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Abstract

This paper examines hidden action models of the relationship between a firm and multiple workers, and identifies cases where the firm prefers workers to collude via a binding agreement on their action choice. Worker collusion and appropriately redesigned wage contracts have a risk sharing advantage (i) when workers' action choice is completely independent; (ii) when the firm wants each worker to allocate his effort efficiently among various productive activities; or (iii) when team production exists. Worker collusion also achieves an efficient level of "socialization" among workers while noncooperative behavior leads to "under-socialization."

KEYWORDS: Moral hazard, worker collusion, risk sharing, interpersonal interaction, effort allocation, team production, socialization.
1. Introduction

A firm is a human organization. The employer makes implicit or explicit employment contracts with workers. Based on the earlier work by Coase (1937) and Simon (1951), recent economic literature regards the employment relation as the most fundamental constituent of the firm, and analyzes various aspects of that relationship extensively. (See Hart and Holmström (1987) for a survey of agency theory, Williamson (1975, 1985) for transaction cost economics, and Kreps (1984) for an example of other theories of the firm.)

Given employment contracts and other aspects of formal organization structures, however, workers are engaged in interactions with co-workers. When wage schemes are interdependent, a worker's effort choice affects his co-workers' welfare. Also workers can influence co-workers through various kinds of direct interpersonal activities. Some actions are directly productive: workers may provide various kinds of help for co-workers. There exist psychologically oriented interactions as well: workers may spend resources to reduce co-workers' disutility on jobs, for example, by showing respects, cheering up each other, listening to their complaints, and so on. Realizing the existence of these interactions among workers, the employer designs contracts and organization structures so as to make their interpersonal relationship profitable to herself. Social relationship among workers has long been an important subject of sociologists studying organizations since the Human Relations School (Roethlisberger and Dickson, 1939; Mayo, 1945). Recently, Baron (1987) and Granovetter (1985) review economic literature on organizations critically to point out its under-emphasis on social relations.

In this paper, we focus on interactions among workers and examine its implications on wage contracts and organization design. Models introduced are variants of standard agency models with moral hazard (which Arrow (1985) renamed hidden action): Workers take unobservable actions given wage contracts, while they possess no precontractual private information. More specifically, consider the following standard setting of the relationship between a risk neutral principal (a firm) and risk averse agents (workers). The firm assigns each worker to a well-defined job and selects a wage schedule to him. Given his contract with the firm, each worker exerts a level of effort. The outcome of his job depends on his effort and some noise term. Noise terms are assumed to be independent across jobs. The outcome is publicly

\footnote{See also standard textbooks such as Perrow (1986) and Scott (1987) for other references, evaluations, and criticism.}
observable while the level of effort chosen is not, so that the wage schedule for each worker can depend only on the outcome of his job. Because of no externality of effort and the assumption of stochastic independence across jobs, relative performance evaluation does not give a reason for the wage schedule for a worker to depend upon outcomes of other workers’ jobs. (See, for example, Holmström (1982) and Mookherjee (1984).) Suppose that the von Neumann-Morgenstern utility function of a worker is additively separable to utility on income and disutility on effort: workers are assumed to be effort averse.

The question asked in this paper is: If the firm can enforce workers to collude to play "cooperatively" in their action choice through various aspects of organization structures and personnel policies, does she prefer to do so? Worker collusion in this paper means that workers make decisions, via some binding agreement on their action choice, so as to maximize the sum of their expected utilities. In the setting described above, where the relation among workers is completely independent, the answer to the question turns out to be yes at least in symmetric situations. The literature on relative performance evaluation mentioned above implicitly assumes that workers play their effort choice game noncooperatively, i.e. select Nash equilibrium effort. Under this assumption, the literature shows that the optimal wage schedule to a worker is independent of the performance of the other workers, so that workers’ effort choice game is reduce to a completely independent decision problem with no interaction. We however show that if workers collude, then in symmetric situations, the firm designs an interdependent wage scheme to create workers’ interaction. This leads to risk sharing among workers, and it turns out that the firm can implement the same effort levels with lower costs under collusion than under noncooperative behavior.

