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SPRCIAL ISSUE PAPER

# On the relation between urban road network and distribution centre location strategy of Franchise retail firms

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## Abstract
Franchise retailers such as supermarkets and convenience stores have achieved greater efficiency in delivering products to their chain stores by locating distribution centres (DCs) within or near cities. In this study, we propose that the reduction of product transportation time due to the construction of intra-urban roads facilitates the location of distribution centres. To this end, we model the inventory management behaviour of franchise retailers and show that distribution centres have two functions: (1) consolidating the inventory holding risk faced by individual chain stores and (2) achieving larger lot sizes when purchasing goods from outside the city. We then analyse the effects of facilitating logistics by improving intra-city roads on reorganizing the location patterns of chain stores and distribution centres in the city and on improving the convenience of shopping for consumers.

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## 1 Introduction

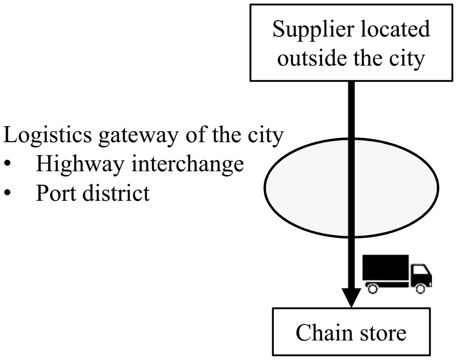
The location of stores that handle food and daily necessities is directly related to the convenience of consumers' lives. In recent years, consumers have been diversifying their product procurement methods through various distribution channels, such as online shopping and home delivery. In this study, we focus on consumers' procurement of goods through stores. If no store is located near a consumer's residence, the consumer is forced to go to a distant store for shopping every time he/she wants to buy something. If a new store were to be located in an area with low shopping accessibility, nearby residents would benefit from a reduction in shopping transportation costs (gasoline and consumer travel time). The location strategy of many small supermarket and convenience store chains in modern cities is to increase sales and profits by improving convenience for consumers.

To locate a large number of chain stores in a city, it is effective to locate a DC at each chain store, which serves as a base for distributing products. Chain stores hold inventories of products. Because demand for products fluctuates on a daily basis, holding inventory always entails the risk of unsold products. It is difficult for a small chain store to bear such a risk. Therefore, the franchise retailer that controls the chain stores locates a DC in or near the city and provides this DC with a warehouse function to hold a large amount of inventory. Because goods are frequently delivered from the DC to the chain stores, the chain stores can operate with only the minimum necessary inventory. In this respect, the logistics centre can be regarded as a base for chain stores to consolidate and assume the risk of holding inventory.

Many modern franchise retailers, such as supermarkets and convenience stores, have adopted a strategy of locating DCs within cities and distributing products to their chain stores. The continuous reduction of logistics costs due to the development of intra-city road networks might contribute to this trend. If the intra-city road network is not developed and the transportation time between the DC and the chain stores is long, even if the chain stores order additional products at short notice, they might run out of stock before the products are delivered to the chain stores. Therefore, it is reasonable for chain stores to increase their stock of merchandise to sell merchandise stably. In a situation where all chain stores have sufficient inventory stock, it would be efficient for chain stores to purchase goods directly from suppliers without going through DCs, as shown in Fig. 1. However, if the intra-city logistics time is sufficiently short, it will be efficient to locate a DC that serves as a logistics hub for the city, as shown in Fig. 2.

Theoretical studies on the location of commercial facilities have been accumulated in the field of urban economics since the pioneering studies of Hotelling (1929), Christaller (1933) and Losch (1940). In Hotelling's theoretical model, the number of stores is given exogenously; thus, analysing the number of stores located in a space is not possible. The central place theory of Christaller (1933) and Losch (1940) proposes a normative theory for the number of commercial cities that should exist in a space. The central place theory assumes a situation in which economies of scale operate in the supply of goods to consumers and

Fig. 1 When adopting direct purchasing



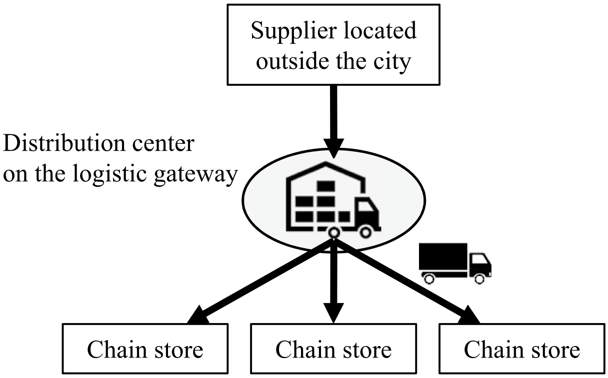




Fig. 2 To locate a distribution centre within a city

clarifies that the cost of society as a whole can be minimized by locating commercial cities responsible for the supply of goods at regular intervals in space. The efficient spatial distance between commercial cities is determined by the trade-off between economies of scale and consumer shopping costs: the more the former prevails, the wider the efficient distance. The ideas of central place theory are carried over to the "new economic geography" model, which analyses urban and industrial agglomeration (Fujita et al. 1999). New economic geography models economies of scale that work in increasing the diversity of goods and explains the spatial agglomeration of economic activities by the trade-off between economies of scale and the cost of transporting goods. However, new economic geography deals with macro agglomeration phenomena such as cities and industries and does not analyse the micro-location of commercial facilities.

Theoretical studies on the phenomenon of commercial agglomeration include those of Eaton and Lipsey (1982) and Wolinsky (1983). In the former study, a situation is assumed in which consumers can save money by purchasing two different products at a single chain store. In this case, two types of stores are distributed in the space: stores that sell two types of products and stores that sell only one type of product. In the latter study, consumers lack sufficient information about the characteristics of the products sold in each store and visit multiple stores to compare the characteristics of the products before deciding which product to purchase. In this case, consumers are shown to first go to distant commercial clusters rather than to stores that exist alone in a neighbourhood. As a result, the inducement to locate in commercial clusters works for stores. In these studies, too, the economies of scale that act on stores are expressed in the form of fixed costs.

In the context with intercity network, the close and frequent intercity connections can create the external productivity benefits analogous to that of agglomeration economies. Klaesson and Johansson (2010) suggest the complementarities between agglomerations and networks in providing benefits that arise from standard market processes, but which are external to individual participants. In the spatial context, networks play a role in facilitating exchange both within and between regional agglomerations. Higher network density makes the contact easier and more frequent or routinized, which in turn enhances network intensity by increasing the frequency of interaction (Johansson and Quigley 2004). Andersson and Johansson (2012) propose a mode where fixed costs give rise to economies of scope. With many variety triplets, a firm can exploit economies of scope by employing the same export link for several different codes or by selling an already developed product to many destinations.

Theoretical studies on the location of commercial facilities commonly analyse the location of stores in terms of the relationship between consumers' shopping costs and the economies of scale at work in stores. However, no theoretical study has analysed the location of commercial facilities by focusing on the road network and logistics used to purchase goods. However, numerous studies have been conducted in the fields of traffic engineering and operations research (OR) on the relationship between urban road networks and logistics. A representative research area is the vehicle routing problem (VRP), which optimizes the number of trucks and truck routes for intra-city distribution services (Eksioglu et al. 2009). Although this model has no direct relationship with the location of DCs, it can evaluate the influence of the shape of the intra-city road network on truck traffic flow and distribution costs.

Another representative field for analysing the relationship between urban road networks and logistics is the facility location model, which optimizes the location of logistics centres and other facilities on the basis of geographical conditions such as where consumers live (Melo et al. 2009). This model analyses the relationship between the road network and the location of facilities within a city and shares the same problem with the present study. However, this model supports the design of a supply chain that minimizes costs under a given demand and cannot analyse the behaviour and benefits on the consumer side. In addition, for the function of DCs, emphasis has been placed on their function as relay points from large trucks that handle mass transportation to small trucks that handle small-lot deliveries.

The supply chain network equilibrium (SCNE) model developed by Nagumey et al. (2002) is a model that analyses the relationship between logistics costs and supply chains from a macro perspective. This model assumes a supply chain consisting of a manufacturer, a retailer and a consumer market and seeks equilibrium among the volume of commodity transactions, commodity prices and network structure in the supply chain under the assumption that each actor maximizes its profit and utility.

The present study analyses the location strategies of franchise retailers for in-city DCs from the perspective of inventory management. To increase customer satisfaction, chain stores need to ensure that products are not out of stock when consumers visit their stores. However, because the demand for products fluctuates daily and higher-than-average demand can occur, chain stores must hold inventory in case demand increases. This inventory is called safety stock. Safety stock is a heavy burden for chain stores that deal in products whose value is quickly depleted because they must dispose of unsold products. Examples of products with fast product value depletion include food products, seasonal clothing, fast fashion and electronic devices that tend to fall out of production. Efficient inventory management is an important management strategy issue for retailers that handle such products. For example, food and beverages are the main products of supermarkets and convenience stores. The above discussion also applies to these familiar chain stores.

In the present study, we model retailers' decision-making regarding the location of a DC in a city. For this purpose, we apply inventory management models used in the field of OR. The most widely known basic inventory management models are the economic order quantity (EOQ) model and the safety stock model (Chopra and Meindl 2012). In the transportation research field, inventory management models have been applied in studies that evaluate the quality (speed and reliability) of logistics lead time, such as the studies of Baumol and Vinod (1970) and Blauwens et al. (2006); de Jong and Ben-Akiva (2007) have applied inventory management models to cargo route selection.

One important contribution of the safety stock model is that it reveals that the total amount of safety stock required can be reduced by sharing safety stock among multiple chains (Chopra and Meindl 2012). For example, excess safety stock from a chain with fewer-than-normal customers can be replenished to a chain with more-than-normal customers. This replenishment would equalize the overall inventory holding risk and reduce the total amount of safety stock. Another way to share safety stock among multiple chain stores is to consolidate the inventory holding risk of chain stores located downstream by placing a DC upstream of the supply chain. However, this method requires a transportation environment that allows chain stores to replenish products from the DC as and when needed, as mentioned earlier. That is, a short transportation time between the DC and chain stores is a requirement for forming a supply chain with the DC at its core.

The novelty of the present study is that it analyses the relationship between transportation time on the road and the location of logistics centres in a city from the perspective of consolidating inventory holding risk upstream in the supply chain. This analysis is only possible with inventory management models in OR, which explicitly expresses the relation between inventory management and lead times of products. Although there are numerous studies on inventory management models in the literature of OR, they do not use the models to analyse how the reduction in transportation time on the road affects the location of logistics centres as well as the coordination of inventory management between upstream and downstream of the supply chain. We perform such analysis by formulating an economic model that incorporates an inventory management model.

