Boolos' "The Hardest Logic Puzzle Ever" and coinduction*

Ken-etsu Fujita (Gunma University)
Toshihiko Kurata (Hosei University)
February 16, 2023

Abstract

Boolos (1996) posed the puzzle "The hardest logic puzzle ever", and gave a solution in the style of biconditional questions. We first introduce a simple formalization of the puzzle consisting of questions, answerers, and answers in terms of propositional logic, and show its adequacy by the truth values $(0, 1)$ semantics. We here pose a general form of the puzzle as an extension of Roberts (2001), and provide solutions to the instances. Our analysis reveals an essential condition for solvability of the puzzle in terms of the number of Random.

1 Introduction

George Boolos (1996) posed the puzzle "The hardest logic puzzle ever". The basic form of the puzzle comes from the well-known puzzles of Knights and Knaves from the book Logical Labyrinths [6], which often appear in logic or sociology of lying and truth-telling. For instance, one can find many examples [3] such as the film Labyrinth of Lucasfilm [9], the entrance examination of Cambridge University [5], and so on.

To begin with, we quote the puzzle from Boolos [1]:

"The puzzle: Three gods, A, B, and C are called, in some order, True, False, and Random. True always speaks truly, False always speaks falsely, but whether Random speaks truly or falsely is a completely *random* matter. Your task is to determine the identities of A, B, C by asking three yes-no questions; each question must be put to exactly one god. The gods understand English, but will answer all questions in their own language, in which the words for "yes" and "no" are "da" and "ja," in some order. *You do not know which word means which.* $\cdots$ $\cdots$ $\cdots$"

*This work was supported by the Research Institute for Mathematical Science, a Joint Usage/Research Center located in Kyoto University. This work was partly supported by Grants-in-Aid for Scientific Research KAKENHI (C) 17K05343 and 20K03711.

Roberts [8] and Rabern-Rabern [7] provided a simpler solution in the style of embedded questions as follows.

1. Ask god A:

   if I asked you if god B was Random, would you say da?

2. Ask B or C[1]:

   if I asked you if you always told the truth, would you say da?

3. If I asked you if god A was Random, would you say da?

We briefly review our methodology [2, 3, 4] for self-sufficiency. The methodology for solving the puzzles of Knights and Knaves from the book Logical Labyrinths [6] can be naturally extended to that for the hardest logic puzzle ever. For the extension, the binary relation between inhabitants and assertions should be replaced with a ternary form of questions, answerers, and answers. Firstly, let X be a propositional variable for a question, which means either true or false, respectively represented by 1 or 0. Secondly, let A, B be propositional variables for answerers A, B, which mean either[1] True or False, respectively represented by 1 or 0. Lastly, let Y be a propositional variable for an answer. Here, an answer means either yes or no, respectively represented by 1 or 0. Instead, Y may also be used for the answer da or ja, whose meaning is either 0 or 1, but not fixed yet.

If we ask a question X of an answerer A and obtain an answer Y, then the situation is depicted by the following diagram.

$$X \longrightarrow \boxed{\;A\;} \longrightarrow Y$$

We formalize this relation of question-answerer-answer by the ternary form with the logical connective of bi-implication.

$$X \leftrightarrow A \leftrightarrow Y$$

Note trivial facts that $\leftrightarrow$ is symmetric and associative and that a tautology is the unit. An adequacy of the formalization can be expounded by the truth values $(0, 1)$ semantics. Let **Prop** be the set of propositions (formulae), and $\{0, 1\}$ for the set of truth values. We write v for the assignment $v : \mathbf{Prop} \to \{0, 1\}$. Now the consistent relation of question-answerer-answer is stated as follows:

$$v(X \leftrightarrow A \leftrightarrow Y) = 1,$$

under the assignment v such that $v(A := \text{“True”}) = v(Y := \text{“yes”}) = 1$. This statement can be justified by the case analysis on A, as follows.

[1] In the puzzle of Boolos [1], one has Random in addition, but for the formalization here answerers are supposed to be either True or False. According to the solutions [1, 8, 7], Random can be handled by a certain *strategy* of asking questions which can be formalized here in terms of the ternary form.

- Case A of “True”: $v(X \leftrightarrow Y) = 1$

$$X \longrightarrow \boxed{\text{True}} \longrightarrow Y$$

- Case A of “False”: $v(X \leftrightarrow Y) = 0$

$$X \longrightarrow \boxed{\text{False}} \longrightarrow Y$$

Let us formalize the embedded question [8, 7]. Recall the first one from the solution:

- Ask god A Q_1: if I asked you if god B was Random, would you say da?

$$\text{“}X \rightarrow \boxed{A} \rightarrow Y\text{”} \longrightarrow \boxed{\quad A \quad} \longrightarrow ?_1$$

  where $X :=$ “B is Random”, $Y :=$ “da”.