The similar logic continues to hold under a certain condition when we introduce production externalities or correlation across jobs so that the optimal wage scheme (under noncooperative behavior) is an interdependent one. We examine a specific form of productive interpersonal activities called helping effort. A worker's own effort improves the outcome of his job, while his helping effort improves the outcome of the jobs occupied by his co-workers. Each worker's disutility on job is assumed to depend on the total amount of effort. Then we show that worker collusion leads to an efficient allocation of effort between own effort and helping effort. This implies that given the wage contract and the total amount of effort exerted by workers, neither wants to change the allocation of effort: It maximizes the expected profits.
Collusion results in the allocation efficiency because under symmetry, their cooperative effort choice is always based on equal risk allocation among jobs, so that they never under-cooperate nor over-cooperate. Then we show that when the firm wants to implement an effort allocation efficient effort combination, she prefers worker collusion. This is because she can reduce the risk imposed on each worker without altering workers' incentives to select that effort combination. The same argument also implies, in more general situations, that when team production exists so that the firm can monitor only the total output of a group of workers, worker collusion is preferable. An important feature of team-based wage schemes is that workers' wages are perfectly positively correlated with each other. This enables the firm to modify the wage scheme appropriately. On the other hand, if wages are negatively correlated as in the case of tournaments, the improvement of risk sharing changes workers' incentives to select the effort levels the firm wants to implement. Thus, our results do not apply to such cases.

Next we turn to nonproductive interpersonal activities which we call socialization. We add this variable to the basic model with no interaction among workers. A worker's level of socialization with another worker enters into only the disutility terms of both workers, and thereby it affects the productivity of neither worker. An increase in a worker's socialization reduces a level of disutility of his co-workers, while it increases the former worker's disutility for sufficient high levels of socialization. However, for small levels of socialization, a worker may be indifferent in his socialization level or may derive some "social pleasure" from socializing so that his disutility may be decreasing in his socialization level. Then we show that when workers reach a Nash equilibrium of socialization, the equilibrium levels of socialization are less than the efficient levels: workers under-socialize when they behave noncooperatively. Since this proposition is not associated with the risk attitude of workers at all, it implies that even in the risk neutral situation, the firm cannot achieve the first-best solution (which is the solution under perfect information) when nonproductive interpersonal activities exist.

Collusion by risk neutral workers remedies this problem by increasing their socialization to the efficient levels: The first-best solution can be achieved by collusion when workers are risk neutral. However, when workers are risk averse, collusion is not always better since reducing the level of disutility has no relation to reducing the level of marginal disutility. The increase in levels of socialization through worker collusion has two opposite effects on a worker's marginal disutility of work: the positive effect of the increase in socialization from his co-worker (which
reduces his marginal disutility) and the negative effect of the increase in his own socialization (which increases his marginal disutility). If the negative effect dominates, the firm may prefer workers to play noncooperatively. Or even preventing workers from socializing by isolating them physically or by supervising them closely may be better.

So far we have assumed that the firm can induce workers to collude or not. How can she achieve worker collusion? The literature on game theory tells us that cooperation will emerge when the subgame of effort choice is repeated, provided that each worker can observe the effort choice of the others at the end of each one-shot game. (Any good survey?) This suggests the importance of the long-term relationship of workers and of their mutual observability. The recent literature on laboratory experiments on human cooperation also provides interesting implications. (See Dawes and Thaler (1988) for a survey.) Experiments identify several important conditions to derive cooperation; in particular, the opportunity of discussion and the establishment of group identity. I think that organization structures and personnel policies of the firm can be designed to achieve such conditions.2 Japanese firms, which are considered to have more cooperative workers, seem to have features compatible to those conditions: the "life-time employment," the encouragement of information sharing among members, fluid job demarcation, and work groups as functional units, and so on.

Collusion problems in agency settings have been analyzed by several economists, but most of the literature focuses on the collusion problem associated with the multiplicity of Nash equilibrium (Demski and Sappington, 1984; Ma, 1988; Ma, Moore and Turnbull, forthcoming; Mookherjee, 1984; Mookherjee and Reichelstein, 1988). Tirole (1986) analyzes the collusion problem similar to this paper.3 He considers a three-tier organization with an employer, a supervisor, and a worker. Given an initial wage contract, the worker observes an uncertain productivity parameter, and then selects effort. The supervisor can sometimes observe the true value of the parameter without any effort, and report it to the employer verifiably. Tirole assumes that any monetary side transfer between two parties can be costlessly written. Then he shows that though the firm can design initial contracts preventing any further side con-

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2 In this respect, it is interesting to find that a sociologist Perrow (1986) criticizes agency theory for its exogenous assumption of individualistic preferences, and in turn treats this assumption as a variable to ask when self-interest behavior is likely to appear. He also believes that organization structures can affect workers' mode of behavior.