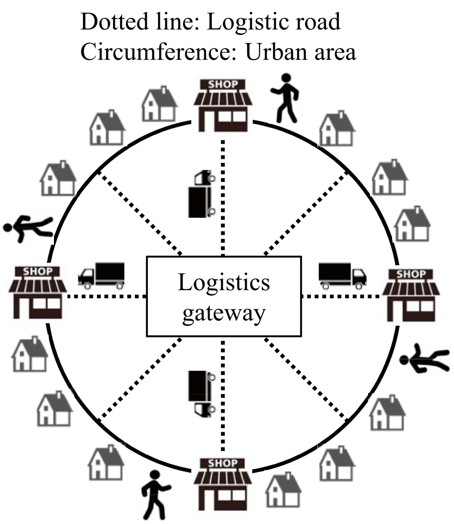
As discussed above, the availability of intra-city roads and the cost of intra-city logistics can influence franchise retailers' strategies for locating intra-city DCs. The location of DCs is then expected to increase the number of chain store locations by reducing the inventory holding costs of chain stores, as well as by increasing the number of shopping trips made by consumers to the nearest chain. On the basis of this argument, we formulate a theoretical model in which a franchise retailer that controls multiple chain stores in a city decides the location of a DC in the city from the viewpoint of inventory holding risk. We then analyse the effects of facilitating logistics by improving intra-city roads on reorganizing the location patterns of intra-city DCs and chain stores and on improving the shopping convenience for consumers. The remaining of the study is structured as follows: In the following section, we formulate the model. In Sect. 3, we analyse the model using comparative statics and discuss the policy implications.

## 2 Model

### 2.1 Assumptions

This study proposes a simple theoretical model to improve the prospects of the discussion. Consider a circumscribed city as shown in Fig. 3. At the centre of the city is a logistics gateway district (e.g. a highway interchange and a port district). The logistics gateway district is connected to the circumference of the city by a number of logistics roads (dashed lines in the figure) extending in a radial pattern. Traffic congestion is not assumed to occur. The circumference is an urban area

**Fig. 3** Assumed urban structure



inhabited by consumers with an equal population density of $e$. The total length of the circumference is denoted by $R$.

Within a city there is a monopolistic firm that operates a chain of stores that sell everyday goods. The firm can locate its chain stores at any location on the circumference of the city. The firm determines the number of chain stores $n$ and the selling price $p$ of the commodity so as to maximize its profit. Because the population density around the circumference is homogeneous, the firm will locate chain stores with equal spacing $R/n$. Figure 3 shows the case when $n = 2$. Consumers travel in a circle to the nearest chain store, incurring shopping transportation costs proportional to the distance between their home and the chain store. The trade area served by one chain store is $\pm R/(2n)$ centred on the location of the chain store.

Also included in the decision-making process of the firm is whether to locate a DC in a logistics gateway district. If the DC is not located within such a district, individual chain stores purchase goods directly from outside the city. If a DC is located within such a district, the DC purchases goods from outside the city and holds inventory and the chain stores purchase goods from the DC. In either case, when the chain stores purchase goods, the goods are transported from suppliers located outside the city to each chain store via a logistics gateway district and then via a dedicated logistics road.

## 2.2 Consumer shopping behaviour and consumer surplus

### 2.2.1 Demand for shopping

The emergence of consumer shopping demand is random, with demand for each product occurring independently according to a Poisson process. Each time a demand arises, the consumer goes to a nearby chain store to purchase one of the commodities because the commodity deteriorates in value quickly or requires space for storage. The arrival rate $\lambda$ of the Poisson process is considered to depend on the price of the commodity and the cost of shopping, and this relationship is represented by a linear demand function:

$$\lambda(l) = \bar{\lambda} - a(p + \tau l) \tag{1}$$

where $\lambda(l)$ is the number of visits to a chain store per unit time for consumers living a distance $l$ to the nearest chain store (hereafter referred to as the arrival rate), $\bar{\lambda}$ is the upper bound on one consumer's demand for the product, $a$ is an exogenous positive constant that represents the sensitivity of the demand for the product to the price and cost of shopping, $p$ is the selling price of the commodity set by the firm and $\tau$ is a constant that represents the consumer's round-trip shopping transportation cost per unit distance. The term $p + \tau l$ denotes the total cost required for a consumer to purchase one commodity. Depending on this total cost, consumers purchase goods less frequently. This Poisson process expresses the risk of fluctuations in the demand for the commodity.

### 2.2.2 Consumer surplus

We define consumer surplus using the demand function in Eq. (1). First, the consumer surplus $cs(l)$ of one consumer living at a point $l$ away from the nearest chain store is

$$\begin{aligned} cs(l) &= \int_{p+\tau l}^{\frac{\bar{\lambda}}{a}} (\bar{\lambda} - ax)dx \\ &= \frac{1}{2}\left[\frac{\bar{\lambda}^2}{a} - 2\bar{\lambda}(p+\tau l) + a(p+\tau l)^2\right] \end{aligned} \tag{2}$$

Thus, the total consumer surplus $CS$ for the entire city is

$$\begin{aligned} CS &= n \cdot 2\int_0^{\frac{R}{2n}} e \cdot cs(l)\mathrm{d}l \\ &= eR\left[\frac{\bar{\lambda}^2}{2a} - \bar{\lambda}\left\{p + \frac{\tau R}{4n}\right\} + a\left\{\frac{p^2}{2} + \frac{p\tau R}{4n} + \frac{1}{24}\left(\frac{\tau R}{n}\right)^2\right\}\right] \end{aligned} \tag{3}$$

The integral of Eq. (3) represents the total consumer surplus of consumers living within the right half of the trade area served by a chain of stores. Multiplying this integral by two yields the total consumer surplus within the trade area served by one chain. Multiplying this value by the number of chain stores $n$ gives the total consumer surplus for the entire city.

### 2.2.3 Nature of consumer surplus

This section discusses the properties of consumer surplus $CS$. As an assumption, we assume that all consumers living in a city have positive demand. This condition is equivalent to a positive demand for consumers living at the boundary of the trade area of neighbouring chain stores. Therefore,

$$\lambda\left(\frac{R}{2n}\right) = \bar{\lambda} - a\left(p + \frac{\tau R}{2n}\right) > 0 \tag{4}$$

is satisfied. Although the number of chain stores $n$ can only be a strictly integer value, in the present study, for simplicity, it is treated as a real value. Under the assumptions shown in Eq. (4), the impact of the product price $p$ and the number of chain stores $n$ on consumer surplus is evaluated as

$$\frac{\partial CS}{\partial p} = -R\left[\bar{\lambda} - a\left(p + \frac{\tau R}{4n}\right)\right] < 0 \tag{5a}$$

$$\frac{\partial CS}{\partial n} = \frac{e\tau R^2}{4n^2}\left[\bar{\lambda} - a\left(p + \frac{\tau R}{3n}\right)\right] > 0 \tag{5b}$$

The consumer surplus can be evaluated on the basis of $p$; it is found to monotonically decrease with respect to the price of the good $p$ and to monotonically increase with respect to the number of chain stores $n$. As the number of chain stores $n$ increases, consumer surplus increases because consumers in the city save money on transportation for shopping.

## 2.3 Inventory control for chain stores

### 2.3.1 Inventory change in the chain store

The chain stores manage their inventory according to the classical inventory management model, the EOQ model and the safety stock model (Chopra and Meindl 2012). The transition process of the number of products in stock in a chain store is explained using Fig. 4. Each time a consumer visits a store and purchases a product, the number of products in stock in the chain store decreases. When the inventory number falls below a certain threshold (reorder point, ROP), the product is ordered upstream (from a supplier outside the region if there is no DC, or from a DC if there is). The variable $Q$ represents the quantity (lot size) ordered at this time. When the ordered goods arrive at the chain store, the inventory number is restored by $Q$. The above process is repeated.

There is a time lag between when a chain store places an order for a product and when the product arrives at the chain store. This time lag is called lead time. Lead time includes the time required to dispatch goods, dispatch trucks, load trucks and run trucks on the road. To avoid running out of stock during this lead time, chain stores need to order goods well in advance. As a result, there are many cases where goods remain in stock in the chain store when the goods are delivered. The expected number of inventory items remaining in the chain store at the time of delivery is called safety stock and is represented by the variable $ss$. The EOQ model determines the lot size $Q$, and the safety stock model determines the safety stock $ss$.

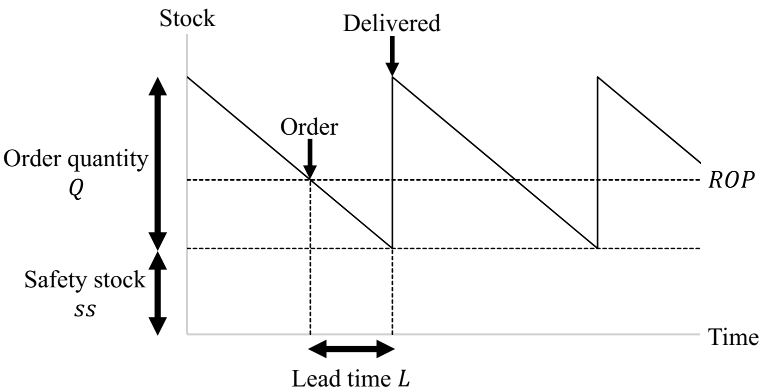



**Fig. 4** Inventory change in the chain store (when demand is constant)

For convenience of explanation, Fig. 4 shows a graph where orders are assumed to be placed at regular intervals; however, this scenario is a special case in which demand has been accidentally kept constant. In the general case of fluctuating demand, as this study assumes in the following analysis, the ordering interval will fluctuate irregularly. This method of ordering is called the fixed-quantity ordering method and is often applied to products with low unit prices and products with small demand fluctuations. Other ordering methods include the periodic ordering method, in which the ordering interval is fixed and the lot size is changed each time. The periodic ordering method is often applied to expensive products and products with large demand fluctuations. In the present study, we adopt the EOQ model and the safety stock model, which are analytically tractable and can express the qualitative relationship between safety stock and lead time. These models enable us to conduct a theoretically prospective analysis.

In real stores, it is common for each item to have a different ordering system because each product item has different characteristics in terms of demand and inventory holding costs. To improve the theoretical perspective, this study adopts the formulation that a chain store handles only one type of item and adopts a fixed-quantity ordering method. However, even if we model a store that handles multiple items and manages the ordering method for each item, the qualitative relationship between safety stock and lead time remains the same and the results of the qualitative analysis in the present study are considered to be equally valid.

#### 2.3.2 EOQ model

The EOQ model is used to determine the lot size $Q$, where $D$ is the expected hourly demand for the product in the trade area served by one chain store, $S$ is the fixed cost per order (logistics costs for shipping, loading, transportation, unloading, etc.) and $h$ is the inventory holding cost of one product per unit time. The inventory holding cost $h$ consists of the capital cost of inventory investment (interest rate) and warehousing costs that occupy space in the chain stores (Chopra and Meindl 2012). In addition, for food products, for which the value of the product is rapidly depleted, the product needs to be discounted or disposed of as time passes after arrival. Such costs of product value depletion are also included in $h$.