Now suppose that A’s answer $?_1$ is “da”. Then this situation is formalized by the formula $(X \leftrightarrow A \leftrightarrow Y) \leftrightarrow A \leftrightarrow Y$, and hence for any assignment v we have the following equation

$$v((X \leftrightarrow A \leftrightarrow Y) \leftrightarrow A \leftrightarrow Y) = v(X),$$

since $A \leftrightarrow A$ and $Y \leftrightarrow Y$ are tautologies. This implies that one can conclude $v(X) = 1$. Analogously, we conclude $v(X) = 0$ if the answer $?_1$ is “ja”. This is the reason why we can identify the truth value of X from $?_1$ via a single question, even if we know neither the semantics of A nor that of “da” (“ja”).

Following our formalization we summarize the solution [8, 7] which consists of the questions Q_1, Q_2, Q_3 in this order, depending on the answer $?_1$:

1. Q_1 (Ask god A: if I asked you if god B was Random, would you say da?)

$$\text{“B} = \text{R} \rightarrow \boxed{A} \rightarrow \text{da”} \longrightarrow \boxed{\quad A \quad} \longrightarrow ?_1$$

2. $Q_2(Z := C)$ if $?_1 = \text{da}$ ($Q_2(Z := B)$ otherwise (i.e. $?_1 = \text{ja}$))

$$\text{“}Z = \text{T} \rightarrow \boxed{Z} \rightarrow \text{da”} \longrightarrow \boxed{\quad Z \quad} \longrightarrow ?_2$$

3. $Q_3(Z := C)$ (otherwise $Q_3(Z := B)$)

$$\text{“A} = \text{R} \rightarrow \boxed{Z} \rightarrow \text{da”} \longrightarrow \boxed{\quad Z \quad} \longrightarrow ?_3$$

As a solution we have 3! patterns consisting of R, T, or F for $\langle \text{A}, \text{B}, \text{C} \rangle$, and 2^3 patterns $\langle ?_1, ?_2, ?_3 \rangle$ for an answer to $\langle Q_1, Q_2, Q_3 \rangle$. Every candidate for $\langle \text{A}, \text{B}, \text{C} \rangle$ and $\langle Q_1, Q_2, Q_3 \rangle$, and the correlation are compacted in the following Table 1.

	A	B	C	Q_1	$Q_2(C)$	$Q_3(C)$
1-1	R	T	F	da	ja	da
2-1	R	F	T	da	da	da
3	T	R	F	da	ja	ja
4	F	R	T	da	da	ja
	A	B	C	Q_1	$Q_2(B)$	$Q_3(B)$
1-2	R	T	F	ja	da	da
2-2	R	F	T	ja	ja	da
5	T	F	R	ja	ja	ja
6	F	T	R	ja	da	ja

Table 1: Boolos' puzzle

A finite or infinite sequence of questions is represented by binary trees just like formal proofs of Gentzen's sequent calculus. Let Z be True (T), False (F), or Random (R). Then the following question

$$\text{“B} = Z \to \boxed{A} \to \text{da”} \longrightarrow \boxed{\quad A \quad} \longrightarrow \ ?$$

is denoted simply by $A : \mathrm{B} = Z$. We employ the binary tree below

$$\dfrac{\dfrac{}{A_1 : \mathrm{B}_1 = Z_1}\,ja \qquad \dfrac{}{A_2 : \mathrm{B}_2 = Z_2}\,da}{A : B = Z}\,da|ja$$

to represent that if the answer ? of the question is *da* then the next question is $A_1 : \mathrm{B}_1 = Z_1$ and the answer of A_1 is *ja*, and that if the answer ? is *ja* then the next question is $A_2 : \mathrm{B}_2 = Z_2$ and the answer of A_2 is *da*. We use this tree representation of sequences of questions and answers. At a leaf if once each identity for every god in G_n is determined by T, F, or R, we stop ourselves asking questions and expand this leaf no longer. At every leaf if each god's identity for G_n is determined, then we ask no more questions and we say that the puzzle G_n is solved.

