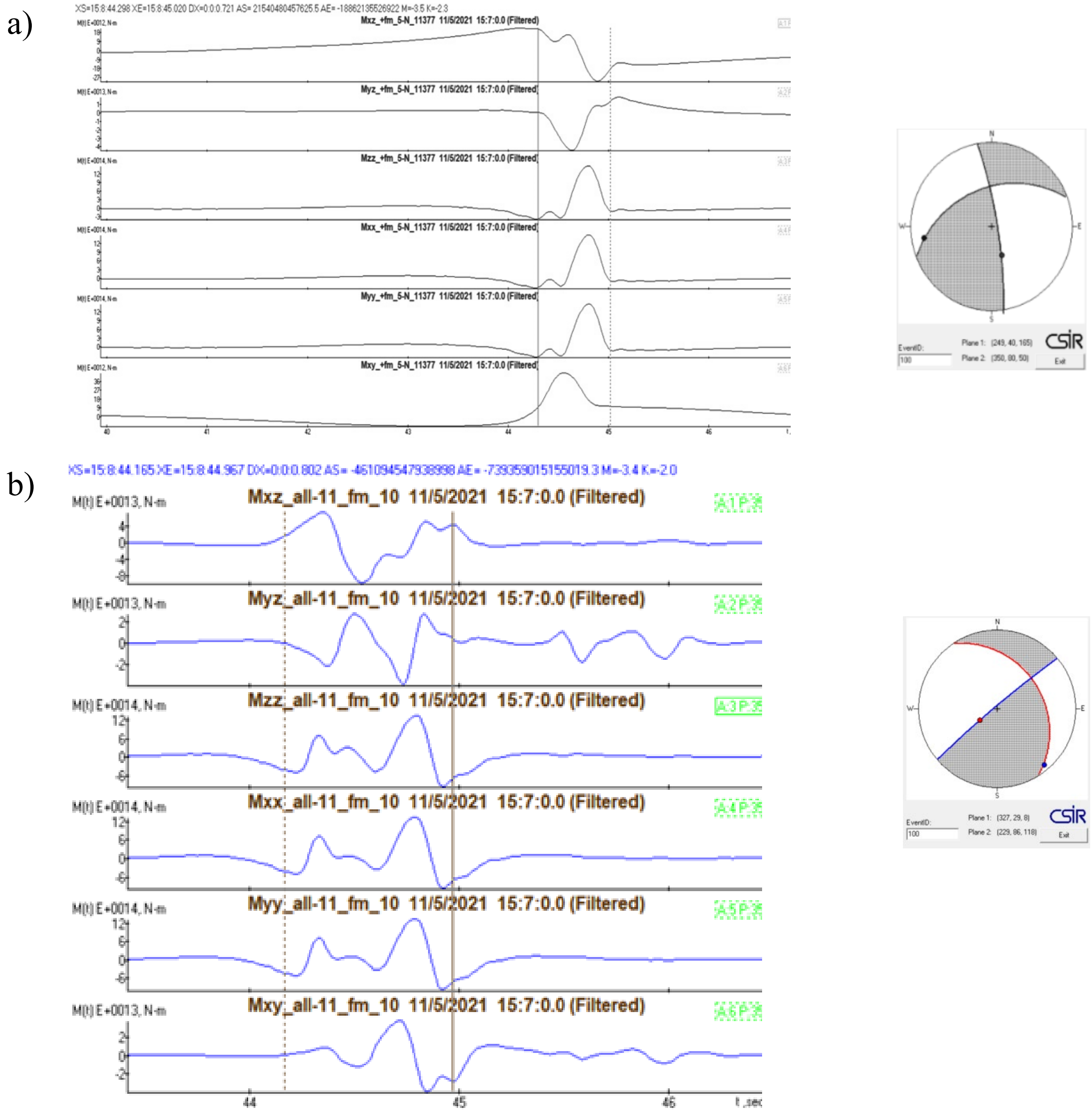

Fig.3. Seismic moment tensor solution (left) and focal mechanism (right) obtained using records from a) four stations: DIG, NRO, OCU, and MNZ and b) from 11 stations, DIG, NRO, OCU, MNZ, TOY, HSD, KM2, KTR, IHS, SMA, and TDO.

The 1D crustal model used in all the inversions of direct waveforms. The duration of direct P- and S- waves at the stations are estimated visually from the records, and accounting for the delays of reflection-conversion phases at the station corresponding to the structure model at a respective epicentral distance and source depth.

To conclude, the seismic moment tensor inversion is important but non-trivial task and the number of observation points influences the precision of the solution. Still, when using appropriate advance technicks and carefull data quality check, it is possible to infer the focal solution from a low number of observational points.

The results of determination of seismic moment tensor and focal mechanism by inversion of only direct waves (P- and S-waves) for the event of May 11, 2021 (t_0= 06:08:44.03 (UTC), 34.8362N, 135.6163E, depth=12.49 km) in Kyoto Osaka Border Region using records at four and eleven stations are presented at the conference: Geodynamics and Geospatial Research 2024 on May 30, 2024 in Riga, Latvia.

4. Acknowledgements

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5. References

Aki, K. & P.G. Richards (2002). *Quantitative seismology – Theory and methods*, University Science Books, Sausalito, California, 520 p.

Haskell, N. A. (1951). The dispersion of surface waves on multilayered media. *Bulletin of the Seismological Society of America*, **43**(1), 1–18.

Malytskyy D. & Kozlovskyy E. (2014). Seismic waves in layered media. *J. of Earth Science and Engineering*, **4**, 311–325.

Malytskyy, D. (2010). Analytic-numerical approaches to the calculation of seismic moment tensor as a function of time. *Geoinformatika*, **1**, 79–85. (In Ukrainian).

Malytskyy, D. (2016). *Mathematical modeling in the problems of seismology*, Naukova Dumka, Kyiv, 277 p. (in Ukrainian).

National Research Institute for Earth Science and Disaster Resilience (2019). NIED F-net, doi: 10.17598/NIED.0005

Thomson, W. T. (1950). Transmission of elastic waves through stratified solid medium. *Journal of Applied Physics,* **21**(2)*,* 89–93.