When the lot size is $Q$, the frequency of ordering is $D/Q$ per unit time. Therefore, the ordering cost per unit time is expressed as $DS/Q$. As confirmed from Fig. 4, the long-term average number of inventory items held by a chain store is $Q/2 + ss$. Therefore, the inventory holding cost per unit time is $hQ/2 + h \cdot ss$. The sum of the ordering cost per unit of time and the inventory holding cost per unit of time is

$$\frac{DS}{Q} + \frac{hQ}{2} + h \cdot ss \tag{6}$$

The lot size $Q^*$ (EOQ) that minimizes Eq. (6) is

$$Q^* = \sqrt{\frac{2DS}{h}} \tag{7}$$

EOQ is the efficient lot size that balances the cost of ordering goods and the cost of holding inventory. EOQ decreases monotonically with the inventory holding cost $h$. That is, it is efficient to reduce the lot size of products with high inventory holding costs and order more frequently to increase the inventory turnover rate in the store. Substituting $Q^*$ into Eq. (6), we obtain the sum of ordering cost and inventory holding cost per unit time borne by one chain store as

$$\sqrt{2hDS} + h \cdot ss \tag{8}$$

#### 2.3.3 Safety stock model

The safety stock model is used to obtain the safety stock $ss$. A chain store places an order for an item when the number of items in stock falls below a certain threshold (ROP). This threshold is represented by the variable $ROP$, and $L$ is the lead time from the time a chain store orders a product until the product arrives at the chain store. During this lead time, demand averages $LD$. Therefore, the expected value of the number of items in stock immediately before the stock is replenished (safety stock) can be approximated by the following equation:

$$ss = ROP - LD \tag{9}$$

During lead times, greater than average demand might occur. Safety stock is the inventory held to prevent consumers from encountering shortages in such a situation (see Fig. 4). However, stock-outs can occur even when safety stock is held.

The probability that no stock-outs occur during the lead time is equal to the probability that the demand that occurs during the lead time is less than the $ROP$. If the variance per unit time of the demand for a product in the trade area served by one chain of stores is $\sigma_D^2$, the variance of the demand that occurs during the lead time is $L\sigma_D^2$. If the demand of individual consumers follows a Poisson distribution, the demand that occurs during the lead time can be approximated as following a normal distribution with mean $LD$ and variance $L\sigma_D^2$. Using this approximation, we obtain the probability of no stock-outs occurring during the lead time as

$$\Phi\left(\frac{ROP - LD}{\sqrt{L}\sigma_D}\right) = \Phi\left(\frac{ss}{\sqrt{L}\sigma_D}\right) \tag{10}$$

The cumulative distribution function of the standard normal distribution is expressed as where $\Phi$ is the cumulative distribution function of the standard normal distribution. The chain store determines $ss$ so that this probability matches the exogenous target value $\alpha$ (e.g. 0.95)

$$ss = \sqrt{L}\sigma_D\Phi^{-1}(\alpha) \tag{11}$$

where $\Phi^{-1}$ is the inverse function of $\Phi$. Equation (11) shows that the burden of safety stock for chain stores increases in an environment with a long lead time $L$. Because the cost of holding safety stock is $h \cdot ss$, if the inventory holding cost $h$ of the products handled by the chain store is high, shortening the lead time can reduce the cost burden on the chain store.

Strictly speaking, Eq. (9) is an approximation and underestimates the safety stock because it ignores the fact that the number of inventories cannot be negative. If the demand generated during the lead time exceeds the $ROP$, the inventory count immediately before the inventory is replenished will be zero. Thus, strictly speaking, the safety stock will be larger than $ROP - LD$ and its exact level $ss'$ is

$$ss' = \int_{-\infty}^{ROP} (ROP - x) f_N(x, LD, L\sigma_D^2) dx \tag{12}$$

where $f_N(x, \mu, \sigma^2)$ is the probability density function of normal distribution $N(\mu, \sigma^2)$. However, the long-term average of the number of stocks held by a chain of stores cannot be expressed exactly using $ss'$. The exact long-run average of the number of stocks will be larger than $Q/2 + ss$ and smaller than $Q/2 + ss'$ because using $Q/2 + ss'$ is equivalent to assuming that stock-outs occur at the same time as the arrival of goods (i.e. that inventory runs out before goods arrive and inventory never remains at zero). The above approximation errors are often ignored in textbooks and literature dealing with inventory control models. To investigate the magnitude of these approximation errors, the present study performs Monte-Carlo experiments as shown in **Appendix** A. The result suggests that the approximation errors are generally not very large, although the analysis is carried out under limited parameters. The qualitative properties of $ss$ expressed by Eq. (9) are similar to the exact long-run average of the safety stock. The present study therefore gives priority to theoretical research and ease of analytical treatment and uses Eq. (9) for the evaluation of safety stock.

### 2.3.4 Chain store inventory management costs and travel time on the road

The EOQ and safety stock models require the expected value $D$ and variance $\sigma_D^2$ of the demand for a product per unit time for a single chain store. These values are obtained as a result of consumer purchase behaviour. The expected value and variance of the demand per unit of time for a single consumer living a distance $l$ to the nearest chain store are both $\lambda(l)$. If the individual consumer's demand for a product follows an independent Poisson process, the expected value and variance of the sum of demands can be expressed as the sum of the expected value and variance of the individual demands. Therefore, the expected value $D$ and variance $\sigma_D^2$ of the demand per unit time in the trade area served by one chain of stores are

$$D = \sigma_D^2 = 2\int_0^{\frac{R}{2n}} e\lambda(l)\mathrm{d}l = \frac{eR(\bar{\lambda} - ap)}{n} - \frac{ea\tau R^2}{4n^2} \quad (13)$$

Substituting Eqs. (11) and (13) into Eq. (6) gives the sum of the ordering cost and inventory holding cost per unit time incurred by one chain store, $C_I(S, L)$ (hereinafter referred to as inventory management cost). The inventory management cost for a chain store depends on the ordering cost $S$ and the lead time $L$ and is expressed as

$$C_I(S, L) = \sqrt{2hDS} + h\sqrt{L}\sigma_D\Phi^{-1}(\alpha) = \sqrt{D}\left[\sqrt{2hS} + h\sqrt{L}\Phi^{-1}(\alpha)\right] \quad (14)$$

The order cost $S$ includes the cost of truck transportation, and the lead time $L$ includes the travel time on the road. Therefore, if the travel time on the radial logistics-only road shown in Fig. 3 is reduced, $S$ and $L$ are expected to decrease. In the following, we explicitly model this relationship. Because $S$ and $L$ are the ordering cost and lead time for the chain to order goods upstream, respectively, these values differ when there is no upstream DC and when there is an upstream DC. The values of $S$ and $L$ when there is no DC are denoted by $S_N$ and $L_N$, and the values when there is a DC are denoted by $S_W$ and $L_W$, respectively. These variables are formulated as follows:

$$S_N = S_0 + \beta T \quad (15a)$$

$$L_N = L_0 + T \quad (15b)$$

$$S_W = \beta T \quad (15c)$$

$$L_W = T \quad (15d)$$

where $S_0$ and $L_0$ are the logistics costs and transport times, respectively, from the extra-city supplier to the city's logistics gateway district, and $T$ is the one-way travel time on a dedicated logistics road in Fig. 3. The $\beta$ is the coefficient that converts the travel time to logistics cost. If there is a DC, $L_0$ is not included in the lead time. Instead, the lead time $L_0$ occurs when the DC purchases goods from suppliers outside the city.

### 2.4 Firm's profit

A monopolist operating a chain of stores has three decision-making targets: whether to locate its DC in a distribution gateway district, the number of chain stores in the city $n$ and the price of the product $p$. Firms make decisions so as to maximize their profits. In the following, we formulate the profit of the firms in two cases: the case where the logistics centre is not located in a distribution gateway district and the case where the logistics centre is located in a distribution gateway district.

### 2.4.1 If the distribution centre is not located in a distribution gateway district

We denote the profit of the firm per unit time without locating the DC by $\pi_N(n, p, T)$, and the equation is

$$\begin{aligned}\pi_N(n,p,T) &\\ &= (p-c)nD - n[C_I(S_N, L_N) + F] \\ &= \left[eR(\bar{\lambda} - ap) - \frac{ea\tau R^2}{4n}\right](p-c) - Fn - \sqrt{neR(\bar{\lambda} - ap) - \frac{ea\tau R^2}{4}} \\ &\quad \cdot \left[\sqrt{2h(S_0 + \beta T)} + h\Phi^{-1}(\alpha)\sqrt{L_0 + T}\right]\end{aligned} \tag{16}$$

where $c$ is the various variable costs required to sell one unit of the product, $nD$ is the total sales of the product in the chain and $F$ is the fixed costs required to locate and operate a chain of stores.

Notably, Eq. (16) does not consider the drop in sales when consumers face stock-outs. Realistically, if consumers face stock-outs, they will either give up purchasing the product or visit the chain store again after the product is in stock. In the former case, part of the demand will not be realized. That is, strictly speaking, the total sales of a product must be expressed as $p\gamma nD$ using the variable $\gamma$ that represents "the fraction of demand $nD$ that does not face stock-outs". In addition, if the impact of $\gamma$ on consumer demand $D$ and consumer surplus $CS$ is modelled, the social cost caused by consumer demand uncertainty (demand risk) can be evaluated more accurately. However, because such a model is difficult to handle analytically, this study proceeds with the analysis under the assumption that $\gamma \simeq 1$. Even under the simplification adopted in the present study, the basic property of social costs caused by demand risk can be expressed (i.e. that social costs can be reduced by consolidating demand risk in one place); thus, the theoretical discussion is more prospective. In addition, the Monte Carlo experiments in **Appendix** A analyses the relationship between $\alpha$ and $\gamma$. The result suggests that the approximation error of $\gamma \simeq 1$ is generally not so large if $\alpha$ is sufficiently large, although the analysis is performed under limited parameters.

