2024

代数幾何学シンポジウム

記録

2024 年度科学研究費補助金 基盤研究 (A)
(課題番号 20H00111　代表 小木曽啓示)
2024 年度科学研究費補助金 基盤研究 (A)
(課題番号 21H04429　代表 並河良典)

於　京都大学理学研究科セミナーハウス
(2024 年 10 月 22 日〜10 月 25 日)

目次

Posters

Kinosaki Algebraic Geometry Symposium 2024 (at Kyoto)

城崎代数幾何学シンポジウム 2024 (京都開催)

October 22nd – 25th, 2024

Location: Kyoto University (North Campus), Science Seminar House (理学研究科セミナーハウス).

Program

	22nd (Tue.)	23rd (Wed.)	24th (Thu.)	25th (Fri.)
10:00–11:00	E. Sato	A.-S. Kaloghiros	A. Kanemitsu	J. Song
11:15–12:15	Y. Sato	N. Takahashi	R. Gotou	M. Tosun
13:45–14:45	H. Akaike		Y. Okuyama	Y. Yasufuku
15:00–16:00	A. Hoshi		J. Li	I. Nakamura
16:00–17:15	Poster Session			

Titles and Abstracts

Eiichi Sato (Kyushu University)

Fano bundles on P^n with fiber bundle structures

I study the structure of Fano bundle E on $Y = P^n$ whose elementary contraction (not a canonical projection) $f : P(E) \to Z$ is of fiber type, moreover, of fiber bundles, though it is expected that there are few examples with such structures. Particularly, I classify Fano bundles on P^n where f has the projective bundle structure. Moreover, I state some results about such bundles in case that Y is not a complex projecitive space.

Yusuke Sato (Osaka Institute of Technology)

Generalized Markov triples and toric surface singularities

A Markov triple is a positive integer solution to the equation $x^2+y^2+z^2 = 3xyz$, which was studied by Markov around 1880. For a Markov triple (a, b, c), the cyclic quotient singularity of type $(a^2, b^2)/c^2$ possesses a T-singularity, making it an important object in the deformation theory of surfaces. As a generalization of the Markov triple, the k-generalized Markov triple, introduced by Gyoda and Matsushita, is a positive integer solution of the equation $x^2+y^2+z^2+k(yz+zx+xy) = (3+3k)xyz$. In this talk, I will introduce the properties of the cyclic quotient singularities obtained from k-generalized Markov triples. This is based on joint work with Yasuaki Gyoda and Shuhei Maruyama.

Hiroto Akaike (Tohoku University)

On the Gonality type invariants and the slope of a fibered 3-fold

For algebraic fibered spaces, a numerical invariant called the slope is defined. In the study of the geography of algebraic fibered spaces, one of the fundamental problems is to clarify the relationship between the geometric properties of the fibers and the lower bound of the slope. For two-dimensional algebraic fibered spaces, i.e., fibered surfaces, slope inequalities have been established for various geometric properties. On the other hand, slope inequalities for fibered 3-folds are less well-known. We focus on slope inequalities in fibered 3-folds. In this study, two birational invariants are focused on. One is an invariant called the covering gonality. The covering gonality is the higher-dimensional generalization of gonality, the classical invariant of projective curves. The other is an invariant newly introduced, called the minimal covering degree. In the talk, I will explain the relationship between the geometric properties of the fibers captured by these two invariants and the slope inequalities of fibered 3-folds.

Akinari Hoshi (Niigata University)

Birational classification for algebraic tori

We give a stably birational classification for algebraic k-tori of dimensions 3 and 4 over a field k. Kunyavskii [Kun90] proved that there exist 15 not stably k-rational cases among 73 cases of algebraic k-tori of dimension 3. Hoshi and Yamasaki [HY17] showed that there exist exactly 487 (resp. 7, resp. 216) stably k-rational (resp. not stably but retract k-rational, resp. not retract k-rational) cases of algebraic k-tori of dimension 4. First, we define the weak stably k-equivalence of algebraic k-tori and show that there exist 13 (resp. 128) weak stably k-equivalent classes of algebraic k-tori T of dimension 3 (resp. 4) which are not stably k-rational by computing some cohomological stably birational invariants, e.g. the Brauer-Grothendieck group of X where X is a smooth k-compactification of T, provided by Kunyavskii, Skorobogatov and Tsfasman [KST89]. We make a procedure to compute such stably birational invariants effectively and the computations are done by using the computer algebra system GAP. Second, we define the p-part of the flabby class $[\widehat{T}]^{fl}$ as a $\mathbb{Z}_p[Sy_p(G)]$-lattice and prove that they are faithful and indecomposable $\mathbb{Z}_p[Sy_p(G)]$-lattices unless it vanishes for $p = 2$ (resp. $p = 2, 3$) in dimension 3 (resp. 4). The $\mathbb{Z}_p$-ranks of them are also given. Third, we give a necessary and sufficient condition for which two not stably k-rational algebraic k-tori T and T' of dimensions 3 (resp. 4) are stably birationally k-equivalent in terms of the splitting fields and the weak stably k-equivalent classes of T and T'. In particular, the splitting fields of them should coincide if $\widehat{T}$ and $\widehat{T}'$ are indecomposable. Forth, for each 7 cases of not stably but retract k-rational algebraic k-tori of dimension 4, we find an algebraic k-torus T' of dimension 4 which satisfies that $T \times_k T'$ is stably k-rational. Finally, we give a criteria to determine whether two algebraic k-tori T and T' of general dimensions are stably birationally k-equivalent when T (resp. T') is stably birationally k-equivalent to some algebraic k-torus T'' of dimension up to 4. This is a joint work with Aiichi Yamasaki (arXiv: 2112.02280).

