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# Ant colony system based routing and scheduling for hazardous material transportation

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**Abstract**

This paper presents a new meta-heuristic algorithm using an ant colony system (ACS) for multi-objective optimisation of hazardous material (HAZMAT) transportation. We focus on the vehicle routing problem with time windows (VRPTW) aspect of HAZMAT transportation problem. A VRPTW formulation considering multiple attributes in application to HAZMAT transportation is provided. ACS in the proposed algorithm works in the framework of pareto-optimisation for routing and integrates a labelling algorithm for finding non-dominated paths for path choice purpose. Validity of the algorithm has been tested by applying it to several VRPTW benchmark problems. Results show that the proposed algorithm performs quite satisfactorily to the wide variety of VRPTW problems.

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*Keywords:* Hazardous material; optimisation; vehicle routing problem with time window; ant colony system

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**1. Introduction**

While industries are the primary users of hazardous materials, reliance on HAZMAT has become a fact of life to all living in industrialized society. HAZMATs of various classes are in use in day to day business activity and owing to the nature of these materials, every production, storage and transportation activity related to their use inherits many risks for both society and the environment. HAZMAT transportation is an economic activity of great importance. While the HAZMAT transport sector is far safer than other transport sectors, HAZMAT transport accidents do happen (Erkut et al., 1995) and their consequences in most cases are serious. The relatively high volumes of HAZMAT shipments and the potential adverse conditions that the hazardous materials accidents may lead to are the main problems in HAZMAT transportation.

Most real life engineering problems are characterized by number of conflicting objectives that are often to be minimised simultaneously to get to optimal state. HAZMAT transportation is a commonly faced multi-objective issue in logistical decision making in which a number of parties with often conflicting priorities and viewpoints are involved. For example, the transporting company has priority to minimise the cost of transportation of these

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materials while the local government and the affected group have intentions to minimise the risk associated with transportation process. This multi-objective requirement makes the problem more complicated to be solved and this has been further worsened by public's increasing sensitiveness towards risk.

This is the reason that HAZMAT transportation has gathered the attention of large number of researchers since past few decades. A number of HAZMAT studies considering multi-objective aspect of HAZMAT transportation have been carried out in the past. However, most of them focused on developing multi-objective models to find paths for HAZMAT shipment for a given origin and destination pair and the practical situation in HAZMAT distribution system where a fleet of vehicles have to service a pre-defined set of customers have been inadequately studied.

Our aim here is to contribute to safety in HAZMAT transportation through the application of proper routing and scheduling decisions. A detailed HAZMAT transportation model considering the needs of a multiple number of stakeholders involved in decision making process and a new ACS based meta-heuristic algorithm that is supported by labelling algorithm for solving this routing problem have been presented. The proposed algorithm explicitly maintains the requirement of multi-objective consideration both during path choice and routing processes of HAZMAT shipment with the use of labelling algorithm to obtain all non-dominated paths for the former process and through use of pareto-front concept in ACS for the later. Before proceeding to the application of this algorithm to real HAZMAT problems, it is necessary to check the performance of the proposed model and its solution algorithm. In regard to lack of standard test datasets in HAZMAT transportation, in this paper we present the performance of the algorithm based on the tests carried out with benchmark problems for the VRPTW in general.

## 2. Literature Review

Routing and scheduling of HAZMAT is a subject of considerable interest to the transportation community. A comprehensive survey of past modelling and analysis techniques in the application to routing and scheduling of HAZMAT shipments has been presented in Erkut et al. (2007). With regard to the multiple number of stakeholders involved in the HAZMAT transportation problem, List et al. (1991) emphasised the need of movement towards multi-objective models in HAZMAT routing. A number of works considering multiple attributes in HAZMAT routing were carried out in the past (Abkowitz and Cheng, 1988; Nozick et al., 1997; Chang et al., 2005). However, most of these studies employed extensions of shortest path algorithms, and much attention has been devoted to finding paths for a given origin-destination pair.

The VRPTW aspects of HAZMAT transportation where a fleet of vehicles are used to service HAZMAT demands of a pre-specified numbers of customers, has received considerably less attention in the literatures. So far, Tarantilis and Kiranoudis (2001) and Zografos and Androutsopoulos (2004) are the only two studies that have explicitly considered the VRPTW prospective of the HAZMAT transportation problem. Both of these studies proposed a bi-objective VRPTW model and applied heuristic techniques to find solutions for routing vehicles carrying HAZMAT. While Tarantilis and Kiranoudis (2001) maintained bi-objective modelling for routing using a List Based Threshold Accepting (LBTA) meta-heuristic algorithm, they used a risk based single objective approach for path selection using Dijkstra's algorithm.