### 2.4.2 When locating a distribution centre in a distribution gateway district

When a DC is located in a city, the DC must also manage inventory. The DC purchases goods from suppliers located outside the city. The DC can always monitor the inventory status of each chain store through the POS system. In this case, the expected value per unit time of demand for goods ordered from chain stores to the DC, $D_W$, and the variance, $\sigma_{DW}^2$, are

$$D_W = \sigma_{DW}^2 = nD = eR(\bar{\lambda} - ap) - \frac{ea\tau R^2}{4n} \tag{17}$$

(In the absence of a POS system, $\sigma_{DW}^2$ is considered to be larger than the value expressed by Eq. (17) because of increased uncertainty due to the inventory status of chain stores being unobservable.) The DC is also considered to manage inventory according to the EOQ model and the safety stock model. Using Eqs. (8), (11) and (17), the inventory management cost for the DC is

$$\begin{aligned} C_W =& \sqrt{2h \cdot D_W \cdot S_0} + h\sqrt{L_0}\Phi^{-1}(\alpha_W)\sigma_{DW} \\ =& \sqrt{eR(\bar{\lambda} - ap) - \frac{ea\tau R^2}{4n}} \cdot \left[\sqrt{2hS_0} + h\Phi^{-1}(\alpha)\sqrt{L_0}\right] \end{aligned} \tag{18}$$

However, $S_0$ and $L_0$ are the ordering cost and lead time for the DC to purchase goods from suppliers outside the city, respectively. The inventory holding cost $h$ and the target value of the probability of running out of stock during the lead time $\alpha$ are assumed to be the same between the chain stores and the DC.

Using the inventory management cost $C_W$ of the DC, the profit of the firm per unit time $\pi_W(n, p, T)$ for locating the DC can be expressed as

$$\begin{aligned} \pi_W(n,p,T) =& (p - c)nD - n[C_I(S_W, N_W) + F] - [C_W + F_W] \\ =& \left[eR(\bar{\lambda} - ap) - \frac{ea\tau R^2}{4n}\right](p - c) - Fn - F_W \\ & - \sqrt{neR(\bar{\lambda} - ap) - \frac{ea\tau R^2}{4}}\left[\sqrt{2h\beta T} + h\Phi^{-1}(\alpha)\sqrt{T}\right] \\ & - \sqrt{eR(\bar{\lambda} - ap) - \frac{ea\tau R^2}{4n}}\left[\sqrt{2hS_0} + h\Phi^{-1}(\alpha)\sqrt{L_0}\right] \end{aligned} \tag{19}$$

However, $F_W$ is the fixed cost of locating and operating the DC.

### 2.4.3 Firm's decision-making

The firm's decision is made as follows. First, the intra-city transport time $T$ is given exogenously. Next, the firm optimizes $\pi_N(n, p, T)$ and $\pi_W(n, p, T)$ for (hypothetically)

$n$ and $p$ under the given $T$ to determine the optimal number of chain stores and product prices in each case when the DC is not located and when it is located. The solution of the optimization problem for $\pi_N(n, p, T)$ is denoted by $n_N(T)$ and $p_N(T)$, and that for $\pi_W(n, p, T)$ by $n_W(T)$ and $p_W(T)$. Finally, the firm compares $\pi_N(n_N(T), p_N(T), T)$ and $\pi_W(n_W(T), p_W(T), T)$. If the former is larger than the latter, the firm does not locate the DC. That is, the following equation holds:

$$\begin{cases} \text{No distribution centre is located} \\ \pi_N(n_N(T), p_N(T), T) \geq \pi_W(n_W(T), p_W(T), T) \end{cases} \tag{20}$$

$$\begin{cases} \text{Distribution centre is located} \\ \pi_N(n_N(T), p_N(T), T) < \pi_W(n_W(T), p_W(T), T) \end{cases} \tag{21}$$

## 3 Comparative static analysis

In this section, we apply analytical and numerical comparative statics analysis to the model formulated in **3.** To analyse the effects of facilitating logistics by improving intra-urban roads on encouraging retailers to locate intra-urban DCs and on improving shopping convenience for consumers. Specifically, we analyse the effect of a reduction in intra-urban transportation time $T$ on firms' decision-making. We also analyse the effect of this change in firms' decision-making on consumer surplus.

### 3.1 Impact of road improvements on the location of distribution centres

We analyse the effect of shorter intra-city transport time $T$ on the location of DCs. First, we compare $\pi_W(n, p, T)$ and $\pi_N(n, p, T)$ under given $n$ and $p$. The $n$ and $p$ are endogenous variables determined by the firms and depend on $T$ and the location of the DC. However, for clarity, we assume that $n$ and $p$ can be treated as fixed values like exogenous variables. Under this assumption, the difference between $\pi_W(n, p, T)$ and $\pi_N(n, p, T)$ is

$$\begin{aligned} \pi_W(n, p, T) - \pi_N(n, p, T) =& \sqrt{\frac{eR(\bar{\lambda} - ap)}{n} - \frac{ea\tau R^2}{4n^2}} \\ & \cdot \left[h\Phi^{-1}(\alpha)\left(n\sqrt{L_0 + T} + n\sqrt{S_0 + \beta T}\right)\right. \\ & \left. - \sqrt{2h}\left(n\sqrt{T} + \sqrt{nL_0} + n\sqrt{\beta T} + \sqrt{nS_0}\right)\right] - F_W \\ =& \sigma_D h\Phi^{-1}(\alpha)\left(n\sqrt{L_0 + T} - n\sqrt{T} - \sqrt{nL_0}\right) \\ & + D\sqrt{2h}\left(n\sqrt{S_0 + \beta T} - n\sqrt{\beta T} - \sqrt{nS_0}\right) - F_W \end{aligned} \tag{22}$$

The resultant value represents the amount of increase in profit that a firm without a DC would gain by locating a DC under a given $n$ and $p$. To analyse the impact of shortening $T$ on this amount, partial differentiation of both sides of Eq. (22) with respect to $T$ yields

$$\frac{\partial}{\partial T}[\pi_W(n,p,T)-\pi_N(n,p,T)] = -\sigma_D h\Phi^{-1}(\alpha)n\left(\frac{1}{2\sqrt{T}}-\frac{1}{2\sqrt{L_0+T}}\right)-D\sqrt{2hn}\left(\frac{\beta}{2\sqrt{\beta T}}-\frac{\beta}{2\sqrt{S_0+\beta T}}\right)<0 \tag{23}$$

From Eq. (23), if $n$ and $p$ are given exogenously, we can confirm that the profit increase due to the location of DCs expands as $T$ shortens.

To examine the benefits of locating a DC, we consider an extreme situation in which the intra-city transport time is $T=0$. In this case, chain stores can purchase goods instantaneously from the DC without incurring ordering costs, thereby eliminating the need to hold inventory. The value of Eq. (22) is

$$\pi_W(n,p,0)-\pi_N(n,p,0) = \left(n-\sqrt{n}\right)\sigma_D h\Phi^{-1}(\alpha)\sqrt{L_0}+\left(n-\sqrt{n}\right)D\sqrt{2hS_0}-F_W \tag{24}$$

In general, $n \geq 1$. Eq. (24) implies that the location of a DC will always increase the profit of a firm if there is no fixed cost $F_W$ of the DC. The first term on the right-hand side of Eq. (24) represents the benefit of the DC's function of "assuming the aggregate inventory holding risk of downstream chain stores," as described in **2. (2)**. The $\sigma_D$ is the standard deviation of the demand for goods from individual chain stores. From Eq. (11), when chain stores individually hold safety stock, each chain store must hold $\sqrt{\sigma_D^2}\Phi^{-1}(\alpha)\sqrt{L_0}$ of safety stock. Because there are $n$ chain stores, the total amount of safety stock held by the company is $n$ times this amount. However, if the DC holds an intensive inventory, the variance of its demand for the product is $n\sigma_D^2$. Thus, the total amount of safety stock held by the company is $\sqrt{n\sigma_D^2}\Phi^{-1}(\alpha)\sqrt{L_0}$. Thus, by assuming the inventory holding risk of multiple chain stores in one lump sum, the DC can reduce the inventory holding risk as a whole and the total amount of safety stock. That is, the first term on the right-hand side of Eq. (24) represents the benefit of reducing safety stock.

Another benefit of locating a DC is represented by the second term on the right side of Eq. (24). This benefit is the effect of reducing ordering costs (logistics costs) by taking advantage of economies of scale by increasing the lot size at the time of purchase through the joint purchase of goods by multiple chain stores. This effect has been considered a benefit of DC location in previous studies on the facility location model (Melo et al. 2009). Thus, by introducing an inventory management model, as introduced in the present study, the benefit of a DC can be expressed as the sum of the benefit of a larger lot size and the benefit of reduced safety stock.

The benefit of the DC expressed in Eq. (24) assumes an ideal environment (when $T=0$) where the intra-city transit time is negligible. When the intra-city transport time cannot be ignored ($T > 0$), each chain store must hold safety stock even if a DC is located in the city. In this situation, the benefit of reducing safety stock because of the location of the DC is expressed by the first term on the right-hand side of the second equality in Eq. (22). The total amount of safety stock without a DC is $\sigma_D \Phi^{-1}(\alpha) n \sqrt{L_0 + T}$. However, the total amount of safety stock when a DC is located is $\sigma_D \Phi^{-1}(\alpha) \left( n\sqrt{T} + \sqrt{nL_0} \right)$. As evident from Eq. (23), the difference between the two decreases with increasing $T$; in particular, the limit of $T \to \infty$ is

$$\lim_{T \to \infty} \pi_W(n, p, T) - \pi_N(n, p, T) = -\sigma_D h \Phi^{-1}(\alpha) \sqrt{nL_0} - D\sqrt{2h}\sqrt{nS_0} - F_W \tag{25}$$

That is, when $T$ is very large, the location of a DC inversely increases the total amount of safety stock, independent of the values of $n$ and $p$. When $T$ is large, even if the DC is located, the chain must hold a large amount of safety stock to prepare for long lead times. In addition, the DC must also hold safety stock and the location of the DC ultimately increases the burden of safety stock. The results of Eqs. (23) and (25) indicate that DCs need to improve intra-city transport times to fulfil their function of increasing the intra-city logistics efficiency.

