Clean Energy Conversion Research Section

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1. Introduction

The heat transport on the magnetic island and stochastic magnetic field are essential in magnetic confinement fusion research because the magnetic island and stochastic magnetic field sometimes simultaneously appear in the fusion plasma. For example, in tokamaks, the magnetic island is generated by the MHD instability of the tearing mode. If two magnetic islands simultaneously appear and overlap in the reversed magnetic shear, the magnetic field may be stochastic. On the other hand, for stellarators, due to the lack of toroidal symmetry, the magnetic field can become naturally stochastic.

For a long time, it has been believed that the temperature gradient cannot be kept if the magnetic field structure is strongly stochastic. Rechester and Rosenbluth theoretically discussed the electron heat transport on the strongly stochastic magnetic field[1]. On the strongly stochastic magnetic field, the diffusion of the magnetic field lines enhances the diffusion process, and then the electron temperature flattens. That is, there is no temperature gradient on the stochastic field. However, in many tokamak and stellarator experiments, although the stochastic magnetic field is expected, a finite temperature gradient is observed from the measurement. Thus, some believe that the finite temperature gradient is evidence where the magnetic field does '*not*' become stochastic. However, this is not true.

Dewer and Hudson studied the chaos in the perturbed magnetic field and successfully constructed the nearby integrable field, the so-called quadratic-flux-minimizing (QFM) surface or ghost surface[2]. Then, they proved that a near-integrable surface is a KAM surface. The heat transport is studied on the analytical and realistic stochastic magnetic field. If some KAM surfaces exist on the stochastic magnetic field, those KAM surfaces can be worked as a transport barrier. In addition, depending on the ratio of the parallel and perpendicular heat conductivity, $\chi_\parallel/\chi_\perp$, the finite temperature gradient can be kept on the stochastic magnetic field.

In this study, we numerically study the heat transport on the stochastic magnetic field of a stellarator. In particular, the heat transport depending on the different diffusions is considered.

2. Model and Numerical Implementation of anisotropic heat diffusion

In this study, we study the anisotropic heat diffusion on the stochastic magnetic field following numerical studies in tokamaks. At first, it assumes that heat transport is dominated by heat conduction. Then, the heat transport equation is considered as follows:

$$\frac{3}{2} n \frac{\partial}{\partial t} T = -\nabla \cdot \mathbf{q} + Q$$

where n and T are electron density and temperature, q is the electron heat flux, and Q is the heat source. Here, the convection term is neglected. For simplicity, the constant density is assumed. So, the heat transport equation is rewritten as

$$\frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q} + Q$$

This is a starting equation.

In this study, the equation is numerically solved. Here, the strong anisotropy is given by the heat flux,

$$\mathbf{q} = -(\chi_\parallel \nabla_\parallel T + \chi_\perp \nabla_\perp T)$$

Gradients of the T along the parallel and perpendicular directions can be defined by,

$$\nabla_\parallel T = \mathbf{bb} \cdot \nabla T$$

and

$$\nabla_\perp T = \nabla T - \nabla_\parallel T = (\mathbf{I} - \mathbf{bb}) \cdot \nabla T$$

where, b is the unit vector of the magnetic field, B/B, and I is the unit tensor.

This study approximates these gradients by the finite difference scheme on the cylindrical coordinate (R, ϕ, Z). In previous studies, the parallel and perpendicular gradients are approximated by metrics of the magnetic field. However, guaranteeing enough accuracy, particularly along the parallel direction, is challenging. Therefore, this study uses the field line tracing method to improve the numerical accuracy of calculating the parallel gradient. At first, the anisotropic heat diffusion equation is simplified in the given magnetic field as

$$\frac{\partial T}{\partial t} = \left(\chi_\parallel \nabla_\parallel^2 T + \chi_\perp \nabla_\perp^2 T\right)$$

Next, two magnetic field lines are traced from a computational grid on (R, ϕ, Z) to b and $-b$

directions, respectively. Here, the equation of the magnetic field lines is defined by

$$\frac{dR}{dl} = B_R/B$$
$$\frac{d\phi}{dl} = B_\phi/RB$$
$$\frac{dZ}{dl} = B_Z/B$$

where the integral variable is l.