Zografos and Androutsopoulos (2004) transformed their bi-objective model to a single objective model using a weighing approach, and performed both routing and path choice based on this single objective transformed model. They employed a insertion based heuristic for optimal routing and Dijkstra's shortest path algorithm for route choice between customers. This model and the heuristic algorithm were used in Zografos and Androutsopoulos (2008) for developing a GIS based decision support system for integrated hazardous materials routing and emergency response decisions. So far, none of the studies till now are able to represent the multi-objective nature of these problems during both the path choice and routing processes.

An effective way to deal with multi-objective optimisation is to use concept of non-dominated paths. However, the necessity of considering non-dominated paths for both these processes complicates the solution process, creating the need for carrying out these processes together as a single step process. In this regard, a single step HAZMAT routing process that uses non-dominated paths based on a labelling algorithm for path choice and an ACS for pareto-optimal routing has been developed and is presented in this paper.

### 3. HAZMAT Vehicle Routing Problem with Time Windows

The HAZMAT Vehicle Routing Problem with Time Windows (HVRPTW) is a typical case of the VRPTW. The VRPTW has a great deal of ongoing research being conducted in the operations research community. Details on the topic including its variants, formulations and solution techniques can be referred from Desrosiers et al. (1995) and Taniguchi et al. (2001). To facilitate prospective readers, attempts have been made to use standard notations used in Taniguchi et al. (2001) during formulation of HVRPTW in this study.

The HVRPTW is defined in a network of nodes and arcs  $(V, A)$  similar to the VRPTW. The vertex set  $V = \{v_1, v_2, v_3, \dots, v_{\hat{V}}\}$  includes the depot vertex, a set of customer vertices  $N = \{n_1, n_2, n_3, \dots, n_{\hat{N}}\}$  and/or some non customer vertices. The arc set  $A = \{a_1, a_2, a_3, \dots, a_{\hat{A}}\}$  includes all possible connections between vertices in  $V$ . The VRPTW in general is formulated as a single objective function. This type of formulation allows it to be solved in two separate steps of path choice and routing. During path choice, a single shortest path for each customer-customer and customer-depot node pair is determined. These pre-determined paths are then used in the routing step to determine the optimal order of customers to be visited by a fleet of vehicles. However, both the path choice and routing in the HVRPTW are multi-objective, creating the necessity to carry out single step routing as discussed previously. Therefore, a set  $P = \{p_1, p_2, p_3, \dots, p_{\hat{P}}\}$  that includes all non-dominated paths between all customer-customer and customer-depot node pairs based on travel time and risk values is defined specifically for the HVRPTW in this study. The HVRPTW is formulated mathematically as

$$\text{Min} \quad Z(X, Y) = [Z_1(X, Y) \quad Z_2(X, Y) \quad Z_3(X, Y)]^T \quad (1)$$

$$\text{Where,} \quad Z_1(X, Y) = \sum_{l=1}^m \delta(x_l, y_l) \quad (2)$$

$$\begin{aligned} Z_2(X, Y) &= \sum_{l=1}^m Z_{rl}(x_l, y_l) \\ &= \sum_{l=1}^m \sum_{i=0}^{\hat{N}_l} (\bar{T}_{n(i), n(i+1)}^{p(i)} + t_{c, n(i+1)} + t_{w, n(i+1)}) \end{aligned} \quad (3)$$

$$\begin{aligned} Z_3(X, Y) &= \sum_{l=1}^m Z_{rl}(x_l, y_l) \\ &= \sum_{l=1}^m \sum_{i=0}^{\hat{N}_l} R_{n(i), n(i+1)}^{p(i)} \end{aligned} \quad (4)$$