In the above discussion, for clarity, we treated $n$ and $p$ as fixed values. In reality, these variables depend on $T$ and the location of the DC, as described in **3. (4)**. In the following discussions, we conduct a comparative statics analysis assuming the general situation where $n$ and $p$ are endogenous variables. In such a situation, a firm decides the location of its DC by comparing $\pi_N(n_N(T), p_N(T), T)$ and $\pi_W(n_W(T), p_W(T), T)$. Here, we define a function that expresses the location benefit of a DC given an intra-city transport time $T$, as follows:

$$f(T) = \pi_W(n_W(T), p_W(T), T) - \pi_N(n_N(T), p_N(T), T) \tag{26}$$

Analytically guaranteeing the monotonicity of the function $f(T)$ is difficult. For this reason, we discuss the qualitative properties of the function $f(T)$ using Eqs. (24) and (25). From Eq. (25), the following equation holds:

$$\lim_{T \to \infty} \pi_W(n_W(T), p_W(T), T) - \pi_N(n_W(T), p_W(T), T) < 0 \tag{27a}$$

$$\pi_N(n_N(T), p_N(T), T) \geq \pi_N(n_W(T), p_W(T), T) \tag{27b}$$

Therefore, $\lim_{T \to \infty} f(T) < 0$ holds. Next, from Eq. (24), the following equation holds:

$$\pi_W(n_N(0), p_N(0), 0) - \pi_N(n_N(0), p_N(0), 0) > 0 \quad (\text{if } n_N(0) \geq 1 \text{ and } F_W \simeq 0) \tag{28a}$$

$$\pi_W(n_W(0), p_W(0), 0) \geq \pi_W(n_N(0), p_N(0), 0) \tag{28b}$$

Therefore, if $n_N(0) \geq 1$ and $F_W$ is sufficiently small, then $f(0) > 0$ holds. From the above result and the fact that $f(T)$ is continuous, the following proposition holds:

**Proposition 1** *When $n_N(0) \geq 1$ and $F_W$ is sufficiently small, there exists a threshold $\tilde{T} > 0$ satisfying Eqs. (29a), (29b).*

$$f(\tilde{T}) = 0 \tag{29a}$$

$$f(T) > 0 \quad (0 \leq T < \tilde{T}) \tag{29b}$$

That is, if $0 \leq T < \tilde{T}$ holds, then the location of a DC generates a profit. This means that a firm will locate a DC if the intra-city transport time $T$ is reduced below a certain level $\tilde{T}$. The possibility that there are multiple $T$ satisfying $f(T) = 0$ has not been eliminated; however, in the event of multiple such $T$, the smallest of them is $\tilde{T}$. This relation can be derived from the continuity of $f(T)$ and $f(0) > 0$.

### 3.2 Impact of road improvements on the number of chain stores and commodity prices

In the following, we analyse the effect of the shortening of $T$ on the number of chain stores $n$ and product price $p$ determined by the firm. For this purpose, we apply comparative statics analysis to the first-order optimization conditions of the firm's profit maximization problem. First, assuming a situation where no DC is located, the first-order optimization conditions of the maximization problem for $n$ and $p$ of the company's profit $\pi_N(n, p, T)$ are

$$\frac{\partial \pi_N}{\partial n}(n,p,T) = \frac{ea\tau R^2}{4n^2}(p-c) - F - (\bar{\lambda} - ap)X(n,p)Z_N(T) = 0 \tag{30a}$$

$$\frac{\partial \pi_N}{\partial p}(n,p,T) = eR(\bar{\lambda} + ac - 2ap) - \frac{ea\tau R^2}{4n} + anX(n,p)Z_N(T) = 0 \tag{30b}$$

Here, $X(n, p)$ and $Z_N(T)$ are functions introduced to simplify the description and are defined as follows:

$$X(n,p) = \frac{eR}{2\sqrt{neR(\bar{\lambda} - ap) - ea\tau R^2/4}} \tag{31a}$$

$$Z_N(T) = \sqrt{2h(S_0 + \beta T)} + h\Phi^{-1}(\alpha)\sqrt{L_0 + T} \tag{31b}$$

By partial differentiation of Eqs. (30a) and (30b) with respect to $n$, $p$ and $T$, the equations of comparative statics analysis are derived as follows:

$$\begin{pmatrix} \frac{\partial^2 \pi_N}{\partial n^2} & \frac{\partial^2 \pi_N}{\partial n \partial p} \\ \frac{\partial^2 \pi_N}{\partial n \partial p} & \frac{\partial^2 \pi_N}{\partial p^2} \end{pmatrix} \begin{pmatrix} dn \\ dp \end{pmatrix} = - \begin{pmatrix} \frac{\partial^2 \pi_N}{\partial n \partial T} \\ \frac{\partial^2 \pi_N}{\partial p \partial T} \end{pmatrix} dT \tag{32}$$

Solving this equation for d$n$ and d$p$, we obtain

$$\begin{pmatrix} dn \\ dp \end{pmatrix} = \left\{ \det \begin{vmatrix} \frac{\partial^2 \pi_N}{\partial n^2} & \frac{\partial^2 \pi_N}{\partial n \partial p} \\ \frac{\partial^2 \pi_N}{\partial n \partial p} & \frac{\partial^2 \pi_N}{\partial p^2} \end{vmatrix} \right\}^{-1} \begin{pmatrix} \frac{\partial^2 \pi_N}{\partial p^2} & -\frac{\partial^2 \pi_N}{\partial n \partial p} \\ -\frac{\partial^2 \pi_N}{\partial n \partial p} & \frac{\partial^2 \pi_N}{\partial n^2} \end{pmatrix} \begin{pmatrix} \frac{\partial^2 \pi_N}{\partial n \partial T} \\ \frac{\partial^2 \pi_N}{\partial p \partial T} \end{pmatrix} (-dT) \tag{33}$$

From the second-order optimization condition of the profit maximization problem, the sign of the determinant on the right-hand side of Eq. (33) is positive. Therefore, we attempt to specify the sign of the remaining part of the right-hand side of Eq. (33). First, from Eqs. (30a) and (30b), the following equation is derived.

$$\frac{\partial^2 \pi_N}{\partial n^2} = -\frac{ea\tau R^2}{2n^3}(p-c) - (\bar{\lambda} - ap)X_n Z_N \tag{34a}$$

$$\frac{\partial^2 \pi_N}{\partial p^2} = -2eaR + anX_p Z_N \tag{34b}$$

$$\frac{\partial^2 \pi_N}{\partial n \partial p} = \frac{ea\tau R^2}{4n^2} + [aX - (\bar{\lambda} - ap)X_p]Z_N = \frac{ea\tau R^2}{4n^2} + [aX + anX_n]Z_N \tag{34c}$$

$$\frac{\partial^2 \pi_N}{\partial n \partial T} = -(\bar{\lambda} - ap)XZ'_N \tag{34d}$$

$$\frac{\partial^2 \pi_N}{\partial p \partial T} = anXZ'_N \tag{34e}$$

where $Z'_N$ is the derivative of $Z_N$, and $X_n$ and $X_p$ are the partial derivatives with respect to $n$ and $p$ of $X$, respectively. Function arguments are omitted. From Eqs. (34a)–(34e), the following equations are derived:

$$\frac{\partial^2 \pi_N}{\partial p^2}\frac{\partial^2 \pi_N}{\partial n \partial T} - \frac{\partial^2 \pi_N}{\partial n \partial p}\frac{\partial^2 \pi_N}{\partial p \partial T} = 2eaRXZ'_N \left[ \bar{\lambda} - a\left( p + \frac{\tau R}{8n} \right) - \frac{anXZ_N}{2eR} \right] \tag{35a}$$

$$-\frac{\partial^2\pi_N}{\partial n\partial p}\frac{\partial^2\pi_N}{\partial n\partial T}+\frac{\partial^2\pi_N}{\partial n^2}\frac{\partial^2\pi_N}{\partial p\partial T}=-\frac{ea\tau R^2}{4n^2}XZ_N'\left[\bar{\lambda}-a\left(p+\frac{\tau R}{2n}\right)\right]\left(1-\frac{4n^2XZ_N}{e\tau R^2}\right) \tag{35b}$$

Notably, we use the conclusion of Eq. (30b) in the optimal solution when deriving Eq. (35b). If the signs of Eqs. (35a) and (35b) can be specified, the signs of $dn$ and $dp$ can be specified. Analytical identification of the signs of Eqs. (35a) and (35b) is generally difficult; however, if the following conditions are satisfied in the optimal solution, then analytical identification becomes possible:

$$\frac{Z_N n\sqrt{D}}{nD}<\frac{\tau R}{2n} \tag{36}$$

The numerator on the left side of this equation represents the total inventory management cost of the firm, $nC_I(S_N, L_N)$. Because $nD$ in the denominator is the total demand for the product, the left side represents the average inventory management cost per unit of demand. Equation (36) implies that this average cost is lower than $\tau R/2n$ (i.e. the transportation cost per shopping trip for consumers living in the most inconvenient location). When Eq. (36) holds for the optimal solution, the signs of Eqs. (35a) and (35b) can be specified. When Eq. (36) holds, the following inequality holds:

$$\frac{anXZ_N}{2eR}<\frac{a\tau R}{8n} \tag{37a}$$

$$\frac{4n^2XZ_N}{e\tau R^2}<1 \tag{37b}$$

Using Eqs. (4), (37a), (37b) and $Z_N'>0$, the sign of Eq. (35a) can be specified to be positive and that of Eq. (35b) negative. From the above, the following proposition is derived:

**Proposition 2** *When Eq.* (36) *holds in the optimal solution, a decrease in intra-city transport time T increases* $n_N(T)$ *and decreases* $p_N(T)$.

Next, a comparative statics analysis is performed for the situation in which the DC is located. In this situation, the first-order optimality conditions of the maximization problem for $n$ and $p$ of the firm's profit $\pi_N(n,p,T)$ are

$$\frac{\partial\pi_W}{\partial n}(n,p,T)=\frac{ea\tau R^2}{4n^2}(p-c)-F-(\bar{\lambda}-ap)X(n,p)\left[Z_W(T)+\frac{Z_0}{\sqrt{n}}\right]-\frac{eRZ_0}{4X(n,p)n\sqrt{n}}=0 \tag{38a}$$

$$\frac{\partial\pi_W}{\partial p}(n,p,T)=eR(\bar{\lambda}+ac-2ap)-\frac{ea\tau R^2}{4n}+anX(n,p)\left[Z_W(T)+\frac{Z_0}{\sqrt{n}}\right]=0 \tag{38b}$$

However, $Z_W(T)$ and $Z_0$ are functions and variables introduced to simplify the description and are defined as follows:

$$Z_W(T) = \sqrt{2h\beta T} + h\Phi^{-1}(\alpha)\sqrt{T} \tag{39a}$$

$$Z_0 = \sqrt{2hS_0} + h\Phi^{-1}(\alpha)\sqrt{L_0} \tag{39b}$$

By partially differentiating Eqs. (38a) and (38b) with respect to $n$, $p$ and $T$ and solving for $dn$ and $dp$, we derive the equations of comparative statics. The proof is given in **Appendix** B. Similar to the proof of **Proposition** 2, we can show that the signs of $dn$ and $dp$ can be specified analytically if the following conditions are satisfied in the optimal solution:

$$\frac{Z_W n\sqrt{D} + Z_0\sqrt{nD}}{nD} < \frac{\tau R}{2n} - \frac{Z_0\sqrt{nD}}{nD}\frac{\bar{\lambda} - ap - \frac{3a\tau R}{8n}}{\bar{\lambda} - ap - \frac{a\tau R}{2n}} \tag{40}$$

The numerator on the left side of this equation represents the total inventory management cost of the firm, $nC_I(S_W, L_W) + C_W$, and the entire left side represents the average inventory management cost per unit of demand. Equation (40) means that this average cost must be below a certain level. When Eq. (40) holds for the optimal solution, the signs of $dn$ and $dp$ can be specified and the following proposition follows:

**Proposition 3** *When Eq.* (40) *holds in the optimal solution, a decrease in intra-city transport time $T$ increases $n_W(T)$ and decreases $p_W(T)$.*

Whether the preconditions of **Proposition** 2 and **Proposition** 3 always hold irrespective of the exogenous parameters is unclear. Therefore, in addition to the analytical comparative statics analysis, we perform a numerical comparative statics analysis in Subsect. 3.4. Within the range of numerical examples presented in section 4, the preconditions of **Proposition** 2 and **Proposition** 3 are confirmed to be satisfied.