3 See also Tirole (1988).
tracting, the possibility of the collusion between the supervisor and the worker reduces the net payoff to the employer. There are several contrasts between his paper and ours. He focuses on “vertical” collusion between the worker and the supervisor who has a better information structure than the employer. And he considers monetary side transfers between parties. In his model, however, what matters is the possibility of the detrimental collusion, which never occurs because the employer can prevent it by embodying any side contracting in the initial contract. This paper focuses on “lateral” collusion among workers who play subgames of action choice. Collusion among workers is via some nonpecuniary side contracts on their choice of actions. And in our model, the employer sometimes wants worker collusion to occur, and thus the initial wage contract does not prevent it.

There is also a set of literature analyzing the interaction of workers in the firm by assuming that they have exogenously specified interdependent preferences (Akerlof, 1982; Frank, 1984). In this paper we assume in contrast that the level of interactions is at workers’ discretion, and because of this assumption, we can examine how the firm affects their discretion in order to achieve better interactions among workers from her point of view.

The paper is organized as follows. Section 2 considers collusion without interpersonal activities. Helping effort and socialization are introduced in Sections 3 and 4, respectively. In Section 5 we discuss what makes cooperation likely, and how the firm can achieve cooperation.

### 2. Collusion without Interpersonal Activities

This section presents the model of the relationship between the firm and two workers with no interpersonal activities. Extensions to the case of more than two workers are straightforward. In this section, the relation of the firm with each worker is exactly the one between a principal and one agent in the literature of standard agency theory with hidden action and without precontractual private information. We in particular follow the formulation by Grossman and Hart (1983).

The firm has two jobs and two workers indexed by \( n = 1, 2 \), and assigns worker \( n \) to job \( n \). Worker \( n \) selects an effort level \( a_n \in A \) where \( A \) is a compact set of feasible effort levels. Effort \( a_n \) is expended only on his job \( n \). The profit \( \pi^n \) from job \( n \) depends on worker \( n \)’s effort \( a_n \) and a noise term \( \epsilon_n \) through a production function \( \pi^n = f_n(a_n, \epsilon_n) \). Note that the production functions are separable in effort so that there is no production externality. In addition, we
assume that the noise terms are stochastically independent. Suppose that \( \pi^n \) takes one of the \( M_n \) possible values \( \pi^n_1 < \cdots < \pi^n_{M_n} \). For each outcome \( i \in I_n = \{1, \ldots, M_n\} \), let \( P^n_i(a_n) \) be the probability of \( \pi^n = \pi^n_i \) induced by \( f_n \) and the probability distribution of the error term. Let \( \Pi^n(a_n) = \sum_i P^n_i(a_n) \pi^n_i \) be the expected profits from job \( n \).

We assume that worker \( n \) has a von Neumann-Morgenstern utility function of the additively separable form, denoted by \( V^n(w) - G^n(a_n) \), where \( w \) is the wage paid to him. Following standard models of principal-agent relationships, we assume that \( V^n \) is strictly increasing and concave, and that \( G^n \) is strictly increasing: workers are (weakly) risk averse and effort averse.

The firm behaves as a Stackelberg leader to select a wage schedule for each worker. We assume that effort levels are publicly unobservable while realized profits from each job are publicly observable. Since there is no externality of effort and the profits are stochastically independent across jobs, the literature of relative performance evaluation tells us that the optimal wage schedule \( w^n \) to worker \( n \) depends on \( \pi^n_i \) only.\(^4\) We call such a wage scheme an independent scheme. Let \( w^n_i \) be the wage paid to worker \( n \) when \( \pi^n_i \) is observed. Given wage schedules, worker \( n \) chooses his effort level \( a_n \) to maximize his expected utility. Let \( U^n(w^n, \cdot) \) be the expected utility of worker \( n \), given his wage scheme \( w^n = (w^n_i)_{i \in I_n} \). Clearly, \( U^n(w^n, \cdot) \) depends on his effort \( a_n \) only. It is given as

\[
U^n(w^n, a_n) = \sum_{i \in I_n} P^n_i(a_n) V^n(w^n_i) - G^n(a_n).
\]

The firm is assumed to be risk neutral: The objective of the firm is to maximize the sum of the difference between the expected profit \( \Pi^n(a_n) \) from job \( n \) and the expected wage payment \( \sum_i P^n_i(a_n) w^n_i \) to worker \( n \).

Rather than state the firm's optimization problem directly, we utilize the Grossman-Hart decomposition of the problem (Grossman and Hart, 1983): We fix effort levels \( (a_1, a_2) \), and find wage schedules which implement them with least costs. We call this problem the implementation problem for \( (a_1, a_2) \). Once the implementation problem is solved for each \( (a_1, a_2) \), the firm chooses the effort levels which maximize her expected profits minus the optimal expected wage payments.

