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# Inversion algorithm determining sharp boundaries in electrical resistivity tomography

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# ABSTRACT

Blurred resistivity boundaries resulting from smoothness-regularized inversions of electrical resistivity tomography (ERT) data can lead to inaccurate interpretations of sharp

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22 boundary structures. To address this issue, various ERT inversion algorithms have introduced 23 localized adjustments (localized discontinuities) in the regularization operator at positions where 24 sharp boundaries are anticipated. Current approaches rely on prior information about sharp 25 boundary locations, obtained from complementary geophysical, geological, and drilling data, to 26 determine the positions and weights for these regularization adjustments. However, such prior 27 information is frequently insufficient, limiting the application of localized regularization 28 adjustments. Accordingly, we developed a sharp boundary inversion (SBI) algorithm using the 29 Akaike Bayesian Information Criterion (ABIC) that determines the optimal positions and 30 weights for localized regularization adjustments by testing various configurations and selecting the one that minimizes ABIC. A synthetic modeling study demonstrated that the SBI algorithm 31 32 correctly delineated the sharp boundaries of a conductor. Its application to field data 33 demonstrated that it delineated the sharp boundaries of a utility tunnel, and the size and horizontal position of the recovered tunnel were consistent with the estimated dimensions from 34 the blueprint. As it does not rely heavily on prior information, the SBI algorithm can be applied 35 36 to a wide range of geophysical survey data, even when prior knowledge of sharp boundary locations is limited. 37

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INTRODUCTION

Electrical resistivity tomography (ERT), or electrical resistivity surveys, are used to determine subsurface resistivity structures by measuring the electrical potential generated by direct current sources (Revil et al., 2010; Dahlin and Loke, 2018; Binley and Slater, 2020). As resistivity strongly depends on the existence of fluids, clays, and metals, ERT has been used for investigating soil properties (André et al., 2012), groundwater (Asaue et al., 2021), seawater

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intrusion (Misonou et al., 2013), landfills (Chambers et al., 2006), hydrothermal systems (Gresse
et al., 2017), volcanic structures (Soueid Ahmed et al., 2018), landslides (Lapenna et al., 2005),
and cavities (Satitpittakul et al., 2013). Other relevant applications include investigating
geological structures below lakes (Wang et al., 2018), gas hydrates (Goto et al., 2008; Ishizu et
al., 2024a), and massive sulfide deposits in marine environments (Ishizu et al., 2019b).

ERT data are inverted to obtain resistivity structures (Sasaki, 1994; Kemna et al., 2002; Günther et al., 2006). Common ERT inversions find resistivity structures by minimizing the misfit between the measured and modeled data (Amatyakul et al., 2017; Binley and Slater, 2020). As ERT data inversion is an ill-posed problem, regularization terms imposing smoothness constraints are applied to stabilize the solutions (Constable et al., 1987). Smoothness-regularized inversions effectively recover smoothly distributed structures.

However, smoothness-based inversions cannot resolve the sharp resistivity boundaries (Loke et al., 2003). This limitation hinders the investigation of sharp boundary structures such as man-made structures (Hung et al., 2022), faults (Rizzo et al., 2004), and metal deposits (Ishizu et al., 2019b) because the blurred boundaries can lead to mispositioning of these structures. Therefore, recovering sharp resistivity boundaries is essential for accurately assessing the positions and properties of these structures.

Various approaches have been applied to the inversion of ERT and electromagnetic data to recover sharp resistivity boundaries. One approach involves applying localized adjustments (small weight) to the L2-norm smoothness regularization term at positions where sharp boundaries are predicted (Slater and Binley, 2006; Key, 2009). This approach is highly sensitive to the accuracy of prior information about sharp boundary locations (Hermans et al., 2012; Johnson et al., 2012; Zhou et al., 2014). Other inversion approaches use sparse

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regularizations, such as minimum gradient support regularization and L1-norm regularization, to recover sharp resistivity boundaries (Farquharson, 2008; Vignoli et al., 2015; Klose et al., 2022; Deleersnyder et al., 2023). These approaches do not necessarily require prior information on sharp boundary positions; instead, the selection of additional parameters is needed to control the sharpness of the model structure. In this study, we employ localized adjustments to the L2-norm regularization term, as we assume that structures other than those with sharp boundaries exhibit a smooth resistivity distribution.

Previous studies have determined the positions and weights of localized regularization 75 76 adjustments from prior information on sharp boundary locations estimated from other 77 geophysical, geological, and drilling data (Hermans et al., 2012; Johnson et al., 2012; Zhou et al., 2014). For instance, Johnson et al. (2012) used geological data and borehole resistivity data to 78 79 determine the positions and weights of localized regularization adjustments. Doetsch et al. (2012) and Zhou et al. (2014) used subsurface structural information obtained from ground-80 penetrating radar images to determine the positions and weights. However, prior information on 81 82 sharp boundary locations is often inadequate, causing the limited application of this approach. Therefore, an inversion algorithm that determines the correct positions and optimal weights for 83 localized regularization adjustments from insufficient prior information is necessary. 84 Accordingly, this study developed a sharp boundary inversion (SBI) algorithm using the Akaike 85 Bayesian Information Criterion (ABIC) which favors models with higher likelihood and entropy 86 87 (Akaike, 1980) to determine the correct positions and optimal weights of localized regularization adjustments from insufficient prior information. 88

The ABIC has been applied to inversion algorithms of various geophysical data for linear problems, such as geodetic data (Yabuki and Matsu'ura, 1992; Fukahata et al., 2004) and

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91 nonlinear problems, such as Interferometric Synthetic Aperture Radar (Fukahata and Hashimoto, 92 2016), ERT (Uchida, 1993a), magnetotelluric (Uchida, 1993b; Ogawa and Uchida, 1996), and 93 controlled source electromagnetic data (Mitsuhata et al., 2002) to determine the optimal tradeoff 94 parameters of model regularization and data misfit terms. Moreover, to recover sharp boundaries at fixed positions in magnetotelluric inversion, Kimura et al. (2010) used ABIC to determine the 95 96 weights for localized adjustments in the smoothness regularization. Our SBI newly extends the 97 ABIC application to determine both the positions and weights for localized regularization adjustments and the tradeoff parameter. 98

99 This study aimed to develop an SBI algorithm that is applicable even when prior information on sharp boundary locations is limited and demonstrate its effectiveness using 100 101 synthetic and field data. We first describe the SBI algorithm. Next, we used a synthetic dataset 102 from a two-dimensional (2D) ERT survey and demonstrate the effectiveness of the SBI algorithm by comparing the true boundaries of a rectangular anomaly with those determined by 103 104 the SBI algorithm. We applied the SBI algorithm to a field dataset to investigate a utility tunnel 105 with sharp rectangular boundaries. Comparing the size and horizontal location of the tunnel 106 determined by the SBI algorithm with those estimated from the blueprint demonstrated its 107 effectiveness.