**Proposition** 2 and **Proposition** 3 show that a reduction in intra-city transport time $T$ has the effect of increasing the number of chain stores $n$ and lowering commodity prices $p$. To examine the mechanism that led to this result, we focus on the Eq. (14) that represents the inventory management cost for individual chain stores. This equation shows that inventory management costs are subject to economies of scale with respect to product demand $D$. The average inventory management cost per unit of product demand is

$$\frac{C_I}{D} = \frac{1}{\sqrt{D}}\left[\sqrt{2hS} + h\sqrt{L}\Phi^{-1}(\alpha)\right] \tag{41}$$

We can confirm that the value of $D$ is monotonically decreasing for $D$. The reason for this behaviour is that the larger the demand for the commodity, the lower the risk of holding safety stock. As expressed in Eq. (13), there is a proportional relationship between the expected value of product demand and variance. Therefore, the safety stock that each chain store should hold can be expressed as $\sqrt{D}\Phi^{-1}(\alpha)\sqrt{L}$ when the lead time is $L$, implying that the greater the demand for the commodity, the smaller the safety stock per consumer. Another reason why the right-hand side of Eq. (41) monotonically decreases with respect to $D$ is that the larger the demand for a product, the larger the lot size at the time of purchase can be made to take advantage of economies of scale and reduce ordering costs. For the above reasons, reducing the number of chain stores $n$ in order to reduce inventory management costs is desirable because an increase in the number of chain stores causes the demand for the product to be divided among the chain stores. The above effects are stronger when the lead time $L$ and ordering cost $S$ are large. Conversely, the optimal number of chain stores increases as $L$ and $S$ decrease because chain stores can order products from the DC in small quantities, reducing the burden of inventory management costs and allowing operation even if the demand per chain store is small. This relationship is why a decrease in $T$ leads to an increase in $n$. The reason why an increase in $T$ leads to a decrease in $p$ is that a decrease in the marginal cost of inventory management costs (the derivative of $C_I$ with respect to $D$) allows room for price reductions.

In the above discussion, we analysed the shortening effect of $T$ given the presence or absence of a DC. As shown in the previous section, when $T$ is shortened across the threshold $\tilde{T}$, a change occurs from a state in which no DC is located to a state in which a DC is located. In this case, the number of chain stores and product prices change discontinuously from $(n_N(T),\ p_N(T))$ to $(n_W(T),\ p_W(T))$ because of the change in cost structure associated with the location of DCs. We here analyse the direction of this discontinuous change.

First, $(n_N(T),\ p_N(T))$ satisfies the Eqs. (30a) and (30b); thus, two conditional expressions for $(n_N(T),\ p_N(T))$ are obtained. Substituting these expressions into the partial derivatives of $\pi_W$, we obtain the following two expressions:

$$\begin{aligned}\frac{\partial \pi_W}{\partial n}(n_N(T), p_N(T), T) = (\bar{\lambda} - ap_N(T))X(n_N(T), p_N(T))\\ \cdot\left[\sqrt{2h}\left(\sqrt{S_0+\beta T}-\sqrt{\beta T}\right) + h\Phi^{-1}(\alpha)\left(\sqrt{L_0+T}-\sqrt{T}\right)\right.\\ \left.- Y(n_N(T), p_N(T))\left(\sqrt{\frac{2hS_0}{n_N(T)}} + h\Phi^{-1}(\alpha)\sqrt{\frac{L_0}{n_N(T)}}\right)\right]\end{aligned} \tag{42a}$$

$$\begin{aligned}\frac{\partial \pi_W}{\partial p}(n_N(T), p_N(T), T) = -an_N(T)X(n_N(T), p_N(T))\\ \cdot\left[\sqrt{2h}\left(\sqrt{S_0+\beta T}-\sqrt{\beta T}-\sqrt{\frac{S_0}{n_N(T)}}\right)\right.\\ \left.+ h\Phi^{-1}(\alpha)\left(\sqrt{L_0+T}-\sqrt{T}-\sqrt{\frac{L_0}{n_N(T)}}\right)\right]\end{aligned} \tag{42b}$$

Here, $Y(n, p)$ is a function introduced to simplify the description and is defined as follows:

$$Y(n,p) = 2 - \frac{ea\tau R^2}{4neR(\bar{\lambda} - ap)} \tag{43}$$

Because the demand $D$ in Eq. (13) is positive, $Y(n, p)$ always takes a value between 1 and 2. The signs of equations (42a) and (42b) indicate the direction in which $n$ and $p$ should be moved for the firm to increase its profit when the firm's cost structure changes with the location of the DC.

We now consider the signs of these equations. We can confirm that [ ] in Eqs. (42a) and (42b) are positive if $n_N(T)$ is greater than a certain value. From $Y(n,p) \geq 1$, the inside of [ ] in Eqs. (42a) and (42b) is positive if the following two conditions are satisfied:

$$\frac{\sqrt{n_N(T)}}{Y(n_N(T), p_N(T))} > \sqrt{\frac{T}{L_0}} + \sqrt{1 + \frac{T}{L_0}} \tag{44a}$$

$$\frac{\sqrt{n_N(T)}}{Y(n_N(T), p_N(T))} > \sqrt{\frac{\beta T}{S_0}} + \sqrt{1 + \frac{\beta T}{S_0}} \tag{44b}$$

Eqs. (44a) and (44b) are more easily satisfied when $T$ is small. For example, when $T/L_0 = 1$ and $\beta T/S_0 = 1$, from $Y(n,p) < 2$, $n_N(T) > 23.3$ is required for Eqs. (44a) and (44b) to hold. However, when $T/L_0 = 0.1$ and $\beta T/S_0 = 0.1$, Eqs. (44a) and (44b) are satisfied if $n_N(T) > 7.5$. This property will be discussed again in the numerical comparative statics analysis in section 4. From the above analysis, the following proposition is derived:

**Proposition 4** *When a new location of a distribution centre is realized, if Eqs. (44a) and (44b) are satisfied, in the neighbourhood of* $(n_N(T), p_N(T))$, *the firm can increase its profit by increasing n and reducing p.*

**Proposition** 4 shows that if the intra-city transport time $T$ is sufficiently short, then firms will increase the number of chain stores $n$ and reduce the price of goods $p$ by locating new DCs. The mechanism by which this result arises is similar to that of **Proposition** 2 and **Proposition** 3. That is, the lead time for chain stores decreases from $L_0 + T$ to $T$ when the DC is located. As a result, the disadvantage of splitting the product demand is mitigated, leading to an increase in the optimal number of chain stores and a decrease in the marginal cost of inventory management. As a result, prices decrease.

**Proposition** 4 does not necessarily imply that $n_N(T) < n_W(T)$ or $p_N(T) > p_W(T)$ because the sign of the derivative of $\pi_W$ can change in the process from $(n_N(T), p_N(T))$ to $(n_W(T), p_W(T))$. However, if $(n_N(T), p_N(T))$ is sufficiently close to $(n_W(T), p_W(T))$, we can at least say from Proposition 4 that $n_N(T) < n_W(T)$ and $p_N(T) > p_W(T)$).

### 3.3 Impact of road maintenance on consumer surplus

The analysis in the previous subsection shows that, if the average inventory management cost is sufficiently small, a reduction in intra-city transportation time $T$ leads to an increase in $n$ and a decrease in $p$. The analysis also shows that, if the intra-city transport time $T$ is sufficiently short, the shortening of $T$ leads to an increase in $n$ and a decrease in $p$ when a new DC is located in the city. As already confirmed in Subsect. 3.2, the consumer surplus $CS$ in the city increases monotonically for $n$ and decreases monotonically for $p$. Therefore, the following proposition follows from **Proposition** 2–**Proposition** 4:

**Proposition 5**

- When no distribution centre is located and Eq. (36) holds, a decrease in $T$ increases consumer surplus $CS$.
- When distribution centres are located and Eq. (40) holds, a decrease in $T$ increases consumer surplus $CS$.
- When a new location of a distribution centre is realized, consumer surplus $CS$ increases if Eqs. (44a) and (44b) are satisfied and $(n_N(T), p_N(T))$ is sufficiently close to $(n_W(T), p_W(T))$.

This proposition shows that a reduction in intra-city transportation time $T$ not only increases firms' profits but also improves consumer surplus. With the change in firms' decision-making due to shorter $T$, consumers benefit from lower commodity prices and lower shopping transportation costs.

### 3.4 Numerical analysis

In the above, we have shown that **Proposition** 1–**Proposition** 5 are true by analytical comparative statics analysis. However, each proposition is valid only if the preconditions are satisfied and it is not clear whether it is valid for arbitrary exogenous parameter values. Therefore, in this section, a numerical comparative statics analysis is performed to confirm the validity of the propositions and their preconditions, albeit within a limited parameter range.

For numerical calculations, the exogenous parameters are set as follows. The population of the city is assumed to be 100,000; in addition, $e = 1000$ and $R = 100$. The unit time is set to one day. The daily per capita demand for a product is assumed to be at most 0.1 units, and $\bar{\lambda} = 0.1$, $a = 0.05$, $\tau = 0.1$ and $c = 1$ under the assumption of a highly elastic demand situation. Fixed costs per day for chain stores and DCs are set to $F = 10$ and $F_W = 100$, respectively. The lead time from outside the city to the city is assumed to be one day; in addition, $L_0 = 1$, $\beta = 10$ and $S_0 = 10$. The cost of holding inventory per day and per unit is set to $h = 0.1$, and the target probability of no stock-out during the lead time is set to $\alpha = 0.95$. Under these settings, the