Subject to

$$\sum_{n(i) \in x_l} D(n(i)) = W_l(x_l) \leq W_{c,l} \quad (5)$$

$$\sum_{l=1}^m \hat{N}_l = \hat{N} \quad (6)$$

$$t_{l, n(i)} \leq f_{n(i)} \quad (7)$$

## Notations:

$Z(X,Y)$	: three dimensional objective vector
$Z_1(X,Y)$	: total number of vehicles in use
$Z_2(X,Y)$	: total scheduling time of all the vehicles in operation
$Z_3(X,Y)$	: total risk exposure associated with transportation process
$X$	: order of visiting customers for all vehicles $X = \{x_l \mid l = 1, m\}$
$x_l$	: order of visiting customers for vehicle $l$ $x_l = \{n(i) \mid i = 0, \hat{N}_l\}$
$n(i)$	: $i^{\text{th}}$ customer visited by a vehicle; $n(0)$ is depot node
$\hat{N}_l$	: total number of customers visited by vehicle $l$ , $\hat{N}_l + 1$ is 0
$m$	: maximum number of vehicles available
$Y$	: order of visiting paths for all vehicles $Y = \{y_l \mid l = 1, m\}$
$y_l$	: order of using paths for vehicle $l$ $y_l = \{p(i) \mid i = 0, \hat{N}_l\}$
$p(i)$	: $i^{\text{th}}$ path used by a vehicle at customer $n(i)$ while visiting next customer $n(i+1)$
$\delta(x_l, y_l)$	: = 1; if vehicle $l$ is used = 0; otherwise
$Z_{il}(x_l, y_l)$	: total scheduling time incurred by vehicle $l$
$\bar{T}_{n(i)n(i+1)}^{p(i)}$	: average travel time of a vehicle at customer $n(i)$ while visiting next customer $n(i+1)$ using path $p(i)$
$t_{c,n(i)}$	: service time at customer $n(i)$
$t_{w,n(i)}$	: waiting time at customer $n(i)$ = $(e_{n(i)} - t_{l,n(i)})$ ; if $t_{l,n(i)} < e_{n(i)}$ = 0; otherwise
$e_{n(i)}$	: start time window at customer $n(i)$
$f_{n(i)}$	: end time window at customer $n(i)$
$t_{l,n(i)}$	: service start time of vehicle $l$ at customer $n(i)$
$Z_{rl}(x_l, y_l)$	: risk associated with vehicle $l$ during transportation process
$R_{n(i),n(i+1)}^{p(i)}$	: risk associated with a vehicle at customer $n(i)$ during use of path $p(i)$ for visiting customer $n(i+1)$ $= \sum_{v(j)v(j+1) \in p(i)} AR_{v(j)v(j+1)} \cdot EP_{v(j)v(j+1)}$
$AR_{v(j)v(j+1)}$	: Probability of HAZMAT accident on arc connecting vertices $v(j)$ and $v(j+1)$
$EP_{v(j)v(j+1)}$	: Exposure population on arc connecting vertices $v(j)$ and $v(j+1)$
$D_{n(i)}$	: demand at customer $n(i)$
$W_l(x_l)$	: load of vehicle $l$
$W_{c,l}$	: capacity of vehicle $l$

Equation (1) represents the objective function of the model that minimises the total number of vehicles in use, the total scheduling time of all the vehicles in operation and the total risk exposure associated with transportation

process. Besides traditional decision variable  $X$ , this objective function also depends upon another decision variable  $Y$  for proceeding path choice process. Equations (2) to (4) show the details in the calculation of each objective. Hard time window constraints have been incorporated during calculation of second objective value of scheduling time. Risk calculations are based on expected consequence definition of risk also referred as traditional risk in the model developed by Erkut and Ingolfsson (2004).

In reference to the model, risk associated with a path due to an undesirable HAZMAT accident is a measure of probability of occurrence of the event and its consequence. Though a number of consequences in relation to a HAZMAT accident are possible, safety for human life counts for top priority. Thus exposure population for each link is used as a consequence of HAZMAT incident during risk formulation in Equation (4). The exposure population for a link is the number of people lying within  $\lambda$  distance from the link segment. The distance  $\lambda$  is dependent upon the HAZMAT class being transported and has been defined with the assumption that all persons within this distance from the accident spot are subjected to the same consequence of life loss while the consequences outside this distance have been ignored. Detail on this threshold distance  $\lambda$  is available in Batta and Chiu (1988). The total load in any vehicle cannot exceed the capacity of the vehicle. Equation (5) represents this capacity constraint for all vehicles. Equation (6) is the customer number constraint to the model that enforces the total number of customers visited by all vehicles to be equal to the total customers to be serviced. The late time window constraint for the model is represented by Equation (7) in the formulation.

#### 4. Ant Colony System for HVRPTW

While a number of exact, heuristics and meta-heuristics algorithms already exist in the literature for solving VRPTW in general, solution of the HVRPTW requires an algorithm that can effectively identify a set of non-dominated solutions. Non-dominated solutions, often referred as pareto-optimal solutions are represented by  $(X^*, Y^*)$  when there is not any  $(X, Y)$  which satisfies  $Z_i(X, Y) \leq Z_i(X^*, Y^*)$  at  $i = 1, \dots, k$  and  $Z_i(X, Y) < Z_i(X^*, Y^*)$  at arbitrary  $j$ . A few algorithms in the past applied the concept of pareto-optimisation for solving the multi-objective VRPTW, however; the algorithms were developed with a key focus on optimal routing and the path choice in almost all cases were carried out independently as a single objective process. In this section, we present a new meta-heuristic solution algorithm for solving the HVRPTW. Figure 1 presents the flowchart of the algorithm.