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# **METHOD**

We first describe the incorporation of ABIC into an ERT inversion. Then, we explain theSBI algorithm using ABIC.

# Incorporating ABIC into inversion with localized regularization adjustments

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Although previous studies have used ABIC only to determine tradeoff parameters (Uchida, 1993a; Mitsuhata et al., 2002; Fukahata et al., 2004), we incorporated ABIC into an ERT inversion to determine both the positions and weights for the localized regularization adjustments and the tradeoff parameter. We modified an Occam-algorithm inversion code for ERT data developed by Ishizu et al. (2019a), using a finite difference forward solver with rectangular elements, to apply the ABIC.

120 The data misfit term S[m] of the ERT inversion is defined as follows (Constable et al.,
121 1987):

$$S[\mathbf{m}] = \|\mathbf{W}(\mathbf{d} - \mathbf{F}[\mathbf{m}])\|^{2}(1)$$

where **d** is a vector of measured apparent resistivity data, **F**[**m**] is a vector of the forward modeled response from the conductivity model **m**, and **W** represents the data weight matrix with diagonal element equal to  $\mathbf{W}_{ii} = 1/\varepsilon_i$  where  $\varepsilon_i$  is the error level of the *i*th data point. The dimensions of **d** and **m** are *N* and *M*, respectively. We applied natural log transformation to the apparent resistivity data of **d** and the conductivity model of **m** (Vachiratienchai and Siripunvaraporn, 2013).

Obtaining the model by minimizing equation 1 is an ill-posed problem for ERT data. Westabilized the solution of equation 1 with the L2-norm model regularization term

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$$R[\mathbf{m}] = \|\mathbf{Cm}\|^2(2)$$

 $\mathbf{C} = \mathbf{W}_{x}\mathbf{R}_{x} + \mathbf{W}_{z}\mathbf{R}_{z}(3)$ 

132 where **C** is the weighted first-derivative roughness operator described as

where  $\mathbf{R}_x$  and  $\mathbf{R}_z$  are the first-derivative roughness operators in the *x* and *z* directions, respectively, and  $\mathbf{W}_x$  and  $\mathbf{W}_z$  represent weights applied to  $\mathbf{R}_x$  and  $\mathbf{R}_z$ , respectively. Localized regularization adjustments (localized discontinuities) can be applied to recover sharp boundaries

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by setting small weights in  $W_x$  and  $W_z$  at the boundaries (Key, 2009; Johnson et al., 2012). To describe **C**, we considered a five-layered model in the *z*-direction as a simplified representation.  $W_z R_z$  is then expressed as

$$\mathbf{W}_{z}\mathbf{R}_{z} = \begin{bmatrix} w_{z1} & 0 & 0 & 0 & 0 \\ 0 & w_{z2} & 0 & 0 & 0 \\ 0 & 0 & w_{z3} & 0 & 0 \\ 0 & 0 & 0 & w_{z4} & 0 \\ 0 & 0 & 0 & 0 & w_{z5} \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & q \end{bmatrix} (4)$$

141 The values of  $w_{z1}-w_{z5}$  were set to one, except for the positions where localized adjustments were applied. If a localized adjustment was used between the second and third layers, the SBI 142 143 algorithm applies a small value equal to by (localized discontinuity parameter) in  $w_{z2}$ . The regularization term  $R[\mathbf{m}]$ , tuned with an appropriate parameter by, enables the recovery of sharp 144 boundaries. To satisfy the full-rank condition required for ABIC applications (Fukahata, 2012), 145 146 we constrained the minimum value of bv to  $10^{-4}$ . The SBI algorithm determined the optimal bv147 by minimizing ABIC. Another small value of q was applied to the end of the layered model to satisfy the full-rank condition. We set q to  $10^{-2}$  because it satisfies the full-rank condition but 148 149 does not affect inversion results. Although we present the five-layered model to illustrate C, we extended this conceptual framework to a 2D model with dimensions of M. In the 2D application, 150 localized regularization adjustments were performed for both  $\mathbf{W}_x$  and  $\mathbf{W}_z$ . 151

The inversion minimizes the objective function

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$$U[\mathbf{m}] = S[\mathbf{m}] + \lambda R[\mathbf{m}] = \|\mathbf{W}(\mathbf{d} - \mathbf{F}[\mathbf{m}])\|^2 + \lambda \|\mathbf{C}\mathbf{m}\|^2 (5)$$

154 where  $\lambda$  is a tradeoff parameter between data fit and model regularization, referred to as a 155 hyperparameter in the ABIC framework (Akaike, 1980). Alternatively,  $U[\mathbf{m}]$  can be expressed 156 as

$$U[\mathbf{m}] = (\mathbf{d} - \mathbf{F}[\mathbf{m}])^{\mathrm{T}} \mathbf{C}_{d}^{-1} (\mathbf{d} - \mathbf{F}[\mathbf{m}]) + \lambda \mathbf{m}^{\mathrm{T}} \mathbf{C}_{m}^{-1} \mathbf{m}(6)$$

158 where

 $\mathbf{C}_d^{-1} = \mathbf{W}^{\mathrm{T}}\mathbf{W}(7)$ 

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$$\mathbf{C}_m^{-1} = \mathbf{C}^{\mathrm{T}}\mathbf{C} = (\mathbf{W}_x \mathbf{R}_x)^{\mathrm{T}} \mathbf{W}_x \mathbf{R}_x + (\mathbf{W}_z \mathbf{R}_z)^{\mathrm{T}} \mathbf{W}_z \mathbf{R}_z(8)$$

# 162 Owing to the non-linearity of the inversion, we linearized $\mathbf{F}[\mathbf{m}]$ about a model at the *k*th iteration

163  $(\mathbf{m}_k)$  such that

$$\mathbf{F}[\mathbf{m}_{k+1}] = \mathbf{F}[\mathbf{m}_k] + \mathbf{J}_k(\mathbf{m}_{k+1} - \mathbf{m}_k)(9)$$

where  $J_k$  is the  $N \times M$  Jacobian matrix. Using equation 9, the linearized objective function  $U^L$  is expressed as

 $U^{\mathrm{L}}[\mathbf{m}_{k+1}] = \left\| \mathbf{W} (\hat{\mathbf{d}}_k - \mathbf{J}_k \mathbf{m}_{k+1}) \right\|^2 + \lambda \|\mathbf{C}\mathbf{m}_{k+1}\|^2 (10)$ 

168 where

$$\hat{\mathbf{d}}_k = \mathbf{d} - \mathbf{F}[\mathbf{m}_k] + \mathbf{J}_k \mathbf{m}_k(11)$$

170 Alternatively,  $U^{L}[\mathbf{m}_{k+1}]$  can be written as

$$U^{\mathrm{L}}[\mathbf{m}_{k+1}] = \left(\hat{\mathbf{d}}_{k} - \mathbf{J}_{k}\mathbf{m}_{k+1}\right)^{\mathrm{T}}\mathbf{C}_{d}^{-1}\left(\hat{\mathbf{d}}_{k} - \mathbf{J}_{k}\mathbf{m}_{k+1}\right) + \lambda \mathbf{m}_{k+1}^{\mathrm{T}}\mathbf{C}_{m}^{-1}\mathbf{m}_{k+1}(12)$$