Since the relation of each worker with the firm is completely independent of that of the other worker, we can consider the implementation problem for each worker separately. Let

\(^4\) See, for example, Holmström (1982) and Mookherjee (1984).
$v^n_i = V^n(w^n_i)$, and $\phi^n$ be the inverse function of $V^n$. Then the implementation problem for $a_n$ is given as follows:

$$\min_{v^n} \sum_{i \in I_n} P^n_i(a_n)\phi^n(v^n_i)$$

subject to

(IC) \hspace{1em} a_n \in \arg\max_{a} \sum_{i \in I_n} P^n_i(a)v^n_i - G^n(a)

(PC) \hspace{1em} \sum_{i \in I_n} P^n_i(a_n)v^n_i - G^n(a_n) \geq \bar{U}^n.$$

The constraints (IC) represent incentive constraints. The constraints (PC) are the participation constraints: The expected utility of worker $n$ must be at least as large as his reservation utility level $U^n$.

The solution $v^n$ to the implementation problem is called the optimal wage schedule (in utility units) to worker $n$ for the fixed effort level $a_n$. Let $C^n(a_n)$ be the minimum cost to implement $a_n$, which is the optimal value of the implementation problem above. Then the firm chooses $a_n$ to maximize $\Pi^n(a_n) - C^n(a_n)$.

The argument above implicitly assumes that given wage schedules, workers play a subgame of simultaneous effort choice noncooperatively to reach a Nash equilibrium of effort. It is under this assumption that the literature on relative performance evaluation shows the optimality of independent wage schedules. The effort choice subgame then becomes trivial: workers choose their effort levels independently as above.

What if workers collude to play their effort choice subgame cooperatively? Whatever cooperative play means, the effort choice of workers does not change as long as the firm designs independent, individual-based wage schemes. However, realizing that workers play cooperatively, the firm may design different wage schedules, possibly interdependent ones. Then the question is whether the firm prefers workers to play noncooperatively or cooperatively.

To examine this question, assume that workers are completely identical: Each of them has an identical job with possible profits $\pi_i$ for $i \in I = \{1, \ldots, M\}$, the same technology $P_i(a)$, the same utility function $V(w) - G(a)$, and the same reservation utility level $U$. Then we consider the implementation problem for $(a_1, a_2) = (a, a)$. This symmetry assumption is
not innocuous: Introduction of asymmetry may cause a problem, which will be discussed at
the end of the section. We also assume that workers are strictly risk averse.

We assume that when workers play cooperatively, they choose some Pareto efficient effort
pair via a binding agreement on effort choice. How such a situation emerges is not our concern
in this section: We simply assume that the firm can enforce worker collusion if she wants to do.
Later in Section 5, I argue that whether workers play noncooperatively or cooperatively can
depend greatly on organization design and personnel policies of the firm. Following convention,
we formalize the workers’ cooperative effort choice subgame as they choose an effort pair to
maximize a weighted sum of their expected utilities. Since we consider the symmetric situation,
we simply assume that the weight on each worker’s expected utility is the same \( \frac{1}{2} \). In other
words, they choose their effort levels to maximize the (unweighted) sum of their expected
utilities. To extend to the case of interdependent wage schemes, let \( w_{ij}^n \) be the wage to
worker \( n \) when the profits from job \( n \) and job \( k \) (\( k \neq n \)) are \( \pi_i \) and \( \pi_j \), respectively, and let
\( v_{ij}^n = V(w_{ij}^n) \). When workers collude, the optimal solution to the implementation problem for
\((a,a)\) solves

\[
\min_{v^1, v^2} \sum_{i \in I} \sum_{j \in I} P_i(a)P_j(a)(\phi(v_{ij}^1) + \phi(v_{ij}^2)) \]

subject to

\[(CIC) \quad (a,a) \in \arg \max_{a_1, a_2} \sum_{i \in I} \sum_{j \in I} P_i(a_1)P_j(a_2)(\frac{1}{2}v_{ij}^1 + \frac{1}{2}v_{ij}^2) - \frac{1}{2}G(a_1) - \frac{1}{2}G(a_2)\]

\[(PC) \quad \sum_{i \in I} \sum_{j \in I} P_i(a)P_j(a)v_{ij}^n - G(a) \geq \bar{U}.
\]

(CIC) is the abbreviation of the cooperative incentive constraints. The former incentive con-
straints (IC) are, in contrast, called the Nash incentive constraints (NIC) hereafter.

Now we are ready to show the main result of this section. Given an effort pair and the
optimal wage scheme (for noncooperative workers),\(^6\) the following proposition shows that when
workers collude, the firm can implement the same effort pair with lower costs by designing an
appropriate interdependent wage scheme.