The proposed algorithm derives its basic structure from an Ant Colony System (ACS) for multi-objective VRPTW and uses a labelling algorithm to find set of non-dominated paths for path choice purpose. Ant Colony Optimisation (ACO) is a meta-heuristic approach inspired by the foraging behaviour of real ant colonies. Dorigo and Stutzle (2004) provide a complete explanation on the topic. Bullnheimer et al. (1998) for the first time used an Ant System (AS), a variant of ACO for solving vehicle routing problems. Later, Gambardella et al. (1999) and Baran et al. (2003) presented the MACS-VRPTW and the MOACS-VRPTW respectively for solving the multi-objective VRPTW. A good convergence to pareto-front surface was observed with the MOACS-VRPTW approach. Therefore, the ACS in this study derives its main idea from this approach with regard to the desirability of using pareto-optimal concept while dealing with multiple objectives in the HVRPTW.

##### 4.1. Pheromone trails initialization

To each path in set  $P$ , an initial pheromone value  $\tau_0$  evaluated using Equation (8) is assigned.

$$\tau_0 = \frac{1}{(|N|)_{nn} * (Z_2)_{nn} * (Z_3)_{nn}} \quad (8)$$

Here,  $(|N|)_{nn}$ ,  $(Z_2)_{nn}$  and  $(Z_3)_{nn}$  are the average number of nodes (total number of customers and average number of vehicles), the total scheduling time and the total risk value of the initial solution created using the nearest neighbourhood ( $nn$ ) heuristic. This solution is then initialized as the first member of pareto-optimal set  $S$ .

#### 4.2. Solution construction

Ants in the proposed algorithm construct solution selecting paths from  $P$  and thereby inserting the corresponding customer nodes. The labelling algorithm is carried out at all customer nodes and the depot node to determine the set  $P$ . Basic knowledge on labelling algorithm is described in detail in Ahuja et al. (1993). The labelling algorithm used here is based on a template labelling algorithm proposed by Irnish and Villeneuve et al. (2003) for the shortest path problem with resource constraints.