172 To minimize equation 12 with a fixed  $\lambda$ , the derivative of  $U^{L}$  for  $\mathbf{m}_{k+1}$  was set to zero, solving 173 the following equation to obtain the optimal solution  $\mathbf{m}_{k+1}^{*}$  such that

174 
$$\left[\lambda \mathbf{C}_m^{-1} + \mathbf{J}_k^{\mathrm{T}} \mathbf{C}_d^{-1} \mathbf{J}_k\right] \mathbf{m}_{k+1}^* = \mathbf{J}_k^{\mathrm{T}} \mathbf{C}_d^{-1} \hat{\mathbf{d}}_k (13)$$

Now, we introduce the ABIC based on the Bayes' theorem (Akaike, 1980). Given that the observed data contain Gaussian noise and relative values of the data covariance matrix  $C_d$  are available, the probability density function of the data with multidimensional normal distribution is written as

$$p(\mathbf{d}|\mathbf{m}_{k+1}) = (2\pi\sigma_0^2)^{-N/2} |\mathbf{C}_d^{-1}|^{1/2} \exp\left(-\frac{1}{2\sigma_0^2} (\hat{\mathbf{d}}_k - \mathbf{J}_k \mathbf{m}_{k+1})^{\mathrm{T}} \mathbf{C}_d^{-1} (\hat{\mathbf{d}}_k - \mathbf{J}_k \mathbf{m}_{k+1})\right) (14)$$

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180 where  $\sigma_0^2$  is correction factor of  $\mathbf{C}_d$  (Mitsuhata et al., 2002; Fukahata et al., 2004). The 181 probability density function of the *a priori* information on the model parameters is also 182 expressed as

$$p(\mathbf{m}_{k+1}) = (2\pi\rho^2)^{-M/2} |\mathbf{C}_m^{-1}|^{1/2} \exp\left(-\frac{1}{2\rho^2} \mathbf{m}_{k+1}^{\mathrm{T}} \mathbf{C}_m^{-1} \mathbf{m}_{k+1}\right) (15)$$

184 where  $\rho^2$  represent variances of the *a priori* information. Using Bayes' theorem, the posterior 185 distribution  $p(\mathbf{m}_{k+1}|\mathbf{d})$  can be formulated as (Yabuki and Matsu'ura, 1992)

186 
$$p(\mathbf{m}_{k+1}|\mathbf{d}) \propto p(\mathbf{d}|\mathbf{m}_{k+1})p(\mathbf{m}_{k+1}) = (2\pi\sigma_0^2)^{-N/2} (2\pi\sigma_0^2)^{-M/2} (\lambda)^{M/2} |\mathbf{C}_d^{-1}|^{1/2} |\mathbf{C}_m^{-1}|^{1/2} \exp\left(-\frac{U^{\mathrm{L}}[\mathbf{m}_{k+1}]}{2\sigma_0^2}\right) (16)$$

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$$\lambda = \frac{\sigma_0^2}{\rho^2} (17)$$

189 ABIC is defined as

 $ABIC = -2\log L + 2N_{\rm P}(18)$ 

191 where *L* is the marginal likelihood and  $N_P$  is the number of hyperparameters (Akaike, 1980). In 192 conventional smoothing inversion,  $\lambda$  is the only hyperparameter ( $N_P = 1$ ). In contrast, the number 193 of hyperparameters for the SBI algorithm varies depending on the number of boundaries for 194 localized regularization adjustments because their positions and weights are themselves 195 hyperparameters determined by ABIC minimization. If the SBI algorithm determines four 196 boundary positions (top, bottom, left, and right sides of a rectangle), their weight (*bv*), and  $\lambda$ ,  $N_P$ 197 = 6. Following Yabuki and Matsu'ura (1992), *L* is expressed as

$$L = \int p(\mathbf{d}|\mathbf{m}_{k+1})p(\mathbf{m}_{k+1})d\mathbf{m}_{k+1} = (2\pi\sigma_0^2)^{-N/2} (\lambda)^{M/2} |\mathbf{C}_d^{-1}|^{1/2} |\mathbf{C}_m^{-1}|^{1/2} |\mathbf{J}_k^{\mathsf{T}} \mathbf{C}_d^{-1} \mathbf{J}_k + \lambda \mathbf{C}_m^{-1}|^{-1/2} \times \exp\left(-\frac{U^{\mathsf{L}}[\mathbf{m}_{k+1}^*])}{2\sigma_0^2}\right).$$
(19)

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199 To minimize ABIC, we maximize *L*, which is a function of  $\sigma_0^2$ ,  $\lambda$  and  $\mathbf{C}_m^{-1}$ . The maximum *L* for 200  $\sigma_0^2$  is obtained as  $\sigma_0^2 = \frac{U^{L}[\mathbf{m}_{k+1}^*]}{N}$  by solving  $\frac{\partial L}{\partial \sigma_0^2} = 0$ . The equations for  $\frac{\partial L}{\partial \lambda} = 0$  and  $\frac{\partial L}{\partial \mathbf{C}_m^{-1}} = 0$  cannot 201 be solved analytically. We maximize *L* for  $\lambda$  and  $\mathbf{C}_m^{-1}$  using a search algorithm described in the 202 next section. By substituting  $\sigma_0^2 = \frac{U^{L}[\mathbf{m}_{k+1}^*]}{N}$  to equation 19, *L* is expressed as

203 
$$L = \left(2\pi \frac{U^{L}[\mathbf{m}_{k+1}^{*}]}{N}\right)^{-N/2} (\lambda)^{M/2} |\mathbf{C}_{d}^{-1}|^{1/2} |\mathbf{C}_{m}^{-1}|^{1/2} |\mathbf{J}_{k}^{T}\mathbf{C}_{d}^{-1}\mathbf{J}_{k} + \lambda \mathbf{C}_{m}^{-1}|^{-1/2} \exp\left(-\frac{N}{2}\right) (20)$$

204 Consequently, the linearized ABIC version (ABIC<sup>L</sup>) is defined as

ABIC<sup>L</sup> $(\lambda, \mathbf{C}_m^{-1}) = N \log (U^L [\mathbf{m}_{k+1}^*]) - M \log \lambda - \log |\mathbf{C}_m^{-1}| + \log |\mathbf{J}_k^T \mathbf{C}_d^{-1} \mathbf{J}_k + \lambda \mathbf{C}_m^{-1}| + 2N_P + Q(21)$ where *Q* is independent of  $\lambda$  and  $\mathbf{C}_m^{-1}$ . Instead of ABIC<sup>L</sup>, we used the quasi-linearized ABIC version of ABIC<sup>QL</sup> (from now on simply called ABIC) to mitigate linearization errors (Mitsuhata et al., 2002) expressed as