If the length L of the field line tracing from the computational grid can be defined, the parallel gradient at $L/2$ can be defined as

$$\frac{\partial T}{\partial l}\bigg|_{l=+\frac{L}{2}} = \frac{T|_{l=+L} - T|_{l=0}}{\Delta l}$$

where l is the arch length of the field line from the grid and $T|_{l=+L}$ is the temperature at $L = +l$. Here, the $l = 0$ is the grid. Thus, the parallel diffusion on the grid can be defined as

$$\frac{\partial^2 T}{\partial l^2}\bigg|_{l=0} = \frac{\frac{\partial T}{\partial l}\big|_{l=+\frac{L}{2}} - \frac{\partial T}{\partial l}\big|_{l=-\frac{L}{2}}}{\Delta l}$$

The magnetic field line is traced by the 4th-order Runge-Kutta method, and the 4th-order scheme interpolates the temperature. At the beginning of the simulation, the field line is traced, and two end points of the field line at $l = +L$ and $-L$ starting from $l = 0$ are stored in the memory. On the other hand, the perpendicular diffusion on the cylindrical coordinate may be simplified as

$$\nabla_\perp^2 T \sim \frac{\partial^2 T}{\partial R^2} + \frac{\partial^2 T}{\partial Z^2}$$

where the metric along the R direction is ignored. The second-order finite difference scheme approximates the second derivate, and the derivative along the R direction is defined as

$$\frac{\partial^2 T}{\partial R^2} = \frac{T_{i+1,j,k} - 2T_{i,j,k} + T_{i-1,j,k}}{\Delta R^2}$$

The time evolution of the anisotropic heat diffusion is integrated by the 4th-order Runge-Kutta method.

In this study, the heat source is given by the Gaussian profile on the magnetic axis. The plasma response effect is not included, and the magnetic field is fixed in time.

3. Results

We applied a new code that implemented the above ideas to a stellarator, L/M= 2/10 Heliotron configuration.

The $\chi_\parallel/\chi_\perp$ is scanned from 10^6 to 10^{10} in figure 1. A top panel shows enlarged T/T_0 profiles on Z=*const.* plane, and a bottom panel shows a Poincare plot at the horizontally elongated cross section for the vacuum standard configuration. Two purple circles indicate the stochastic magnetic field region. Due to the increased $\chi_\parallel/\chi_\perp$, the T/T_0 profile is changed, and the small flattening of the temperature appears from $\chi_\parallel/\chi_\perp = 10^7$. However, for the cases of $\chi_\parallel/\chi_\perp = 10^8$ - 10^{10}, the change of the profile is not large. According to Fitzpatrick's theory[3], the flattening width of the temperature on the magnetic island is proportional to the following equation,

$$\frac{W_d}{r_s} \sim \left(\frac{\chi_\perp}{\chi_\parallel}\right)^{\frac{1}{4}} \left(\frac{1}{\epsilon_s n s}\right)^{\frac{1}{2}} \sim \left(\frac{\chi_\perp}{\chi_\parallel}\right)^{\frac{1}{4}}$$

If the $\chi_\parallel/\chi_\perp$ is 10^8, the W_d/r_s is roughly 10^{-2}. The flattening size in this study is consistent with the theoretical prediction.

5. Summary

We numerically study the anisotropic heat diffusion on the stochastic magnetic field in the Heliotron configuration. The smooth temperature profile appears if the perpendicular diffusion is significant, although the magnetic field becomes stochastic. However, if the ratio of $\chi_\parallel/\chi_\perp$ increases to 10^8, the small flattening of the temperature profile appears on the stochastic magnetic field. In that region, the connection length of the magnetic field line is sufficiently long. Therefore, the finite temperature gradient can be kept.

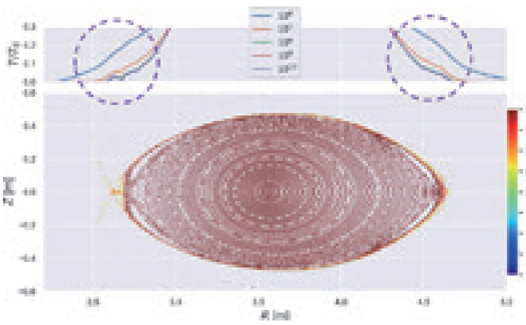

Fig. 1: A comparison of the anisotropic heat diffusion with the different $\chi_\parallel/\chi_\perp$ is shown. The $\chi_\parallel/\chi_\perp$ is scanned from 10^6 to 10^{10}. A top panel shows enlarged T/T_0 profiles on Z = *const*. plane, and a bottom panel shows a Poincaré plot at the horizontally elongated cross section for the vacuum standard configuration. Two purple circles indicate the stochastic magnetic field region. Due to the increased $\chi_\parallel/\chi_\perp$, the T/T_0 profile is changed.

References

[1] Rechester AB, Rosenbluth MN. Phys. Rev. Lett. 1978; 40: 38.

[2] Hudson SR, Dewar RL. Phys. Lett. A 1998; 247: 246.

[3] Fitzpatrick R. Phys. Plasmas 1995; 2: 825.