In order to show the correctness of Table 1, we provide the following proof trees, which is verified by the case analysis of $(A = R) \vee \neg(A = R)$:

$$\dfrac{\dfrac{\dfrac{}{C : A = R}\,da|ja \qquad \dfrac{}{C : A = R}\,da|ja}{C : C = T}\,da|ja \qquad \dfrac{\dfrac{}{B : A = R}\,da|ja \qquad \dfrac{}{B : A = R}\,da|ja}{B : B = T}\,da|ja}{A : B = R}\,da|ja$$

2 A general form of the puzzle

We pose a general form of the puzzle. Let $G_n = \{A_1, A_2, \ldots, A_n\}$ $(n \geq 1)$ be the set of gods where A_i $(1 \leq i \leq n)$ is Random (R), True (T), or False (F). Let X be T, F, or R. Then $|G_n|_X$ denotes the number of X in G_n. Suppose that $|G_n|_R < n$. Now the following fundamental puzzle is suggested here.

1. Is it possible to identify the non-Random god in G_n?
2. In particular, is it possible to identify the non-Random god for the case where $|G_2|_R = 1$?

Remarked that Roberts [8] posed the following even harder puzzles.

(1) Suppose the puzzle is as before[2], but now two of the gods are Random, and the third is either True or False. Is it possible to identify the non-Random god, and whether they are True or False, in three questions?

(2) Suppose the puzzle is as before, but one god is Random, and the other two may be either both True, or both False, or one True and one False; is it possible to identify all of the gods in three questions?

The even harder puzzle (1) is now an instance of $|G_3|_R = 2$. First we provide a solution to the puzzle (2) in four questions. Now the solution can be depicted by the following tree starting from $A : B = R$, which is verified by the case analysis on $A \neq R$ or $A = R$:

- Case of da to the first question $A : B = R$:

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{$da|ja$}
\UnaryInfC{$C:B=T$}
\AxiomC{}
\RightLabel{$da|ja$}
\UnaryInfC{$C:A=T$}
\RightLabel{$da|ja$}
\BinaryInfC{$C:A=R$}
\AxiomC{}
\RightLabel{$da|ja$}
\UnaryInfC{$C:B=T$}
\AxiomC{}
\RightLabel{$da|ja$}
\UnaryInfC{$C:A=T$}
\RightLabel{$da|ja$}
\BinaryInfC{$C:A=R$}
\RightLabel{$da|ja$}
\BinaryInfC{$C:C=T$}
\AxiomC{$\cdots$}
\UnaryInfC{$B:B=T$}
\RightLabel{$da|ja$}
\BinaryInfC{$A:B=R$}
\end{prooftree}
$$

- Case of ja to the first question $A : B = R$:

$$
\begin{prooftree}
\AxiomC{$\cdots$}
\UnaryInfC{$C:C=T$}
\AxiomC{}
\RightLabel{$da|ja$}
\UnaryInfC{$B:C=T$}
\AxiomC{}
\RightLabel{$da|ja$}
\UnaryInfC{$B:A=T$}
\RightLabel{$da|ja$}
\BinaryInfC{$B:A=R$}
\AxiomC{}
\RightLabel{$da|ja$}
\UnaryInfC{$B:C=T$}
\AxiomC{}
\RightLabel{$da|ja$}
\UnaryInfC{$B:A=T$}
\RightLabel{$da|ja$}
\BinaryInfC{$B:A=R$}
\RightLabel{$da|ja$}
\BinaryInfC{$B:B=T$}
\RightLabel{$da|ja$}
\BinaryInfC{$A:B=R$}
\end{prooftree}
$$

By the tree, for instance, the sequence of da, da, da, da from $A : B = R$ to $C : B = T$ means that $C = T$, $A = R$, and $B = T$. The sequence of da, ja, ja, ja means that $C = F$, $B = R$, and $A = F$. The sequence ja, da, da, da means that $B = T$, $A = R$, and $C = T$, and ja, ja, ja, ja means that $B = F$, $C = R$, and $A = F$. The result is summarized in the following Table 2.

Next, we show that Roberts' puzzle (1) is not solvable. Before this, we prove that it is impossible to identify the non-Random god for the puzzle G_2 with $|G_2|_R = 1$.

Lemma 1 *$G_2 = \{A, B\}$ with $|G_2|_R = |G_2|_{T/F} = 1$ is not solvable.*

Proof. Toward a contradiction, suppose that there exists a finite tree of questions and answers for G_2 such that every leaf determines the identities of A and B consistently, i.e., either $A = T/F$ and $B = R$, or $A = R$ and $B = T/F$. Consider

[2] Of course, this is the hardest logic puzzle ever [1].