209 
$$\operatorname{ABIC}^{\operatorname{QL}}(\lambda, \mathbf{C}_m^{-1}) = N \log(U[\mathbf{m}_{k+1}^*]) - M \log \lambda - \log|\mathbf{C}_m^{-1}| + \log|\mathbf{J}_k^{\mathrm{T}} \mathbf{C}_d^{-1} \mathbf{J}_k + \lambda \mathbf{C}_m^{-1}| + 2N_{\mathrm{P}} + Q_{\mathrm{P}}(22)$$

210 and  $U[\mathbf{m}_{k+1}^*]$  is computed as

$$U[\mathbf{m}_{k+1}^*] = (\mathbf{d} - \mathbf{F}[\mathbf{m}_{k+1}^*])^{\mathrm{T}} \mathbf{C}_d^{-1} (\mathbf{d} - \mathbf{F}[\mathbf{m}_{k+1}^*]) + \lambda \mathbf{m}_{k+1}^*^{\mathrm{T}} \mathbf{C}_m^{-1} \mathbf{m}_{k+1}^* (23)$$

ABIC in the SBI algorithm depends on  $\lambda$  and  $\mathbf{C}_m^{-1}$  (positions and weights for localized adjustments in the regularization operator), whereas ABIC used in the previous studies depended only on  $\lambda$  (Uchida, 1993a; Mitsuhata et al., 2002).

To evaluate the inversion performance, we considered an L1-norm misfit between the inverted and true models as follows:

Model misfit = 
$$\sum_{i=1}^{M} |m_i - m_i^{true}| (24)$$

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where  $m_i$  and  $m_i^{true}$  are, respectively, the *i*th conductivity parameters of the inverted and true models. The root-mean-square (rms) data misfit is defined to measure the fit of the model response to the observed data

rms data misfit = 
$$\sqrt{\frac{(\mathbf{d} - \mathbf{F}[\mathbf{m}])^{\mathrm{T}}\mathbf{C}_{\mathbf{d}}^{-1}(\mathbf{d} - \mathbf{F}[\mathbf{m}])}{N}}.(25)$$

223 Algorithm to delineate sharp boundaries using ABIC

The SBI algorithm determined the optimal positions and weights for localized regularization adjustments, assuming *a priori* information about the target structure: a rectangular shape but unknown size or depth. It is summarized as follows (Figure 1). First, we implemented an inversion with conventional smoothness regularization (Flow 1) and located the initial sharp boundaries based on the change in the first derivative of the smooth model (Flow 2). It is computed by applying  $\mathbf{R}_x$  and  $\mathbf{R}_z$  to the conductivity model of **m**. The initial sharp boundaries were assigned at positions where the roughness exhibited high values.

In Flow 3, we conducted inversion analyses with localized regularization adjustments and 231 232 searched for the optimal left and right boundaries by minimizing ABIC through a grid search of 233 the boundary positions, while the top and bottom of the sharp boundaries were fixed to the initial 234 locations determined in Flow 2. Simultaneously, we determined an optimal by by trying various values, ranging from  $10^{-4}$  to one (Flow 3). The same by value was assigned for all the localized 235 236 regularization adjustments. In Flow 4, the optimal top and bottom boundaries and bv were 237 determined, fixing the left and right boundaries determined in Flow 3. Finally, the model with 238 the minimum ABIC in Flow 4 was selected as optimal. Note that in each inversion run, we also

searched for the optimal  $\lambda$  that minimized ABIC using forty log10-scaled trial  $\lambda$  values with a 0.1 spacing.

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# **RESULTS AND DISCUSSION**

# 243 Synthetic modeling study for a rectangular structure

A synthetic ERT dataset was generated using the test model (Figure 2). The survey 244 245 employed a dipole-dipole configuration with a 2 m electrode spacing over a 54-m long profile on 246 a flat ground, resulting in N = 172 apparent resistivity data points. The test model included a 10-247  $\Omega$ m conductor with sharp rectangular boundaries (x = 20–26 m and z = 1.5–6 m) embedded in a 248 100- $\Omega$ m homogeneous half-space structure. The apparent resistivity data were computed via 249 forward modeling, and 2% Gaussian noise was added. A 2% error level was applied to all data, namely  $\varepsilon_i = 0.02 \times |d_i|$  for i = 1, ..., N. The model was discretized into  $74 \times 21$  cells in the x-250 251 and z-directions, resulting in M = 1554. A grid spacing of 1 m was used horizontally (x-direction: 252 0 to 54 m). Vertically (z-direction), a finer spacing of 0.5 m was used for the upper 2 m, and a 253 coarser spacing of 1 m was used for depths between 2 and 10 m. The grid size was gradually 254 increased to the boundary positions in the x- and z-directions. A 100- $\Omega$ m initial model was used 255 for all synthetic data inversions. The data and model settings were chosen based on existing field 256 data.

The SBI algorithm started with a conventional smoothing inversion (no localized regularization adjustments). The smoothing inversion converged after four iterations and recovered a conductor with blurred outlines (Flow 1; Figure 3a). The ABIC value in equation 22 and rms data misfit in equation 25 for the smooth model were 1322.5 and 0.73, respectively. Initial boundaries were suggested to be located at x = 20 m, x = 26 m, z = 1.5 m, and z = 5 m

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based on the maximum rate of change in the first derivative of the smooth model, while the true boundaries were x = 20, x = 26 m, z = 1.5, and z = 6 m (Flow 2; Figures 3b and 3c). The initial boundaries were quite close to the true model boundaries. To enhance the realism of the test, we used slightly shifted initial boundaries of x = 19, x = 27 m, z = 1 m, and z = 7 m.

266 In Flow 3, we first implemented the inversion using localized regularization adjustments for the initial locations of x = 19 m, x = 27 m, z = 1 m, and z = 7 m. The inversion 267 268 with  $bv = 10^{-2}$  vielded an ABIC value of 1270.7 and a rms data misfit of 0.83 (Figure 4a). Although the ABIC was lower than that of the smoothness inversion, the outline of the recovered 269 270 conductor was still blurred because of incorrect positions for the localized discontinuity. In the 271 Flow 3, we estimated the correct left and right boundaries (x = 20 m and x = 26 m) with  $bv = 10^{-10}$ <sup>2</sup> (Figure 4b) and obtained a model with an ABIC of 1186.5 and a rms data misfit of 0.88. After 272 273 fixing the left and right boundaries at x = 20 m and x = 26 m based on Flow 3, we optimized the 274 top and bottom boundaries to minimize the ABIC value (Flow 4). The inversion with the correct top and bottom boundaries (z = 1.5 m and z = 6 m) with  $bv = 10^{-3}$  obtained the optimal model 275 276 with the minimum ABIC value of 978.3 and a rms data misfit of 1.04 (Figure 4c). The model misfit in equation 24 for the optimal model was 9.2, significantly lower than the conventional 277 smoothness inversion result of 164.2. These results demonstrate that the SBI algorithm 278 279 effectively determines the correct positions and appropriate weights for localized regularization adjustments and delineates sharp boundaries. 280