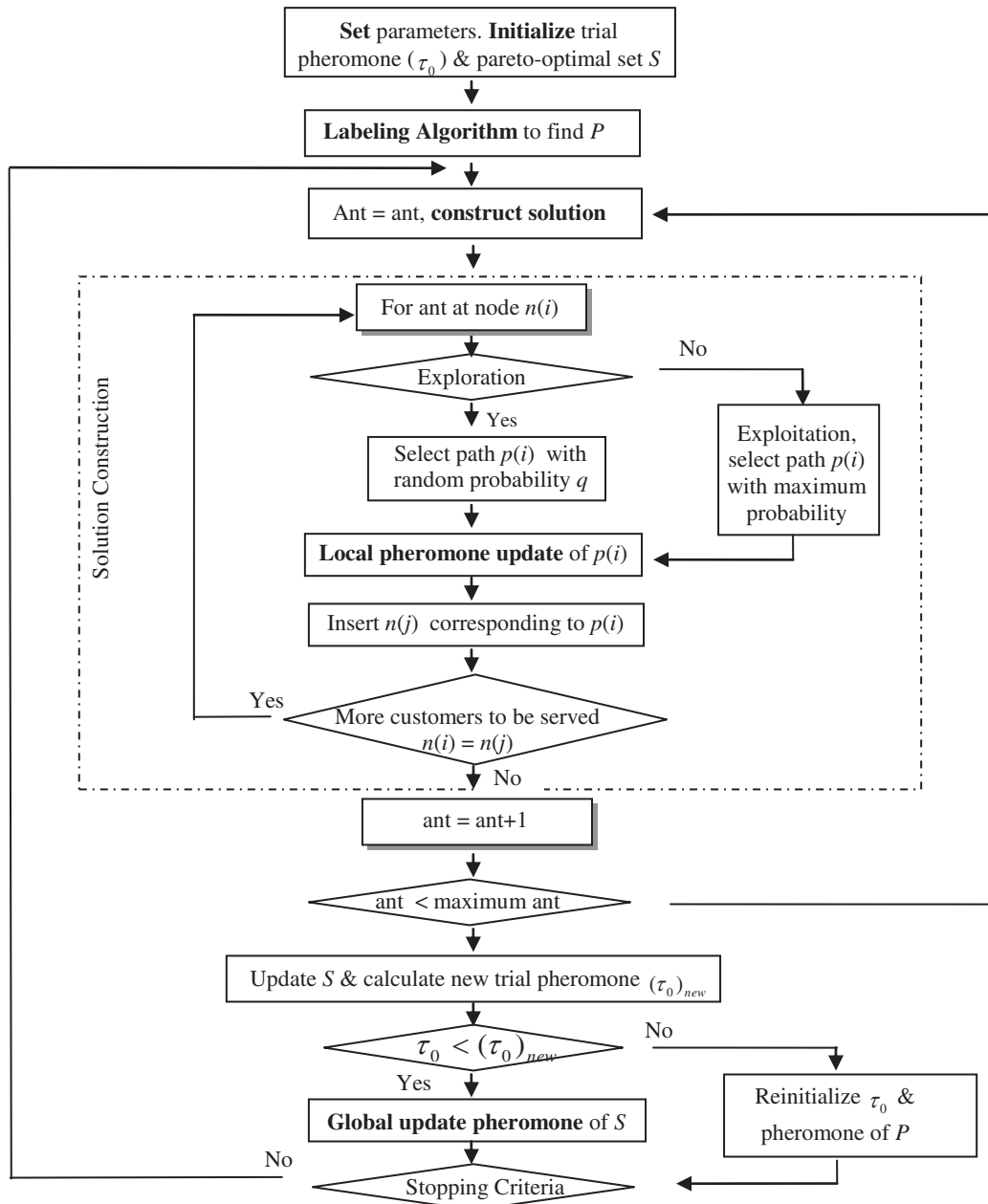


Figure 1 Flowchart of proposed ACS-based meta-heuristic algorithms

Each ant at customer node  $n$  ( $i$ ) selects a path  $p$  ( $i$ ) among  $P'$  that is the set of all non-dominated paths from this customer node to all feasible sets of customer nodes ( $N'$ ) based on the pseudorandom proportional rule. It should be noted that this set  $P'$  is the subset of the set  $P$ . Selection of path  $p$  ( $i$ ) and thereby the insertion of corresponding customer node  $n$  ( $j$ ) based on the expression presented in Equation (9), if  $q \leq q_0$ .  $q$  is a random number such that  $0 \leq q \leq 1$ , and  $q_0$  is a parameter that defines relative importance of exploration and exploitation.

$$\max_{p(i) \in P'} (\tau_{p(i)} \cdot [\eta_{p(i)}]^{\theta^n \beta} \cdot [\nu_{p(i)}]^{\theta^v \mu}) \quad (9)$$

The choice otherwise is made randomly based on a probability value presented in Equation (10).

$$\Pr(p(i)) = \frac{\tau_{p(i)} \cdot [\eta_{p(i)}]^{\theta^n \beta} \cdot [\nu_{p(i)}]^{\theta^v \mu}}{\sum_{p(i) \in P'} \tau_{p(i)} \cdot [\eta_{p(i)}]^{\theta^n \beta} \cdot [\nu_{p(i)}]^{\theta^v \mu}} \quad (10)$$

$\tau_{p(i)}$ ,  $\eta_{p(i)}$ ,  $\nu_{p(i)}$  are the pheromone and heuristic values relating to the scheduling time and risk value of path  $p$  ( $i$ ), respectively.  $\beta$  and  $\mu$  are the parameters that define the relative influence of the time and risk objectives.  $\theta^n$  and  $\theta^v$  are the ant specific weights/preferences for normalizing time and risk objectives respectively.  $\Pr(p(i))$  is the probability of path  $p$  ( $i$ ) being chosen. The calculation of the time-related heuristic value is similar to the one for delivery time in the MOACS-VRPTW that depends on the waiting time and time window at customer nodes. The risk-related heuristic value depends on the risk value associated with path  $p$  ( $i$ ) that connects customer node  $n$  ( $i$ ) to  $n$  ( $j$ ), and is evaluated as shown in equation (11).

$$\nu_{p(i)} = \frac{1}{R_{n(i),n(j)}^{p(i)}} \quad (11)$$

#### 4.3. Local search

While the local search procedures improve the quality of solutions, they are time consuming processes. Unlike in traditional ACS where local search is carried out for each ant solution, we employ an insertion based local search only to those solutions belonging to the pareto-optimal set  $S$ , which is updated at each iteration. Details on local search procedures including their development, analysis and application can be found in Hoos and Stutzle (2004). The insertion local search used in this study utilizes insertion neighbourhoods of a typical solution. All nodes in solution  $\psi$  that belongs to set  $S$  are given a chance to be inserted into the same vehicle route or into the route of other vehicles without violating feasibility requirements. The newly obtained solutions are then checked for improved objective function values. Figure 2 shows two typical insertion neighbours, IN-1 and IN-2 obtained inserting a node of solution  $\psi$  to position in the same vehicle route and in another vehicle route respectively.