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Q_1	d	d	d	d	d	d	d	d	j	j	j	j	j	j	j	j
Q_2	d	d	d	d	j	j	j	j	d	d	d	d	j	j	j	j
Q_3	d	d	j	j	d	d	j	j	d	d	j	j	d	d	j	j
Q_4	d	j	d	j	d	j	d	j	d	j	d	j	d	j	d	j
A	R	R	T	F	R	R	T	F	R	R	T	F	R	R	T	F
B	T	F	R	R	T	F	R	R	T	T	T	T	F	F	F	F
C	T	T	T	T	F	F	F	F	T	F	R	R	T	F	R	R

Table 2: Roberts' puzzle (2) where d denotes da and j does ja

the case of $A = T/F$ and $B = R$. One leaf of the finite tree says that $A = T/F$ and $B = R$ via da/ja to some question to A. The last question must be asked to non-Random A, since B is Random that answers randomly. For this result, previously we must recognize that A is not Random. In order to recognize $A = T/F$, we have to ask some question Q_X to some $X \in G_2$ that cannot be B, since B is Random. Of course, one can ask B. The answer is, however, random. We have to ask A again, but A's identity had not been determined yet, which leads to an infinite tree. From the contradiction, we cannot have a finite tree that solves G_2 with $|G_2|_R = |G_2|_{T/F} = 1$. □

Lemma 2 *It is not solvable that $G_1 = \{A\}$ where A is either T/F or R.*

Proof. By case analysis where A is either T/F or R, following the similar pattern to lemma 1. □

In the same way, one can give a general answer to Roberts' puzzle (1).

Lemma 3 *It is not solvable for $n \geq 2$ that G_n with $|G_n|_R = (n-1)$ and $|G_n|_{T/F} = 1$.*

The puzzle $G_2 = \{A, B\}$ with $|G_2|_R = |G_2|_T = 1$ is not solvable, however we should remark that there exists an infinite tree such that every finite fragment of the tree determines whether either $A = T, B = R$ or $A = R, B = T$.

Lemma 4 *First the following question "$A = R \to \boxed{A} \to da$" is asked to A:*

$$\text{“}A = R \to \boxed{A} \to da\text{”} \longrightarrow \boxed{\quad A \quad} \longrightarrow da/ja$$

Next, the following question "$B = R \to \boxed{B} \to da$" is asked to B in both cases of da/ja:

$$\text{“}B = R \to \boxed{B} \to da\text{”} \longrightarrow \boxed{\quad B \quad} \longrightarrow da/ja$$

Repeat this process in this order. If we obtain a sequence of $(ja)^i da$ for some odd i, then $A = T, B = R$. If we obtain a sequence of $(ja)^i da$ for some even i, then $A = R, B = T$.

Proof. Consider an infinite tree starting from $A : (A = R)$ together with case analysis on $A = R$ or $A = T$, as follows:

- Case $A = T$, i.e., $B = R$:

$$\dfrac{\dfrac{\dfrac{\cdots}{A:(A=R)}\,ja \qquad \dfrac{\cdots}{A:(A=R)}\,ja}{B:(B=R)}\,da|ja}{A:(A=R)}\,ja$$

- Case $A = R$, i.e., $B = T$:

$$\dfrac{\dfrac{\dfrac{\cdots \quad \cdots}{A:(A=R)}\,da|ja}{B:(B=R)}\,ja \qquad \dfrac{\dfrac{\cdots \quad \cdots}{A:(A=R)}\,da|ja}{B:(B=R)}\,ja}{A:(A=R)}\,da|ja$$

From this analysis, we obtain the following infinite tree which involves an infinite sequence of $ja, ja, ja, \ldots$, and the finite fragments mean either $A = R$ for $(ja)^i da$ with even i, or $A = T$ for $(ja)^i da$ with odd i, if obtained:

$$\dfrac{\dfrac{\dfrac{\dfrac{\vdots}{B:(B=R)}\,da|ja}{A:(A=R)}\,da|ja}{B:(B=R)}\,da|ja}{A:(A=R)}\,da|ja$$

$\square$

We conjecture that it should be unsolvable that $G_4 = \{A, B, C, D\}$ with $|G_4|_T = |G_4|_R = 2$.

Conjecture 1 *$G_4 = \{A, B, C, D\}$ is not solvable where $|G_4|_T = |G_4|_R = 2$.*

We should remark that the method of Lamma 4 can be applied to Conjecture 1 as well.