The inversion results revealed several features. The optimal *bv* value was found to be 10<sup>-3</sup>, but the ABIC value was relatively insensitive to *bv* within the range of 10<sup>-2</sup> to 10<sup>-4</sup>, indicating that a localized search for *bv* could be avoided (Figure A1). The lowest ABIC values were obtained with the largest  $\lambda$  (Figures 4 and A1), indicating that the model regularization with

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285 the given localized discontinuities (Figure 4c) is strongly consistent with the true model 286 consisting of two homogeneous domains of 10  $\Omega$ m and 100  $\Omega$ m, except for the sharp boundaries. 287 Figure 5 presents cross-plots of ABIC as a function of the left boundary positions, rms misfits,  $\lambda$ . 288 and by. The smallest ABIC was achieved at the true left boundary position x = 20 m, with the 289 largest  $\lambda$  and largest rms misfit (Figure 5a and 5b), and the smallest by (Figure 5c). As the 290 boundary position deviated from the true position,  $\lambda$  diminished and the rms misfit slightly 291 decreased, while by increased, resulting in a larger ABIC value. The inversion utilizing the 292 correct boundary prioritized the regularization term over the data misfit term, resulting in the 293 smallest ABIC value with the largest  $\lambda$ , the largest rms misfit, and the smallest by. Figure 6 294 illustrates the variation of the ABIC and rms misfit with respect to the left boundary location,  $\lambda$ , 295 and by. For each inversion result, the  $\lambda$  minimizing the ABIC exceeds that minimizing the rms 296 misfit. This difference is especially pronounced for the correct left boundary of 20 m with bv = $10^{-3}$  (Figure 6d). Additionally, with the correct boundary location, the rms misfit increases 297 298 gradually with  $\lambda$  (Figure 6d), reflecting the consistency between the model regularization and the 299 inverted model.

In the above inversions, the model discretization for inversion perfectly matched the 300 301 outlines of the true conductor; however, discretization errors commonly occur. The SBI 302 algorithm was applied to a scenario where the vertical grid closest to the true top boundary at z =303 1.5 m was shifted between 1.1 m and 1.9 m. The model discretization for the left, right, and 304 bottom boundaries remained consistent with those of the true conductor. The SBI algorithm 305 produced models with ABIC values of 1124.8 and 1136.8 for the vertical grids of 1.2 m and 1.8 306 m, respectively (Figure 7a and 7b). ABIC values increased as the grid deviated from 1.5 m 307 (Figure 7c), resulting in a minimum ABIC of 978.3 for the correct vertical grid of 1.5 m (Figure

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4c). This finding indicates that the SBI algorithm can also optimize the model discretization forsharp boundaries.

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# Application to field data with a rectangular structure

A field ERT dataset was collected along a flat promenade on a Kyoto University 312 313 campus using a Super Sting R8 instrument (Advanced Geosciences Inc., Texas, USA) on 314 October 26, 2017. During the ERT survey, we used a dipole-dipole configuration with 2 m electrode spacing over a 54-m profile, obtaining N = 172 data points (Figure 8). The target was a 315 316 utility tunnel with sharp rectangular boundaries. Its size and horizontal position were determined 317 from an existing blueprint, with a horizontal width of 3.5 m, vertical thickness of 4.4 m, and a center at x = 14 m. The blueprint also indicates a depth of less than 4.5 m for the tunnel's top, but 318 319 the exact depth remains unknown. The tunnel frame, made of reinforced concrete, exhibits lower 320 resistivity than the surrounding soil. In contrast, a cavity with effective infinite resistivity is 321 present within the tunnel. Borehole surveys conducted near the ERT profile indicated that the 322 survey area contains sand and clay layers from the surface to a depth of 4.6 m (Kyoto Prefecture Office, 2005). Sedimentary rocks primarily made of shale are distributed below 4.6 m (Kyoto 323 324 Prefecture Office, 2005). The weather was sunny during the survey, with precipitation occurring 325 before the survey date. The Automated Meteorological Data Acquisition System reported an 326 accumulated precipitation of 182 mm in Kyoto over the five days preceding the survey. These 327 meteorological conditions resulted in elevated moisture levels near the surface, lowering the 328 resistivity than what is typically observed during dry intervals.

329 The observed apparent resistivity data revealed a low resistivity zone of 10–40  $\Omega$ m 330 between x = 10 and x = 20 m (Figure 9a). This low resistivity zone corresponded to the

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331 conductive feature associated with the tunnel. A 100- $\Omega$ m initial model was used for all field data 332 inversions. A 2-% error level was applied to all data. Model discretization was the same as that 333 used in the earlier synthetic model.

334 The SBI algorithm started with the smoothing inversion and converged after three 335 iterations, recovering a conductor with blurred outlines, an ABIC value of 1488.2, and a rms data misfit equal to 1.76 (Flow 1; Figure 9b). The recovered conductor was consistent with the low 336 337 apparent resistivity data. The change in the first derivative of the smooth model provided an initial location of sharp boundaries as a rectangular shape at x = 11 m, x = 17 m, z = 1.5 m, and z 338 339 = 7 m (Flow 2; Figure 9c and 9d). In Flow 3, we first implemented the inversion using localized regularization adjustments for the initial location of x = 11 m and x = 17 m; z = 1.5 m and z = 7340 m; the inversion with  $bv = 10^{-1}$  obtained an ABIC of 1471.1 and rms data misfit of 1.82 (Figure 341 342 10a). Then, Flow 3 specified the left and right boundaries as x = 12 m and x = 16 m with  $bv = 10^{-10}$ <sup>1</sup>, fixing the top and bottom at the initial location as z = 1.5 m and z = 7 m (Figure 10b), the 343 344 ABIC was 1461.1, and a rms data misfit was 1.78. While the left and right boundaries were fixed 345 at x = 12 m and x = 16 m, Flow 4 specified the top and bottom of the sharp boundaries of z = 2 m and z = 7 m, respectively, with a minimum ABIC of 1449.8 and rms data misfit of 1.80 (Figure 346 347 10c).

The size of the tunnel determined by the SBI algorithm (Figure 10c; horizontal width of 4 m, x = 12-16 m; vertical thickness of 5 m, z = 2-7 m) was consistent with the estimated size based on the blueprint (horizontal width of 3.5 m; vertical thickness of 4.4 m). The horizontal location determined using the SBI algorithm (x = 12 and x = 16 m) aligned with the estimated location (x = 12.25 and x = 15.75 m). Furthermore, the resistivity values of the tunnel shown in Figure 10c demonstrated greater uniformity than those presented in Figures 9b and 10a–10b.