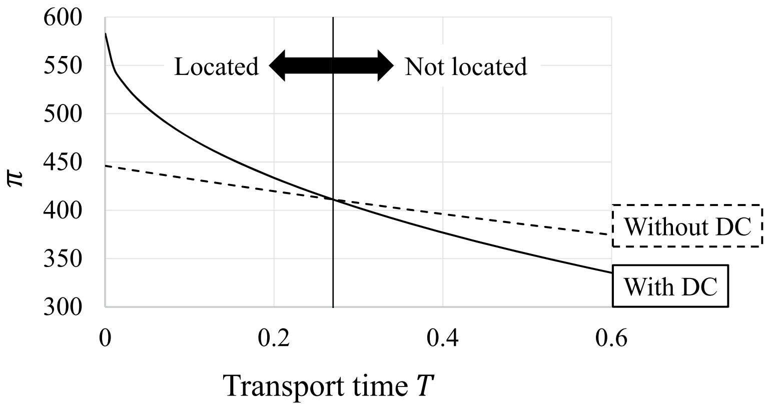



**Fig. 5** Relationship between intra-city transport time $T$ and firms' profit $\pi$

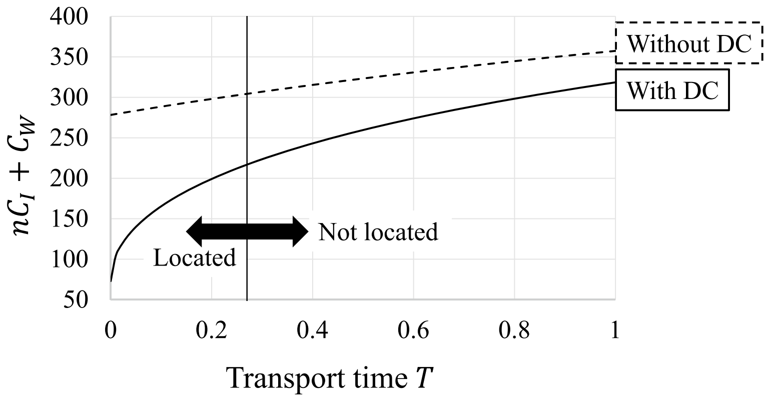



**Fig. 6** Relationship between intra-city transport time $T$ and total inventory management costs of firms

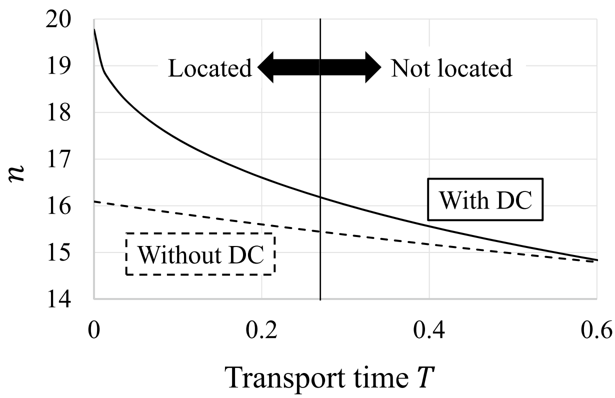



**Fig. 7** Relationship between intra-city transport time $T$ and number of chain stores $n$

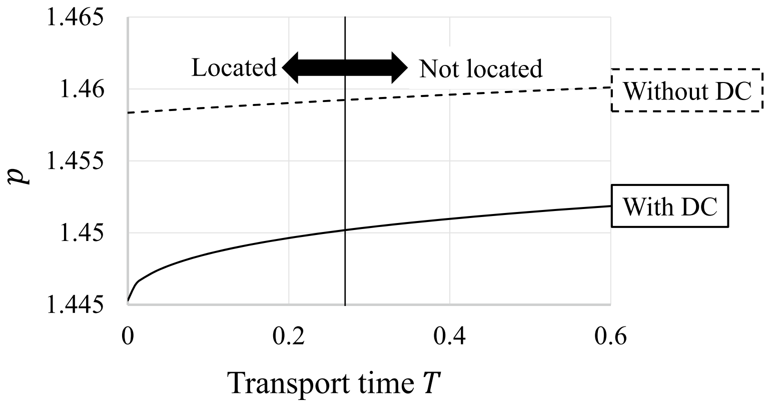



**Fig. 8** Relationship between intra-city transport time $T$ and commodity price $p$

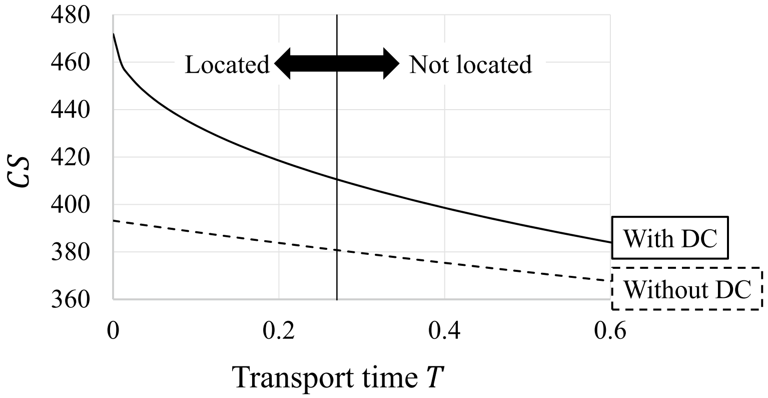



**Fig. 9** Relationship between intra-city transport time $T$ and consumer surplus $CS$

price of the product set by the firm is approximately $p = 1.45$, so the inventory holding cost of $h = 0.1$ means that the value of the product is assumed to be zero in 15 days. Under these settings, $T$ is varied from 1 (day) to 0 (day) and the accompanying changes in each variable are analysed.

Figure 5 shows the relationship between the intra-city transport time $T$ and the profit $\pi$ of the firms. The solid line shows the case where a DC is located ($\pi_W$), and the dashed line shows the case where no DC is located ($\pi_N$). In the other graphs, the solid and dashed lines correspond to the presence or absence of a DC. As shown in **Proposition** 1, we can confirm that there exists a threshold $\tilde{T}$ at which profit is higher if a DC is located. Under the parameter settings in this section, $\tilde{T} = 0.26$, Fig. 6 shows the relationship between the intra-city transport time $T$ and the total inventory management cost $nC_I + C_W$ of the firm. The results confirm that the reduction in inventory management cost, which is a benefit of locating a DC, becomes stronger as $T$ is shortened.

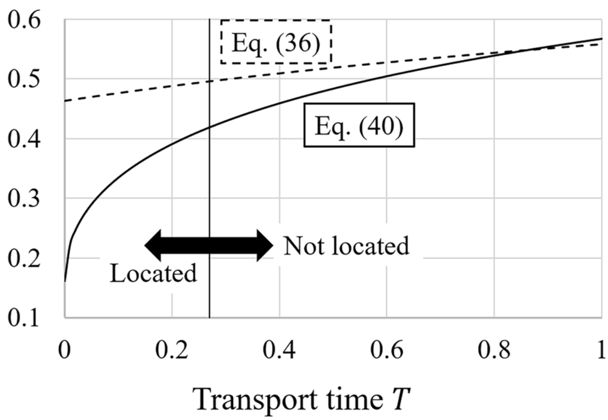



**Fig. 10** Relationship between $T$ and the value of the left-hand side divided by the right-hand side of Eqs. (36) and (40)

The relationship between intra-city transport time $T$ and the number of chain stores $n$, commodity price $p$ and consumer surplus $CS$ is shown in Fig. 7, Fig. 8 and Fig. 9, respectively. As shown in **Proposition** 2, **Proposition** 3 and **Proposition** 5, given the location of the DC, the shortening of $T$ leads to an increase in $n$ and a decrease in $p$, increasing $CS$.

The preconditions for **Proposition** 2 and **Proposition** 3, Eqs. (36) and (40), hold for all numerical cases used to create the graphs in Fig. 5 and Fig. 6. Figure 10 is a graph showing the relationship between $T$ and the value of the left-hand side of Eq. (36) divided by the right-hand side of Eq. (40). The results confirm that the left-hand side is smaller than the right-hand side for all $T$. Because the unit inventory management cost increases with increasing length of $T$, both graphs in Fig. 10 are monotonically increasing with increasing $T$. The unit inventory management cost at $T = 1$ is 0.197 when the DC is not located in the city and 0.174 when it is located in the city, which are 13.5% and 12.0% of the commodity price, respectively. This result illustrates that the assumptions of **Proposition** 2 and **Proposition** 3 are satisfied when inventory management costs are not extremely large.

Focusing on the point $T = 0.26$, which is the threshold at which a DC is located, we can confirm that the location of a DC leads to an increase in $n$ and a decrease in $p$, which is the result expected under **Proposition** 4. However, the number of chain stores is smaller when the DC is located in the range of $T \geq 0.64$. If a new DC is located in a situation where the fixed cost of locating a DC is low and $T$ is long, the number of chain stores will decrease, contrary to the prediction of **Proposition** 4. Thus, the assumption of **Proposition** 4 holds only when $T$ is sufficiently short. Irrespective of the length of $T$, $p$ is lower and $CS$ is higher when a DC is located.

Finally, we consider the effect of the location of DCs on consumer surplus. When $T$ is long, the change in the number of chain stores associated with the location of a DC is small and the benefit to consumers is primarily in the form of lower product prices. However, as $T$ becomes shorter, the increase in the number of chain stores associated with the location of the DC increases and the savings in shopping

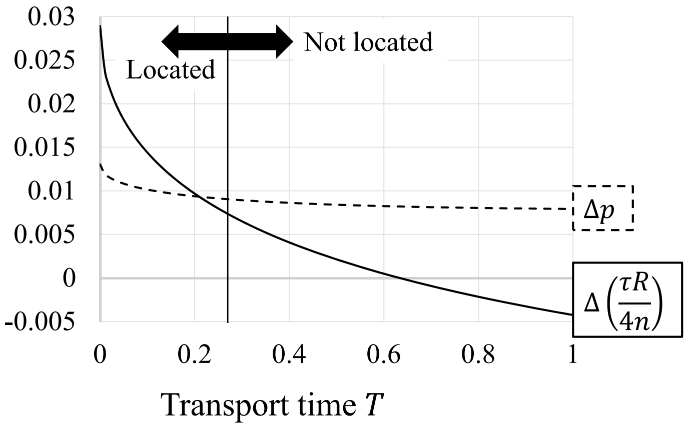



**Fig. 11** Transportation costs and price reductions as function of $T$ associated with the location of DCs

transportation costs contribute to consumer surplus. This property is confirmed by the graph in Fig. 11. This graph shows the reduction in transportation cost per purchase $\tau R/(4n)$ and product price $p$ associated with the location of a DC. The transportation costs are those of an average consumer. These graphs confirm that as $T$ becomes shorter, the savings in shopping transportation costs is the main benefit that DCs bring to consumers. The reason for this result is that, as discussed in Eq. (41), when $T$ is short, chain stores can order products from the DC in smaller quantities, reducing the burden of inventory management costs and enabling them to operate with smaller demand per store.

### 3.5 Policy implications

The results of the above analysis lead to several policy implications regarding the impact of intra-urban road development on the location patterns of DCs and chain stores in the city. First, even if intra-city road improvements contribute only to logistics facilitation, they improve consumer shopping convenience and increase consumer surplus by increasing the number of chain stores in the city. Second, the location of DCs in a city increases the number of chain stores within the city and improves consumer shopping convenience, provided that intra-city transport times are sufficiently short. Therefore, the location of logistics centres in a city leads to an increase in consumer surplus. The necessary conditions to realize such an environment include the development of logistics districts; however, a particularly important condition is suggested to be the shortening of transportation time through the development of intra-city roads. The strategy of consolidating inventory holding risks from downstream chain stores to upstream DCs is adopted by many modern retailers such as supermarkets and convenience stores, and the development of inner-city roads contributes to the establishment of such a strategy. Third, the reorganization of DCs and chain stores within a city due to the construction of intra-city roads can lead to changes in consumers' shopping transportation behaviour. As the number of chain stores increases, consumers will purchase goods closer to where they live, increasing the number of short-distance shopping trips. Thus, changes in logistics might indirectly affect consumers' private trips.