#### 4.4. Pheromone update

An ACS employs two typical pheromone update procedures that are the local and global update of pheromone. To each path used by ant for constructing solution, local pheromone update is carried out as given in Equation (12) where  $\rho$  is the evaporation coefficient which powers exploration process by evaporating trail pheromone values for these used paths.

$$\tau_{p(i)}^{new} = (1 - \rho) \tau_{p(i)}^{old} + \rho \tau_0 \quad (12)$$



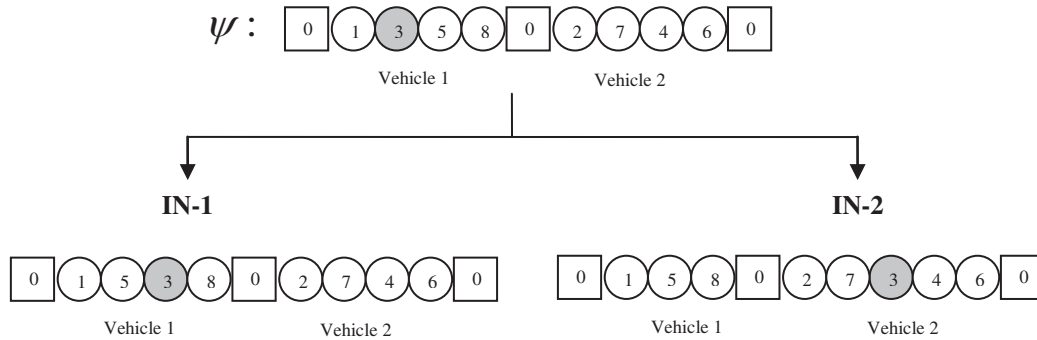


Figure 2 Insertion neighbours

Trail pheromone value based on the newly updated pareto-optimal set  $S$  is calculated in each iteration. If set  $S$  with better pheromone value than the initialized trail pheromone value is obtained, pheromone value equal to this new trail pheromone value is assigned to all paths in set  $P$ . Global update of pheromone is otherwise carried out to each path belonging to set  $S$  as shown in Equation (13) before proceeding to the next iteration.

$$\tau_{p(i)}^{new} = (1 - \rho)\tau_{p(i)}^{old} + \rho / [(Z_2)_{\psi} \cdot (Z_3)_{\psi}] \quad \text{where } p(i) \in \psi \quad (13)$$

#### 4.5. Overall procedure

Finally, the overall steps in the proposed ACS-based meta-heuristic algorithm for the HVRPTW can be summarized in the following steps:

- Step 1: Set parameters. Initialize trail pheromone value and pareto-optimal set  $S$  based on initial solution using the nearest neighbourhood search. Iteration = 1, initialize pheromone values for all paths based on the trail pheromone value.
- Step 2: Perform the labelling algorithm at each customer node to find set  $P$ .
- Step 3: For ant = 1 to number of ants, construct solution.
  - Construct solution:
    - A: depotno =1,  $i=0$  and place ant at depot node,  $n(i)$  = depot node.
    - B: Identify feasible set of customer nodes from  $n(i)$  that is  $N'$  and set of non-dominated paths to these nodes  $P'$ . If  $N'$  is empty, add all non-dominated paths from  $n(i)$  to depot node to  $P'$ .
    - C: Select path  $p(i)$  from  $P'$ .
      - If exploitation, choose path with maximum heuristic and pheromone product value.
      - Else, choose randomly using probability of selection.
    - D: Local update of pheromone value of path  $p(i)$ .
    - E: If customer node corresponding to selected path  $p(i)$  is depot node, depotno = depotno +1,  $i = 0$  and  $n(i)$  = depot node. Else,  $i = i + 1$  and  $n(i)$  = customer node corresponding to selected  $p(i)$ .
    - F: If more customers to be served go to step B.
    - G: Total vehicles = depotno-1
- Step 4: If ant < number of ants, ant = ant +1 and go to Step 3.
- Step 5: Update  $S$  based on dominance rule.
- Step 6: Apply Insertion local search to each solution in  $S$ .
- Step 7: Find new trail pheromone value based on average objective values of current  $S$ .
- Step 8: If previous trail pheromone value < new trail pheromone,



trail pheromone = new trail pheromone and initialize pheromone value of all paths based on this trail pheromone value.