Anne-Sophie Kaloghiros (Brunel University London)

On a 3-dimensional K-moduli space of Fano 3-folds

I will discuss joint work with Ivan Cheltsov, Maksym Fedorchuk and Kento Fujita. Family 4.1 in the Mori-Mukai classification of Fano 3-folds consists of hypersurfaces of multi degree (1,1,1,1) in $(\mathbb{P}^1)^4$ a product of four copies of $\mathbb{P}^1$. Smooth members of the family are K-polystable and belong to a 3-dimensional component of the K-moduli space of smoothable Fano 3-folds of anticanonical degree 24, which I will describe in this talk. Our description of the K-moduli is informed by the appearance of such Fano 3-folds in surprising contexts: as geometric avatars of entangled states of 4 qubits in quantum computing on the one hand, and as moduli spaces of parabolic vector bundles on elliptic curves on the other.

Nobuyoshi Takahashi (Hiroshima University)

One dimensional sheaves on log Calabi-Yau surfaces

Let S be an algebraic surface and B an anticanonical divisor on S. In studying the enumeration of curves on S with contact conditions with B, we are lead to consider moduli spaces of 1-dimensional sheaves on S with conditions on the restriction to B. I will give a short survey on results on related moduli spaces, and talk about my attempts at understanding the geometry of these spaces.

Akihiro Kanemitsu (Tokyo Metropolitan University)

Mukai pairs and associated K3 surfaces

A Mukai pair (X, E) is a pair of a smooth projective variety X and an ample vector bundle E with $c_1(X) = c_1(E)$. Such pairs are classified when the rank of E is at least $\dim X - 2$. When the rank of E is $\dim X - 2$, Mukai pairs are related to K3 surfaces; the zero locus of a section of E is a K3 surface. In this talk, we discuss the structure of these K3 surfaces from the viewpoint of their moduli.

Rin Gotou (Osaka Institute of Technology)

Combinatorial stratifications of geometric moduli spaces of dynamical systems over $\mathbb{P}^1$

The moduli space of the dynamical systems $\mathrm{rat}_d := \mathrm{Rat}_d \,/\, \mathrm{PGL}_2$ is the quotient of the moduli space of self-rational maps of degree d on $\mathbb{P}^1$ with respect to the conjugation action of $\mathrm{Aut}(\mathbb{P}^1) = \mathrm{PGL}_2$. We consider compactifications of rat_d respecting geometric behavior of dynamical systems. We give VGIT-constructions of compactified moduli spaces and a stratification of moduli spaces using marking-weighted trees, analogous to the moduli spaces $\overline{M}_{0,n}$ of stable rational curves.

Yûsuke Okuyama (Kyoto Institute of Technology)

Minimal resultant locus and its moduli theoretic characterization in non-archimedean dynamics

Let ϕ be a rational function (of degree more than 1) on the projective line $\mathbb{P}^1$ over an algebraically closed and complete non-trivial and non-archimedean valued field K, which is an endomorphism of $\mathbb{P}^1$. The degree of the reduction of ϕ modulo the maximal ideal in the ring of K-integers is less than or equal to that of ϕ, and we say ϕ has a good reduction if the equality holds. A conjugacy of ϕ under some projective transformation of $\mathbb{P}^1$ can have a good reduction even if so does not ϕ. The minimal resultant locus for ϕ is a dynamical equivariant which measures how far ϕ is from having a

good reduction, up to conjugations of it under projective transformations. In this talk, after reviewing the foundational moduli theoretic works by Rumely, Szpiro–Tepper–Williams, Silverman, ... on the minimal resultant locus (to characterize the minimum resultant locus as the potential GIT-semistable locus), we introduce the hyperbolic resultant function for ϕ on the Berkovich projective line over K and the intrinsic depths of the intrinsic reduction of ϕ at each point of the Berkovich projective line. The main result is the moduli theoretic characterization of the minimal resultant locus of ϕ using the Berkovich hyperbolic geometry.