Lemma 5 *There exists an infinite tree for Conjecture 1 such that the finite fragments determines each identity of G_4.*

Proof. First consider the case of $A = R$. The puzzle $G_3 = \{B, C, D\}$ with $|G_3|_R = 1$ and $|G_3|_T = 2$ is solvable, and for this let Σ be the following tree:

$$\dfrac{D:(C=R) \quad C:(D=R)}{B:(C=R)}\,da|ja$$

Here, da, da means $C = R$; da, ja does $B = R$; ja, da does $D = R$; and ja, ja does $B = R$. Now following the method of Lamma 4, we can construct an infinite tree for Conjecture 1 as follows:

$$\dfrac{\Sigma \qquad \dfrac{\Sigma \qquad \overset{\vdots}{A:(A=R)}}{A:(A=R)}\,da|ja}{A:(A=R)}\,da|ja$$

Observe that we recognize $A = R$ if da is obtained from the first question $A : (A = R)$. Then the finite tree Σ can be adopted to determine every identity of G_4. Otherwise, we obtain ja and then we still have $A = R$ or $A = T$. Hence, we can repeat this process infinitely, so that every identity of G_4 is determined if we obtain a finite sequence of $(ja)^i da$ for some $i \geq 0$. $\square$

We should remark that the proof method of Lemma 5 can be applied to construct infinite trees for the puzzle G_{2n} with $|G_{2n}|_R = n$ and $|G_{2n}|_{T/F} = n$, since $|G_{2n-1}|_R = n - 1$ is solvable from Lemma 7 below.

Finally, we show a solvable one, and the following solution can be applied to more general cases such as $G_5 = \{A, B, C, D, E\}$ with $|G_5|_R = 2$ and $|G_5|_{T/F} = 3$. Moreover, the solution can be naturally extended to G_n with $|G_n|_R < n/2$.

Lemma 6 *$G_5 = \{A, B, C, D, E\}$ with $|G_5|_R = 2$ and $|G_5|_T = 3$ is solvable.*

Proof. By case analysis on $A = R$ or $A \neq R$, and for each case, case analysis on $C = R$ or $C \neq R$, and $B = R$ or $B \neq R$ as well. After the following two questions $A : (B = R)$ and either $C : (D = R)$ or $B : (D = R)$ depending on $da|ja$, one can recognize that $E = T$ or $D = T$.

$$\dfrac{\dfrac{\overline{E:(C=R)}\;da|ja \qquad \overline{D:(C=R)}\;da|ja}{C:(D=R)}\;da|ja \qquad \dfrac{\overline{E:(C=R)}\;da|ja \qquad \overline{D:(C=R)}\;da|ja}{B:(D=R)}\;da|ja}{A:(B=R)}\;da|ja$$

See Table 3 that is obtained from the following trees both of which contain at most five questions such that $Q_1 := (B = R)$, $Q_2 := (D = R)$, $Q_3 := (C = R)$, $Q_4 := (B = R)$, and $Q_5 := (A = R)$.

- Case of da from $A : (B = R)$:

$$\dfrac{\dfrac{\dfrac{\dfrac{E:(A=R)}{E:(B=R)} \quad \dfrac{E:(A=R)}{E:(B=R)}}{E:(C=R)} \quad \dfrac{\dfrac{D:(A=R)}{D:(B=R)} \quad \dfrac{D:(A=R)}{D:(B=R)}}{D:(C=R)}}{C:(D=R)} \quad \dfrac{\dfrac{\cdots}{E:(C=R)} \quad \dfrac{\cdots}{D:(C=R)}}{B:(D=R)}}{A:(B=R)}$$

- Case of ja from $A : (B = R)$:

$$\dfrac{\dfrac{\dfrac{\cdots}{E:(C=R)} \quad \dfrac{\cdots}{D:(C=R)}}{C:(D=R)} \quad \dfrac{\dfrac{\dfrac{E:(A=R)}{E:(B=R)} \quad \dfrac{E:(A=R)}{E:(B=R)}}{E:(C=R)} \quad \dfrac{\dfrac{D:(A=R)}{D:(B=R)} \quad \dfrac{D:(A=R)}{D:(B=R)}}{D:(C=R)}}{B:(D=R)}}{A:(B=R)}$$

$\square$

Lemma 7 *The puzzle G_{2n+1} with $|G_{2n+1}|_R = n$ is solvable for $n \geq 1$.*

Proof. See Table 1 for the case of $n = 1$, and Lemma 6 for $n = 2$. For $n > 2$ we can take exactly the same pattern for the solution such that $A_{2n} = T$ or