Else, carry out global update of pheromone for all paths of solutions in  $S$ .

Step 9: If iteration < maximum iterations, iteration = iteration + 1 and go to Step 3.

## 5. Solomon Benchmark Instances

Applying the heuristic algorithm to a set of benchmark problems and comparing the deviation of the final outcomes from the best known solutions has been a common practice for testing new algorithms. Due to the lack of standard set of problems in the multi-objective HVRPTW, the proposed ACS-based algorithm has been tested in application to Solomon benchmark instances (Solomon, 1987) for the VRPTW. Numerical tests have been performed on R, C and RC type of benchmark instances. While customers in C type of problems are clustered, those in R type of problems are randomly distributed, and those in RC type are of mixed type where customers are both clustered and randomly distributed. Customer locations in all problems of each type are same but the problems differ in time windows of the customers. All instances provided contain 100 numbers of customers, a central depot, capacity constraint and time window constraints. However, smaller instances can be generated taking the first 25 or 50 customers. In this study, two problems from each type are selected randomly and are tested for cases with 25, 50 and 100 customers respectively.

## 6. Results and Discussions

The proposed algorithm has been executed using Borland C compiler in Core 2 Duo desktop PC of 2.67GHz with 2GB RAM. Parameters values of  $m=10$ ,  $\theta^n = 1$ ,  $\beta = 1$ ,  $\rho = 0.1$ ,  $q_0 = 0.9$  that are the same as in the MOACS-VRPTW and the MACS-VRPTW were used during execution. Appropriate adjustments have been carried out to the algorithm for solving the VRPTW instead of the HVRPTW. For instance, the risk term was removed and instead of total scheduling time, overall distance was calculated. Thus, the objective function has been reduced to a bi-objective function of minimising the total number of vehicles in use and the total distance. Although this algorithm has been particularly designed to deal with multiple objectives in the HVRPTW, it is equally applicable to a single objective as well as a bi-objective VRPTW.

Two problems were randomly selected from each type (C, R and RC), and were run for 10,000 iterations. The results presented are best among 20 test trials. For problems with 25 customers, the results observed were same for all test runs. Table 1 presents a comparison of solutions obtained from the proposed ACS-based algorithm and the best known solutions from exact approaches for C problems for cases with 25, 50 and 100 customers. It should be clear to readers that these exact approaches use a single objective approach for minimising the total distance travelled and the results have been used just as a reference for comparison. Table 2 and Table 3 present similar comparisons for the case of R and RC type of problems respectively. As observable from Table 1, the proposed algorithm is found to yield solutions that are close to best known exact solutions with negligible deviations for all customer numbers cases in C type of problems. Though the results for R and RC type problems with 100 customers are not as good as for C type, the solutions obtained are satisfactory with deviations ranging from 7% to 11% that can be considered practicable for such large size problems. Moreover, the deviations for these R and RC problems with 25 customers are almost negligible similar to that in C problems. Even for 50 customer cases, the observed deviations are very small with a maximum value of 3.3%.

An example of the trend in travelled distance reduction over generations has been presented in Figure 3 for RC107 problem with 50 customers. Only the distance values in relation with solutions using six vehicles are shown. Distance improvements with more vehicles have also been observed in some generations. However to maintain consistency of the graph, those results have been excluded. As shown in Figure 3, the solution using six vehicles which is also the observed optimal vehicle number for this problem was first observed at 183rd iteration for this particular test run. Gradual reductions in total distance travelled were observed up to 1292nd iteration. Most test runs for problems with 25 and 50 customers were found to yield optimal solution before 5000th iteration. Thus slight saving in computational time can be achieved by reducing maximum number of iterations for such smaller problems. However, we maintained a value of 10000 as the maximum number of iterations in this study for applicability to all problem sizes.

Table 1 Comparison of proposed ACS based results with exact results for C problems

Test Problem	Number of Customers	Number of Vehicles		Distance		% Discrepancy
		Proposed ACS	Best Known	Proposed ACS	Best Known	
C101	25	3	3	191.81	191.3	0.27 (Dist.)
	50	5	5	363.25	362.4	0.23 (Dist.)
	100	10	10	828.94	827.3	0.2 (Dist.)
C107	25	3	3	191.81	191.3	0.27 (Dist.)
	50	5	5	363.25	362.4	0.23 (Dist.)
	100	10	10	828.94	827.3	0.2 (Dist.)