Jennifer Li (Princeton University)

On the cone conjecture for log Calabi-Yau mirrors of Fano 3-folds

Let Y be a smooth projective 3-fold admitting a K3 fibration $f : Y \to \mathbb{P}^1$ with $-K_Y = f^*\mathcal{O}(1)$. We show that the pseudo-automorphism group of Y acts with finitely many orbits on the codimension one faces of the movable cone if $H^3(Y, C) = 0$, confirming a special case of the Kawamata-Morrison-Totaro cone conjecture. In Coates-Corti-Galkin-Kasprzyk 2016, Przyjalkowski 2018, and Cheltsov-Przyjalkowski 2018, the authors construct log Calabi-Yau 3-folds with K3 fibrations satisfying the hypotheses of our theorem as the mirrors of Fano 3-folds.

JuAe Song (Kyoto University)

A geometric interpretation of Krull dimensions of T-algebras

Recently many researches to construct algebraic foundation for tropical geometry appear. One of them is Joó–Mincheva's research on Krull dimensions of B-algebras, where B is the Boolean semifield. They revealed that the tropical Laurent polynomial semiring $T[X_1^{\pm}, ..., X_n^{\pm}]$ in n-variables has Krull dimension n plus one. In this talk, with their technique, we explain that the Krull dimension of $T[X_1^{\pm}, ..., X_n^{\pm}]/C$ for a congruence C having a finite congruence tropical basis and nonempty congruence variety $V(C)$ is $\max\{\dim V(C) + 1, \dim V(C_B)\}$ if C_B also has a finite congruence tropical basis. Here C_B is the congruence obtained from C with the B-algebra homomorphism $T \to B$. This is a joint work with Yasuhito Nakajima.

Meral Tosun (Galatasaray University)

Toric Resolutions of Singularities via Jet Spaces

I will talk about a joint work with B. Karadeniz and C.Plenat. Focusing on hypersurfaces with non-isolated singularities and the finite-dimensional jet spaces of these hypersurfaces, we explore two questions: (1) whether a toric resolution can be constructed using particular irreducible components of the jet spaces, and (2) whether this resolution is minimal. For this, we extend techniques on rational double point singularities in the literature.

Yu Yasufuku (Waseda University)

Asymptotic properties of rational points and intersection theory

In 2020 Ru–Vojta created a birational invariant which controls how close rational points can be

to divisors. Their theory provides an optimal way the famed Diophantine approximation theorem—Schmidt subspace theorem—can be used, and their invariant involves the asymptotic growth of the number of global sections of certain line bundles. The Ru–Vojta theory has been applied to analyze integral points on varieties and GCD's. In this talk, I will go over some of these applications, and then describe some cases for which intersection computations become a key.

Iku Nakamura (Hokkaido University)