Note that, for the sake of analysis, the present study completely distinguishes between roads used by logistics and roads used by consumers. Although a detailed analysis is omitted for reasons of space, the number of chain stores $n$ determined by the firm decreases when $\tau$, which represents the consumer's shopping transportation cost per unit distance, is reduced in this study's model. This result is easily confirmed by substituting $\tau = 0$ into the $\pi_N$ and $\pi_W$ expressions into Eqs. (16) and (19). When $\tau = 0$, the firm's profit decreases monotonically for $n$; thus, it is optimal for the firm to set the number of chain stores to $n = 1$. In this case, the profit is higher if the DC is not located in the city. The reason for this result is that reducing the number of chain stores and taking advantage of economies of scale is a more efficient approach if consumers can travel to distant locations with low transportation costs. In real cities, transportation infrastructure has been developed to lower consumers' shopping transportation costs. Nevertheless, the strategy of locating a large number of small chain stores is adopted by many retailers. This can reflect an increase in consumers' value of time, in addition to the development of an inner-city environment conducive to the location of DCs in the city.

It is also undisputed that the findings of the present study are based on limited assumptions and are not generally valid. In addition, the study discards the problems of road congestion in the road network. To analyse the effect of the location of logistics centres and the development of intra-city road networks on the efficiency of intra-city logistics in a realistic city, it is essential to conduct an empirical analysis using a hybrid equilibrium model that simultaneously considers both the traffic equilibrium model and the behaviour of intra-city logistics companies. Such an empirical study is left as a subject for future study.

## 4 Conclusion

We modelled the inventory management behaviour of franchise retailers by considering the consolidation of product inventory holding risk by DCs and the increase in lot size. We also analysed the effects of facilitating logistics through the development of intra-urban roads on reorganizing the location patterns of chain stores and DCs in the city and on improving shopping convenience for consumers. The results show that the development of intra-urban roads and the facilitation of intra-urban logistics improve the efficiency of the supply chain of franchise retailers and, under certain conditions, lead to an increase in the number of chain store locations and a reduction in consumers' shopping transportation costs. There are several issues that remain to be addressed in this study. First, the theoretical findings obtained in this study were derived for a circular city. For realistic location patterns of DCs and stores in a city, where the distribution of consumers' residential areas and the shape of the road network are considered, it is essential to conduct an empirical analysis using a hybrid equilibrium model that simultaneously considers the behaviour of logistics companies in the city as well as a traffic equilibrium model. Second, this study assumed a monopolistic market with a single franchise company; however, it is also necessary to analyse oligopolistic markets with multiple franchise companies competing or markets with monopolistic competition in which differentiated stores attempt to jointly transport goods. Third, consumers might change their inventory management in their homes depending on the location of convenience stores. Thus, there is a need for further theoretical and empirical research on inventory management on the consumer side. Fourth, in the entire supply chain from producer to consumer, road development might improve the overall social welfare by concentrating the inventory holding risk in consumers' households and in chain stores and DCs at the upstream of the supply chain. Until now, the benefits of road development on freight transportation efficiency have been measured mainly in terms of driver time and vehicle depreciation costs. Inventory holding costs have also been partially measured in terms of inventory capital costs (interest rates) during transport time; however, these costs are extremely small. In future, it is necessary to understand the breakdown of inventory holding costs ($h$ in this study) and their quantitative magnitude and to apply inventory management models such as the one used in this study to improve the efficiency of inventory management and better understand the benefits related to changes in the location points of DCs. Finally, although this study was conducted for retailers, the analytical framework of this study can be extended to many other areas, such as analysis of the effects of logistics infrastructure (including railroads and ports) on supply chains in the manufacturing industry.

## Appendix A: Error evaluation by Monte Carlo experiments

Equations (9) and (12) for evaluating the safety stock are approximations; however, it is difficult to evaluate their errors analytically. Therefore, we evaluate the approximation error by conducting Monte Carlo experiments with specific parameters. The simulation was conducted for the number of inventory items in a single store. The

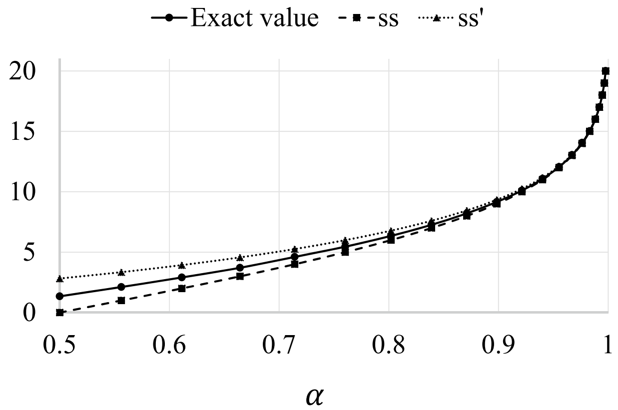



**Fig. 12** Relationship between $\alpha$ and the long-term average of inventory counts (excluding the $Q/2$ part)

demand for goods in this store is assumed to occur according to a Poisson process with a frequency of occurrence of 25 per unit time. That is, the mean demand per unit time is 25 and the standard deviation is 5. Let the lot size of the order $Q$ be 100 and the lead time $L$ be 2. The expected demand during the lead time is 50, which is 50% of the lot size, assuming an environment of high demand uncertainty and inventory turnover. We also assumed that consumers faced with out-of-stock conditions would give up purchasing the product. Under these assumptions, the value of $ROP$ was varied from 50 to 70 in increments of one; for each $ROP$, a one million hour simulation was conducted to calculate the long-term average number of inventory items in the store.

Figure 12 shows a graph of the relationship between the target value $\alpha$ of the probability of no stock-outs during the lead time ($\alpha$ corresponds one-to-one to $ROP$) and the long-term average number of stocks in the store (excluding the part with $Q/2 = 50$). The solid graph shows the values evaluated using the simulation. However, because the number of inventories is always an integer in the simulation and the transition of the number of inventories is staircase-like, there is a difference of 0.5 from the case where the transition of the number of inventories is linear, as in the case of Fig. 4. Therefore, we note that the solid line in Fig. 12 subtracts 0.5 from the long-term average value obtained from the simulation. The same figure shows the relationship between $ss$, $ss'$ and $\alpha$ as dashed lines. Although $ss$ underestimates the long-term average and $ss'$ overestimates it, the error is not large. For example, when $\alpha = 0.898$, the simulation result is 9.15, $ss$ is 9 and the error is 1.6%. When $\alpha = 0.802$, the simulation result is 6.35, $ss$ is 6 and the error is 5.6%.

The fraction of demand that does not face stock-outs, $\gamma$, can also be evaluated using simulations. Figure 13 shows the relationship between $\alpha$ and $\gamma$. This graph shows that, if $\alpha$ is sufficiently large, the approximation error of $\gamma \simeq 1$ is not so large. For example, when $\alpha = 0.898$, $\gamma = 0.996$. Even when $\alpha = 0.802$, $\gamma = 0.992$.

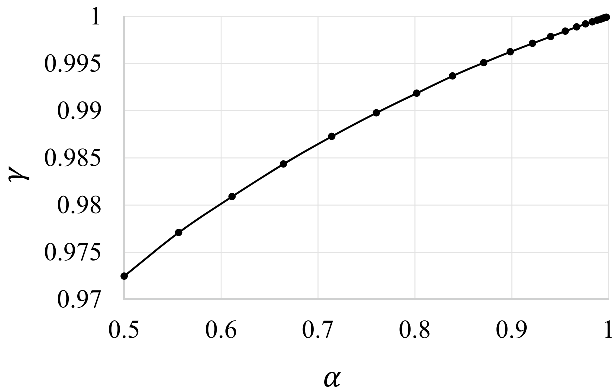



**Fig. 13** Relationship between $\alpha$ and $\gamma$

## Appendix B: Proof of Proposition

The following equation is obtained by the same calculation as in the proof of Proposition 2.

$$\frac{\partial^2 \pi_N}{\partial p^2}\frac{\partial^2 \pi_N}{\partial n \partial T} - \frac{\partial^2 \pi_N}{\partial n \partial p}\frac{\partial^2 \pi_N}{\partial p \partial T} = 2eaRXZ'_W\left[\bar{\lambda} - a\left(p + \frac{\tau R}{8n}\right) - \frac{anX}{2eR}\left(Z_W + \frac{3Z_0}{2\sqrt{n}}\right)\right] \tag{B1a}$$

$$-\frac{\partial^2 \pi_N}{\partial n \partial p}\frac{\partial^2 \pi_N}{\partial n \partial T} + \frac{\partial^2 \pi_N}{\partial n^2}\frac{\partial^2 \pi_N}{\partial p \partial T} = -\frac{ea\tau R^2}{4n^2}XZ'_N\left[\bar{\lambda} - a\left(p + \frac{\tau R}{2n}\right)\right] \cdot \left(1 - \frac{4n^2 XZ_N}{e\tau R^2} - \frac{Z_0}{\sqrt{n}}\frac{8n^2 X}{e\tau R^2}\frac{\bar{\lambda} - ap - \frac{16a\tau R}{7n}}{\bar{\lambda} - ap - \frac{a\tau R}{2n}}\right) \tag{B1b}$$

Note that, in the expansion of the above equation, we use equation (38b) and the following relation:

$$X_n = -\frac{2X^3}{eR}(\bar{\lambda} - ap)$$
$$X_p = -\frac{2X^3}{eR}an\frac{eR}{4nX^2} = \bar{\lambda} - ap - \frac{a\tau R}{4n}$$

When Eq. (40) holds for the optimal solution, the following inequality holds:

$$\frac{anX}{2eR}\left(Z_W + \frac{3Z_0}{2\sqrt{n}}\right) < \frac{a}{8n}\left(\tau R - \frac{Z_0\sqrt{nD}}{nD}\right) \tag{B2a}$$

$$\frac{4n^2 XZ_N}{e\tau R^2} + \frac{Z_0}{\sqrt{n}}\frac{8n^2 X}{e\tau R^2}\frac{\bar{\lambda} - ap - \frac{16a\tau R}{7n}}{\bar{\lambda} - ap - \frac{a\tau R}{2n}} < 1 \tag{B2b}$$

Using the expressions (4), (B2a), (B2b) and $Z'_W > 0$, we can prove Proposition 3.

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