Table 2 Comparison of proposed ACS based results with exact results for R problems

Test Problem	Number of Customers	Number of Vehicles		Distance		% Discrepancy
		Proposed ACS	Best Known	Proposed ACS	Best Known	
R101	25	8	8	618.33	617.1	0.2 (Dist.)
	50	12	12	1056.72	1044.0	1.21 (Dist.)
	100	20	20	1757.49	1637.7	7.31 (Dist.)
R110	25	4	4	446.87	444.1	0.62 (Dist.)
	50	7	7	719.96	697.0	3.29 (Dist.)
	100	12	12	1145.35	1068.0	7.24 (Dist.)

Table 3 Comparison of proposed ACS based results with exact results for RC problems

Test Problem	Number of Customers	Number of Vehicles		Distance		% Discrepancy
		Proposed ACS	Best Known	Proposed ACS	Best Known	
RC102	25	3	3	352.74	351.8	0.27 (Dist.)
	50	7	7	843.47	822.5	2.55 (Dist.)
	100	14	14	1565.45	1457.4	7.41 (Dist.)
RC107	25	3	3	298.95	298.3	0.22 (Dist.)
	50	6	6	651.78	642.7	1.41 (Dist.)
	100	12	12	1336.53	1207.8	10.66 (Dist.)

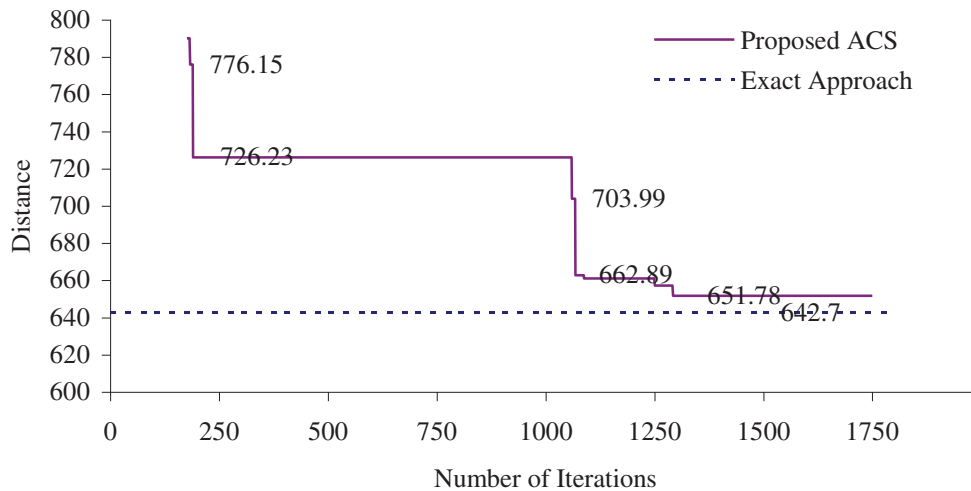


Figure 3 Distance reduction over iterations in RC107-50 for solutions using 6 vehicles

## 7. Conclusion and Future work

Our objective is to contribute to safe transportation of HAZMAT through proper route planning while satisfying the suppliers' need for cost minimization. In contrast to widely available literature in HAZMAT routing that is mainly focused on finding non-dominated paths for shipment of HAZMAT for a given origin and destination pair, we focus on the VRPTW aspect of HAZMAT transportation problem. The proposed ACS-based meta-heuristic algorithm is typically designed to solve the HVRPTW and uses concept of pareto-optimisation to deal with multiple objectives both during the routing and path choice processes.

This paper presents the validation process of the proposed algorithm. Due to lack of benchmark data for the HVRPTW, performance of the proposed algorithm has been tested in the applications to Solomon's benchmark instances for VRPTW in general. Test results show that for the clustered customers' problem, the proposed algorithm outperforms that for even larger problem instances. The results obtained are as competitive as those obtained with exact approaches. Though the results obtained for larger problem instances for randomly scattered and mixed clustered customers problems are not as good as those obtained with exact approaches, the results are found satisfactory for smaller size problems. The overall performance of the algorithm is found quiet satisfactory to the vast majority of problems. Application of the algorithm to a practical size network in the HAZMAT case is the next aim of the study. In regard to the large increment in problem size due to our attempt to carry out path choice and routing together, the stochastic and time varying nature of HAZMAT vehicle routing problem has not been considered in this study. An extension of the work incorporating such natures of HAZMAT transportation problem is the next aspect of this study.

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