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354 Given that the tunnel is an engineered structure, it is expected to display a uniform resistivity 355 value. Therefore, the tunnel resistivity values in Figure 10c are deemed accurate in reflecting this 356 characteristic. The cavity inside the tunnel exhibits an infinite resistivity value. However, the 357 influence of the cavity on the ERT data was masked by the surrounding low-resistivity 358 framework owing to the limited resolution of the ERT data. Consequently, the tunnel was 359 imaged as an entity with overall low resistivity. These results proved that the developed SBI 360 algorithm effectively delineated the sharp boundaries of the tunnel. We infer that the slight size deviation resulted from the survey line being aligned diagonally to the tunnel. 361

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# Synthetic modeling study for non-rectangular structures

364 The SBI algorithm was applied to a rectangular target, assuming that the target shape was rectangular. However, prior information on the target shape is often unavailable. In this 365 366 section, we applied the SBI algorithm to a synthetic dataset generated from a model containing 367 triangular structures, assuming that the structures were rectangular. The purpose of this test was 368 to demonstrate the limitations of the current approach and its potential for broader applicability 369 to complex structures. The test model included two 10  $\Omega$ m triangular structures with upward and 370 downward orientations (A1 and A2), embedded in a 100- $\Omega$ m homogeneous half-space structure 371 (Figure 11a). The triangular structures had a 9-m base, and were 5 m in height and buried at 1 m 372 depth at the top. The survey array, noise level, and model discretization were similar to those 373 used in the previous two models.

The SBI algorithm started with a smoothing inversion for the synthetic data. The inversion produced a smooth model with an ABIC value of 1409.0 and a rms data misfit of 0.72 (Figure 11b). By assuming that the conductors were rectangular with sharp boundaries, we

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identified initial sharp boundaries based on the high derivatives of the smooth model. Initial sharp boundary locations are selected at x = 13 m, x = 16 m, z = 1.5 m, and z = 7 for A1, and x = 3730 m, x = 39 m, z = 1 m, and z = 4 m for A2 (light blue lines in Figure 11c and 11d).

380 The SBI algorithm applied to A1 with  $bv = 10^{-1}$  yielded a model with an ABIC value 381 equal to 1394.4 and a rms data misfit of 0.74 after Flow 4 (Figure 12a). The SBI algorithm applied to A2 with  $bv = 10^{-1}$  resulted in a model with an ABIC of 1366.2 and rms data misfit of 382 383 0.77 after Flow 4 (Figure 12b). The SBI algorithm applied to A1 and A2 with  $bv = 10^{-2}$  produced 384 a model with an ABIC of 1277.2 and rms data misfit of 0.95 after Flow 4 (Figure 12c). The 385 boundaries of A1 and A2 were not correctly recovered in these models (Figure 12a-12c) except 386 for the top boundary of A2. This inaccuracy stemmed from the algorithm's incorrect assumption 387 of rectangular conductor geometries, whereas the true conductors are triangular.

In contrast, the inversion with correct localized discontinuities and  $bv = 10^{-2}$  yielded a model with an ABIC of 1046.3 and a rms data misfit of 0.94 (Figure 12d). The significantly lower ABIC for the correct boundaries suggests that the development of a more advanced search method for finding the optimal boundaries based on ABIC minimization could enable the identification of the complex sharp boundaries, without prior information on the shape and locations of sharp boundaries.

# **Future directions for the SBI algorithm**

Determining the correct positions and optimal weights for localized regularization adjustments remains challenging due to errors and limitations in prior information obtained from geophysical, geological, and drilling data (Hermans et al., 2012; Johnson et al., 2012; Zhou et al., 2014). In contrast, the SBI algorithm determines the correct positions and optimal weights for the

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400 localized regularization adjustments by searching to minimize ABIC. Therefore, it facilitates the 401 application of localized discontinuities in scenarios where prior information on sharp boundary 402 locations is limited, unlike existing methods that determine the positions and weights of localized 403 discontinuities based on prior information.

404 The SBI algorithm was applied to rectangular targets, assuming that the target shape was 405 rectangular. However, target shapes can be more complex than rectangles, and prior information 406 on their shape is often unavailable. One approach for recovering sharp boundaries of a complex 407 structure without prior information is to combine sparse regularization inversions with the SBI 408 algorithm. Sparse inversions require additional parameters that control the sharpness of the 409 model structure and the inversions using different parameter values yield different sharp boundary representations of a target structure. The SBI algorithm may reproduce complex 410 411 boundaries by using boundary representations obtained from the sparse inversions as initial values in the ABIC search. Moreover, modeling techniques using unstructured grids (Günther et 412 413 al., 2006; Ren and Tang, 2010; Ishizu et al., 2019b; Jahandari et al., 2023) are effective in 414 accurately representing complex structures. Future research directions include incorporating an 415 advanced modeling technique into the SBI algorithm and recovering complex sharp boundaries.

Smoothing inversions of other geophysical data such as gravity, magnetic, and electromagnetic data also cannot recover sharp boundaries. We expect the SBI algorithm to be applicable to other geophysical data inversions to delineate sharp boundaries. Existing inversion codes can implement the SBI algorithm by simple modifications such as adding localized weight adjustments to the regularization and computation of ABIC. For example, Ishizu et al. (2022, 2024b) delineated deep-sea massive sulfides using smoothing inversions of controlled source electromagnetic data. However, the blurred boundaries could result in an incorrect evaluation of

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their size. Controlled source electromagnetic inversions employing the SBI algorithm areanticipated to delineate sharp boundaries of massive sulfide deposits.

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# CONCLUSION

We developed an SBI algorithm using ABIC to delineate sharp resistivity boundaries 427 428 of subsurface structures. The delineation of sharp boundaries is imperative for the precise 429 evaluation of the location and attributes of sharp boundary structures. The developed SBI 430 algorithm used localized regularization adjustments and determined the correct positions and optimal weights for these regularization adjustments by searching to minimize ABIC. A 431 synthetic modeling study demonstrated that the SBI algorithm correctly delineated the sharp 432 433 boundaries of a conductor. The ABIC was smallest when the positions of localized regularization 434 adjustments were set correctly. In contrast, traditional smoothing inversion recovered the 435 conductor with blurred outlines. The rms data misfits were 0.73 and 1.04 and final  $\lambda$  were 2.5 436 and 5011.8 for the smooth model and the model obtained by the developed algorithm, 437 respectively. This indicates that ABIC with correctly placed localized discontinuities achieved 438 through regularization adjustment prioritized the regularization term over the data misfit term. The application to field data demonstrated that the SBI algorithm with the smallest ABIC 439 delineated the sharp boundaries of a utility tunnel. The size and horizontal position of the 440 441 recovered tunnel were consistent with the estimated size and horizontal position based on the 442 blueprint. The synthetic and field data studies prove that the developed SBI algorithm effectively 443 delineates sharp boundaries that are difficult to recover using traditional smoothing inversion algorithms. 444

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The SBI algorithm can be used for various geophysical survey data, as it is applicable even when prior information on sharp boundary locations is limited, unlike existing methods that determine the positions and weights of localized regularization adjustments based on prior information. Future research directions include incorporating additional modeling techniques, such as finite element modeling, into the SBI algorithm and applying these additional modeling techniques to delineate more complex sharp boundaries.