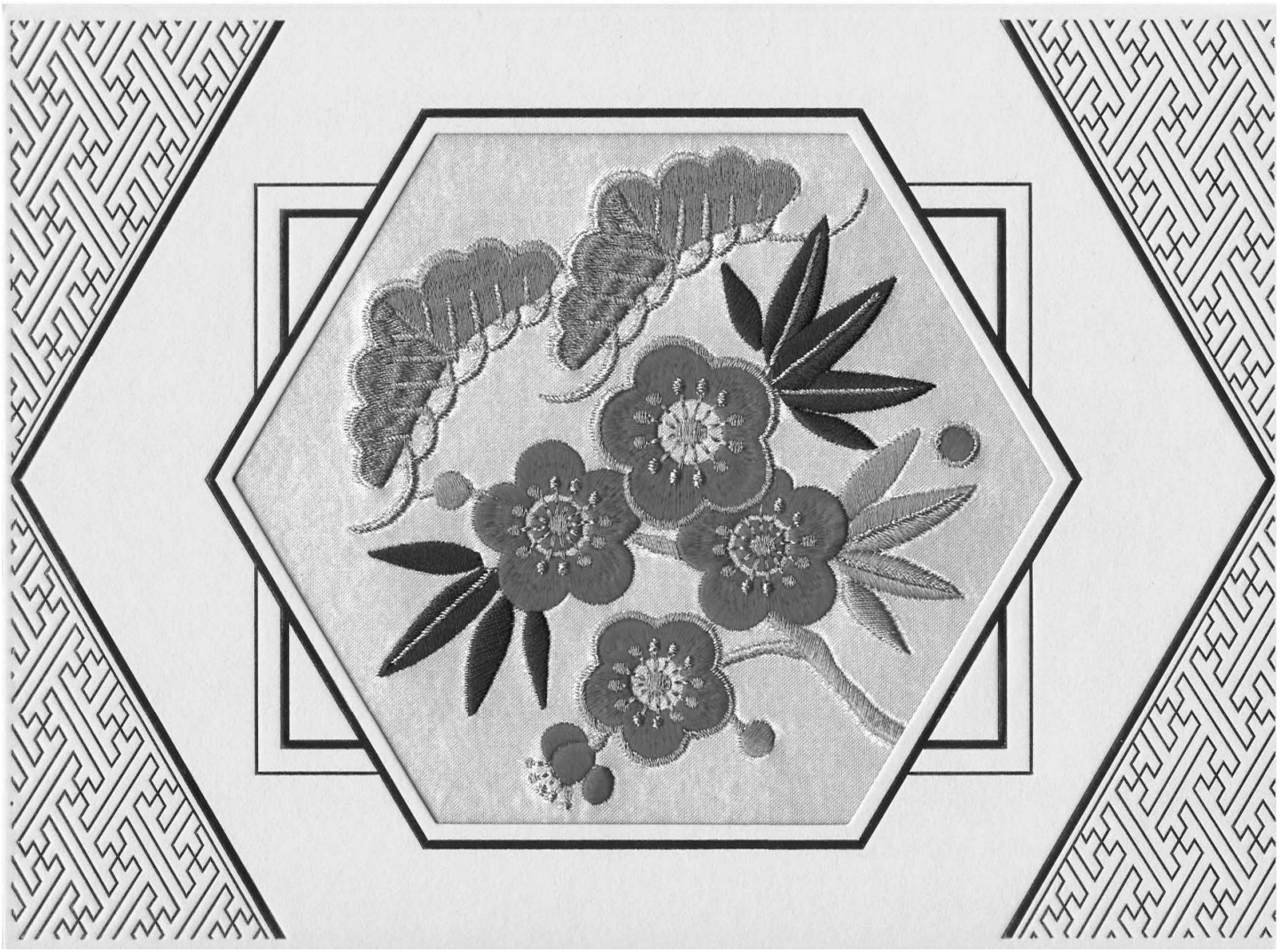
Relative compactification of semiabelian Néron models

(based on a joint work with K. Mitsui in part)

Let R be a complete discrete valuation ring, $k(\eta)$ its fraction field, $S = \operatorname{Spec} R$, $(G_\eta, \mathcal{L}_\eta)$ a polarized abelian variety over $k(\eta)$ with $\mathcal{L}_\eta$ symmetric ample cubical and $\mathcal{G}$ the Néron model of G_η over S. Suppose that $\mathcal{G}$ is semiabelian over S. Then there exists a *unique* relative compactification $(P, \mathcal{N})$ of $\mathcal{G}$ such that (α) P is Cohen-Macaulay with $\operatorname{codim}_P(P \setminus \mathcal{G}) = 2$ and (β) $\mathcal{N}$ is ample invertible with $\mathcal{N}_{|\mathcal{G}}$ cubical and $\mathcal{N}_\eta = \mathcal{L}_\eta^{\otimes n}$ for some positive integer n. The totally degenerate case has been studied in [Mitsui-Nakamura, arXiv:2021.08113]. The partially degenerate case and the case where R is a Dedekind domain are discussed in [Nakamura, arXiv:1406.0174 math.AG]

Organizers: Yohsuke Matsuzawa, Yusuke Nakamura, Kazuhiko Yamaki.
Local organizers: Shu Kawaguchi, Ken-Ichi Yoshikawa.
Supported by

- JSPS KAKENHI Grant-in-Aid (A) 20H00111 (Keiji Oguiso).
- JSPS KAKENHI Grant-in-Aid (A) 21H04429 (Yoshinori Namikawa).



つたや旅館様より頂いたご祝電

お祝い

お届け台紙名　『祝福松竹梅』
お届け日　　２０２４年１０月２１日

城崎代数幾何学シンポジウム２０２４　御中

代数幾何学シンポジウムのご開催を心よりお祝い申し上げます。貴シンポジウムのますますのご発展と、先生方のご多幸、ご健勝をお祈り申し上げます。

兵庫県豊岡市城崎町湯島４８５
城崎温泉つたや旅館（つたや晴嵐亭）
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２０２４ 代数幾何学シンポジウム 記録

発行年月　2025 年 1 月
発行元　京都大学数理解析研究所
〒 606-8502 京都市左京区北白川追分町