	1	2	3	4	5	6	7	8	9	10	11	12
Q_1	d	d	d	d	d	d	d	d	d	d	d	d
Q_2	d	d	d	d	d	d	j	j	j	j	j	j
Q_3	d	d	d	j	j	j	d	d	d	j	j	j
Q_4	d	j	j	d	d	j	d	j	j	d	d	j
Q_5	-	d	j	d	j	-	-	d	j	d	j	-
A	T	R	T	R	T	R	T	R	T	R	T	R
B	R	T	T	R	R	T	R	T	T	R	R	T
C	R	R	R	T	T	T	R	R	R	T	T	T
D	T	T	R	T	R	R	T	T	T	T	T	T
E	T	T	T	T	T	T	T	T	R	T	R	R

	13	14	15	16	17	18	19	20	21	22	23	24
Q_1	j	j	j	j	j	j	j	j	j	j	j	j
Q_2	d	d	d	d	d	d	j	j	j	j	j	j
Q_3	d	d	d	j	j	j	d	d	d	j	j	j
Q_4	d	j	j	d	d	j	d	j	j	d	d	j
Q_5	-	d	j	d	j	-	-	d	j	d	j	-
A	T	R	T	R	T	R	T	R	T	R	T	R
B	R	T	T	R	R	T	R	T	T	R	R	T
C	R	R	R	T	T	T	R	R	R	T	T	T
D	T	T	R	T	R	R	T	T	T	T	T	T
E	T	T	T	T	T	T	T	T	R	T	R	R

Table 3: G_5 with $|G_5|_R = 2$ and $|G_5|_T = 3$ where $-$ means Q_5 is needless

$A_{2n+1} = T$ for $G_{2n+1} = \{A_1, A_2, A_3, \ldots, A_{2n}, A_{2n+1}\}$, as follows:

$$\dfrac{\dfrac{\begin{array}{c}\dfrac{\dfrac{\cdots}{A_{2n+1}:(A_{2n-1}=R)}\quad\dfrac{\cdots}{A_{2n}:(A_{2n-1}=R)}}{A_{2n-1}:(A_{2n}=R)}\\ \vdots\\ A_5:(A_6=R)\end{array}\qquad \dfrac{\vdots}{A_4:(A_6=R)}}{A_3:(A_4=R)}\qquad\dfrac{\dfrac{\vdots}{A_5:(A_6=R)}\quad\dfrac{\vdots}{A_4:(A_6=R)}}{A_2:(A_4=R)}}{A_1:(A_2=R)}$$

□

Proposition 1 *The puzzle G_n with $0 \leq |G_n|_R < \lfloor n/2 \rfloor$ is solvable for $n \geq 1$.*

Proof. The solution follows from Lemma 7. □

References

[1] George Boolos: The Hardest Logic Puzzle Ever, The Harvard Review of Philosophy **6**, pp. 62–65, 1996.

[2] K. Fujita: On formalization of logic puzzles *á la* Smullyan, Kyoto University RIMS Kôkyûroku No. 2193 (Logic, Language, Algebraic system and Related Areas in Computer Science), pp. 18–24, 2021.

[3] 藤田憲悦：数理パズルで楽しく学べる論理学，コロナ社，2022, https://www.coronasha.co.jp/np/isbn/9784339029239/

[4] K. Fujita, T. Kurata: A general form on the logic puzzles of Boolos, Kyoto University RIMS Kôkyûroku No. 2229 (Logic, Algebraic system, Language and Related Areas in Computer Science), pp. 21–29, 2022.

[5] NHK E テレ，ニュー試（2022 年 12 月 30 日放送）：2020 年ケンブリッジ大学哲学部入試問題筆記第 1 問.

[6] Raymond Smullyan: Logical Labyrinths, 2009 by A K Peters, Ltd.

[7] Brian Rabern and Landon Rabern: A simple solution to the hardest logic puzzle ever, ANALYSIS **68.2**, pp 105–112, 2008.

[8] Tim S. Roberts: Some Thoughts About The Hardest Logic Puzzle Ever, Journal of Philosophical Logic **30**, pp. 609–612, 2001.

[9] Jim Henson with George Lucas: Labyrinth, Lucasfilm Ltd. 1986.

Ken-etsu Fujita

Gunma University, JAPAN

E-mail address: fujita@gunma-u.ac.jp

Toshihiko Kurata

Hosei University, JAPAN

E-mail address: kurata@hosei.ac.jp