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# **LIST OF FIGURES**

Figure 1. Schematic diagram of the developed SBI algorithm using ABIC minimization to
 determine correct positions and optimal weights for localized regularization adjustments to
 delineate sharp boundaries.

Figure 2. Test model for generating synthetic datasets. A 10-Ωm conductor with sharp
boundaries is embedded in a 100-Ωm homogeneous half-space structure. The conductor shape
is a rectangle of 20–26 m and 1.5–6 m for the *x*- and *z*-directions, respectively. Magenta lines
represent the outlines of the conductor. The vertical exaggeration is 2.

Figure 3. Smooth model and its roughness. (a) Inverted model of synthetic data obtained using smoothness regularization. Magenta lines represent the true outline of the conductor (x = 20 m and x = 26 m; z = 1.5 m and z = 6 m). (b) and (c) show the absolute of the roughness of the smooth model in (a) for the *x*- and *z*-directions, respectively. Red lines indicate our initial sharp boundary location (x = 19 m and x = 27 m; z = 1 m and z = 7 m).

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Figure 4. Inverted models of synthetic data obtained from the SBI algorithm. (a) Inverted model with localized regularization adjustments for x = 19 m and x = 27 m; z = 1 m and z = 7 m (initial location). (b) Inverted model with localized regularization adjustments for x = 20 m and x = 26 m; z = 1 m and z = 7 m (Flow 3). (c) Inverted model with localized regularization adjustments for x = 20 m and x = 26 m; z = 1 m and z = 7 m (Flow 3). (c) Inverted model with localized regularization adjustments for x = 20 m and x = 26 m; z = 1.5 m and z = 6 m (Flow 4; optimal model). Magenta lines represent the true outline of the conductor. The weight  $bv = 10^{-2}$  is assigned for (a) and (b);  $bv = 10^{-3}$  is assigned for (c).

Figure 5. Cross-plots of ABIC as a function of the left boundary discontinuity location for regularization adjustments, (a)  $\lambda$ , (b) rms misfit, and (c) *bv*. Localized regularization adjustments are also applied for true positions for right, top, and bottom boundaries.

Figure 6. Plots of the ABIC and rms misfit as functions of the left boundary discontinuity 632 633 location for regularization adjustments,  $\lambda$ , and by. (a) Smoothness inversion result, and (b), (c), and (d) SBI results with left boundary positions of 18 m, 19 m, and 20 m, respectively. Within 634 each panel of (b)–(d), the left, center, and right plots correspond to by values of  $10^{-1}$ ,  $10^{-2}$ , and 635 636 10<sup>-3</sup>, respectively. Localized regularization adjustments are also implemented for the true positions of the right, top, and bottom boundaries. The black dot indicates the minimum ABIC 637 638 point, while the black dashed line represents the  $\lambda$  value associated with the minimum ABIC. 639 The black solid line indicates the  $\lambda$  value associated with the minimum rms misfit.

Figure 7. Inverted models of synthetic data obtained from the SBI algorithm using vertical grids not aligned with a true top position of z = 1.5 m. (a) Inverted model with localized regularization adjustments for z = 1.2 m (nearest top vertical grid of z = 1.2 m). (b) Inverted model with localized regularization adjustments for z = 1.8 m (nearest top vertical grid of z =

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1.8 m). (c) Relationship between ABIC and nearest grid position. Localized regularization adjustments are also applied for x = 20 m, x = 26 m, and z = 6 m.

Figure 8. Location of an ERT survey along a promenade on a campus of Kyoto University. The survey target is a utility tunnel with sharp rectangular boundaries. A blueprint indicates its horizontal width and vertical thickness as 3.5 m and 4.4 m, respectively. Dashed black lines indicate its horizontal location estimated from the blueprint, and the center is 14 m. The thick black line indicates the 54 m survey profile.

Figure 9. Observed ERT data, smooth model of the field data, and its first-derivative roughness. 651 (a) Observed apparent resistivity data. The vertical axis represents pseudo depth in units of 652 653 nLevel, where nLevel is the distance between the source and receiver electrodes divided by 654 our basic electrode spacing of 2 m. (b) Inverted model using smoothness regularization. Black lines with a double arrow indicate the horizontal location of the tunnel (x = 12.25 - 15.75 m) 655 656 determined based on the blueprint and the center is 14 m. (c) and (d) show the absolute of the 657 first-derivative roughness of the inverted model in (b) for the x- and z-directions, respectively. 658 Red lines indicate our initial sharp boundary location (x = 11 m and x = 17 m; z = 1.5 m and z = 1.5659 7 m).

Figure 10. Inversion results of the field data obtained from the SBI algorithm. (a) Inverted model with localized regularization adjustments for an initial location of x = 11 m and x = 17 m; z = 1.5 m and z = 7 m. (b) Inverted model with localized regularization adjustments for x = 12 m and x = 16 m; z = 1.5 m and z = 7 m after Flow 3. (c) Inverted model with localized regularization adjustments for x = 12 m and x = 16 m; z = 2 m and z = 7 m after Flow 4.  $bv = 10^{-1}$  is assigned for obtaining the models shown in Figure 9a–c. Black lines with double

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arrows indicate the horizontal location of the tunnel (x = 12.25 - 15.75 m) determined based on the blueprint, and the center is 14 m.

- Figure 11. True model including the triangular structures, smooth inversion result, and its first-668 669 derivative roughness. (a) Two 10  $\Omega$ m triangular structures with upward and downward 670 orientations (A1 and A2), are embedded in a 100  $\Omega$ m homogeneous half-space structure. (b) Inverted model using smoothness regularization. Magenta lines represent the true outline of 671 672 the conductors. (c) and (d) present the absolute of the first-derivative roughness of the inverted model in (b) for the x- and z-directions, respectively. Light blue lines indicate our initial sharp 673 boundary locations (A1: x = 13 m and x = 16 m; z = 1.5 m and z = 7 m, and A2: x = 30 m and x 674 =39 m; z = 1 m and z = 4 m).675
- Figure 12. Inversion results of the synthetic data generated from the model, including the
  triangular structures (Figure 11a). Inverted models obtained by the SBI algorithm for (a) A1,
  (b) A2, and (c) both A1 and A2. (d) Inverted model with localized regularization adjustments
  for true sharp boundaries. The parameter *bv* was set to 10<sup>-1</sup> to obtain the models depicted in
  Figure 11a and 11b, and to 10<sup>-2</sup> in Figure 11c and 11d.
- Figure A1. Inverted models obtained with *bv* ranging from  $10^{-4}$  to 1 and localized regularization adjustments for x = 20 m and x = 26 m; z = 1.5 m and z = 6 m. bv = (a) 1, (b)  $10^{-1}$ , (c)  $10^{-2}$ , (d)  $10^{-3}$ , and (e)  $10^{-4}$ . Magenta lines represent the true outline of the conductor. (a) and (d) are the same for Figures 3a and 4c.

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Figure 1. Schematic diagram of the developed SBI algorithm using ABIC minimization to determine correct positions and optimal weights for localized regularization adjustments to delineate sharp boundaries.

220x161mm (300 x 300 DPI)





Figure 2. Test model for generating synthetic datasets. A  $10-\Omega m$  conductor with sharp boundaries is embedded in a  $100-\Omega m$  homogeneous half-space structure. The conductor shape is a rectangle of 20-26 mand 1.5-6 m for the x- and z-directions, respectively. Magenta lines represent the outlines of the conductor. The vertical exaggeration is 2.

188x113mm (300 x 300 DPI)





Figure 3. Smooth model and its roughness. (a) Inverted model of synthetic data obtained using smoothness regularization. Magenta lines represent the true outline of the conductor (x = 20 m and x = 26 m; z = 1.5 m and z = 6 m). (b) and (c) show the absolute of the roughness of the smooth model in (a) for the x- and z-directions, respectively. Red lines indicate our initial sharp boundary location (x = 19 m and x = 27 m; z = 1 m and z = 7 m).

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Figure 4. Inverted models of synthetic data obtained from the SBI algorithm. (a) Inverted model with localized regularization adjustments for x = 19 m and x = 27 m; z = 1 m and z = 7 m (initial location). (b) Inverted model with localized regularization adjustments for x = 20 m and x = 26 m; z = 1 m and z = 7 m (Flow 3). (c) Inverted model with localized regularization adjustments for x = 20 m and x = 26 m; z = 1 m and z = 7 m (Flow 3). (c) Inverted model with localized regularization adjustments for x = 20 m and x = 26 m; z = 1.5 m and z = 6 m (Flow 4; optimal model). Magenta lines represent the true outline of the conductor. The weight by = 10-2 is assigned for (a) and (b); by = 10-3 is assigned for (c).

211x304mm (300 x 300 DPI)



Figure 5. Cross-plots of ABIC as a function of the left boundary discontinuity location for regularization adjustments, (a)  $\lambda$ , (b) rms misfit, and (c) bv. Localized regularization adjustments are also applied for true positions for right, top, and bottom boundaries.

403x145mm (300 x 300 DPI)



Figure 6. Plots of the ABIC and rms misfit as functions of the left boundary discontinuity location for regularization adjustments,  $\lambda$ , and bv. (a) Smoothness inversion result, and (b), (c), and (d) SBI results with left boundary positions of 18 m, 19 m, and 20 m, respectively. Within each panel of (b)–(d), the left, center, and right plots correspond to bv values of 10–1, 10–2, and 10–3, respectively. Localized regularization adjustments are also implemented for the true positions of the right, top, and bottom boundaries. The black dot indicates the minimum ABIC point, while the black dashed line represents the  $\lambda$  value associated with the minimum ABIC. The black solid line indicates the  $\lambda$  value associated with the minimum rms misfit.

547x594mm (600 x 600 DPI)





Figure 7. Inverted models of synthetic data obtained from the SBI algorithm using vertical grids not aligned with a true top position of z = 1.5 m. (a) Inverted model with localized regularization adjustments for z =1.2 m (nearest top vertical grid of z = 1.2 m). (b) Inverted model with localized regularization adjustments for z = 1.8 m (nearest top vertical grid of z = 1.8 m). (c) Relationship between ABIC and nearest grid position. Localized regularization adjustments are also applied for x = 20 m, x = 26 m, and z = 6 m.

211x312mm (300 x 300 DPI)



Figure 8. Location of an ERT survey along a promenade on a campus of Kyoto University. The survey target is a utility tunnel with sharp rectangular boundaries. A blueprint indicates its horizontal width and vertical thickness as 3.5 m and 4.4 m, respectively. Dashed black lines indicate its horizontal location estimated from the blueprint, and the center is 14 m. The thick black line indicates the 54 m survey profile.

224x177mm (300 x 300 DPI)



Figure 9. Observed ERT data, smooth model of the field data, and its first-derivative roughness. (a) Observed apparent resistivity data. The vertical axis represents pseudo depth in units of nLevel, where nLevel is the distance between the source and receiver electrodes divided by our basic electrode spacing of 2 m. (b) Inverted model using smoothness regularization. Black lines with a double arrow indicate the horizontal location of the tunnel (x = 12.25–15.75 m) determined based on the blueprint and the center is 14 m. (c) and (d) show the absolute of the first-derivative roughness of the inverted model in (b) for the xand z-directions, respectively. Red lines indicate our initial sharp boundary location (x = 11 m and x =17 m; z = 1.5 m and z = 7 m).

212x440mm (300 x 300 DPI)



Figure 10. Inversion results of the field data obtained from the SBI algorithm. (a) Inverted model with localized regularization adjustments for an initial location of x = 11 m and x = 17 m; z = 1.5 m and z = 7 m. (b) Inverted model with localized regularization adjustments for x = 12 m and x = 16 m; z = 1.5 m and z = 7 m after Flow 3. (c) Inverted model with localized regularization adjustments for x = 12 m and x = 16 m; z = 2 m and z = 7 m after Flow 4. bv = 10-1 is assigned for obtaining the models shown in Figure 9a-c. Black lines with double arrows indicate the horizontal location of the tunnel (x = 12.25-15.75 m) determined based on the blueprint, and the center is 14 m.

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Figure 11. True model including the triangular structures, smooth inversion result, and its first-derivative roughness. (a) Two 10  $\Omega$ m triangular structures with upward and downward orientations (A1 and A2), are embedded in a 100  $\Omega$ m homogeneous half-space structure. (b) Inverted model using smoothness regularization. Magenta lines represent the true outline of the conductors. (c) and (d) present the absolute of the first-derivative roughness of the inverted model in (b) for the x- and z-directions, respectively. Light blue lines indicate our initial sharp boundary locations (A1: x = 13 m and x = 16 m; z = 1.5 m and z = 7 m, and A2: x = 30 m and x = 39 m; z = 1 m and z = 4 m).

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Figure 12. Inversion results of the synthetic data generated from the model, including the triangular structures (Figure 11a). Inverted models obtained by the SBI algorithm for (a) A1, (b) A2, and (c) both A1 and A2. (d) Inverted model with localized regularization adjustments for true sharp boundaries. The parameter bv was set to 10–1 to obtain the models depicted in Figure 11a and 11b, and to 10–2 in Figure 11c and 11d.

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Figure A1. Inverted models obtained with bv ranging from 10–4 to 1 and localized regularization adjustments for x = 20 m and x = 26 m; z = 1.5 m and z = 6 m. bv = (a) 1, (b) 10–1, (c) 10–2, (d) 10–3, and (e) 10–4. Magenta lines represent the true outline of the conductor. (a) and (d) are the same for Figures 3a and 4c.

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# DATA AND MATERIALS AVAILABILITY

Data associated with this research are available and can be obtained by#xD;#xA;contacting the corresponding author.