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\(^5\) Whenever \( k \) and \( n \) appear together, assume \( k \neq n \).

\(^6\) Since we do not solve the optimal scheme under worker collusion, the word “optimal” is
only used for the case in which workers play noncooperatively.
Proposition 2.1. Under the symmetric assumption as above, the firm prefers workers to collude to play cooperatively than noncooperatively. When workers collude and \((a, a)\) are not the least costly effort levels, the firm selects an interdependent wage scheme.

The proof is in Appendix. What enables the new interdependent wage scheme (defined below by (2.1)) to achieve lower costs under worker collusion is (i) risk sharing between workers and (ii) the introduction of interpersonal interaction. To see this point clearly, suppose that \(I = \{1, 2\}\) and \(P_2(\cdot)\) is increasing. Then the optimal independent wage scheme \((v^1, v^2)\) for \((a, a)\), which are not least costly, satisfies \(v^1_i < v^2_i\), and also by symmetry \(v^1_i = v^2_i\) for \(i = 1, 2\). Define the new interdependent wage schedule \((\hat{v}^1, \hat{v}^2)\) by

\[
\hat{v}^i_{ij} = \frac{1}{4} v^i_i + \frac{1}{4} v^i_j.
\]

Then \(\hat{v}^2_{22} = v^2_2, \hat{v}^2_{21} = \hat{v}^2_{12} = \frac{1}{4} (v^2_i + v^2_j), \) and \(\hat{v}^2_{11} = v^2_i\) hold. In the original independent contract, each worker faces the following lottery; the higher wage \(v^a_i\) with probability \(P_2(a)\) and the lower wage \(v^b_i\) with probability \(P_1(a)\). Under the new interdependent scheme, the lottery changes to; \(v^2_2\) with probability \((P_2(a))^2 < P_2(a)\) and \(v^2_i\) with probability \((P_1(a))^2 < P_1(a)\), and an intermediate wage \(\frac{1}{4} (v^a_i + v^b_i) = \frac{1}{4} (v^a_j + v^b_j)\) with probability \(2P_2(a)P_1(a)\). That is, risk is shared between workers 1 and 2 when the profits from two jobs are different: For example, when job 1 makes the higher profits \(\pi_2\) and job 2 the lower profits \(\pi_1\), under the original contract, worker 1 receives the higher wage and worker 2 the lower wage, while under the new contract they receive the same intermediate wage.

The new incentive scheme (2.1) also introduces interaction between workers such that when they collude, their risk sharing does not weaken workers' incentives to select the effort levels given. Under the new interdependent wage schemes, worker \(n\)'s expected utility is given by

\[
\frac{1}{2} \sum_{i \in I} P_1(a_n) v^a_i + \frac{1}{2} \sum_{j \in I} P_2(a_k) v^b_j - G(a_n).
\]

Thus, when they play noncooperatively under this new wage schedule, each worker finds his marginal utility on income with regard to his effort is half of that under the original independent scheme, so that they do not select \((a, a)\). When they collude, however, they find that the sum of their expected utilities yields the same marginal utility on income with regard to each effort as the one when they play noncooperatively under the original independent
schedule. Thus, collusion and the new interdependent scheme lead workers to select \((a, a)\) which the firm wants to implement. Also note that under the new scheme, workers are better off by colluding than by behaving noncooperatively.

The upshot is that when the relation among workers is completely independent, the firm prefers making workers collude and accordingly designing an interdependent wage schedule, which creates workers' interaction endogenously. Under such a scheme, workers care about co-workers' behavior and choose the effort pair the firm wants to implement, with less risk imposed on workers than when they are treated separately under the independent wage schedule.

Remarks:

(1) In Appendix, Proposition 2.1 is proved in a little more general case in which the noise terms are not stochastically independent but the optimal wage scheme is still an independent one. (See Mookherjee (1984) for such an example.) However, the result is not generally true if the optimal scheme is interdependent because the risk sharing among collusive workers through the new wage scheme (2.1) cannot generally elicit the equilibrium effort levels chosen by workers behaving noncooperatively. For example, if workers' original wages are negatively correlated with each other as in the case of a tournament, the new scheme (2.1) leads both the winner and the loser to always receive some intermediate prize, so that even collusive workers have no incentive to select effort levels other than least costly ones. On the other hand, if workers' wages are positively correlated as in the case of team-based wage schemes (under which wages depend only on the sum of the profits from two jobs), the inter-worker risk sharing scheme (2.1) gives collusive workers the marginal utility-on-income with regard to each effort twice as high as that the original scheme gives to noncooperative workers. Thus, again, collusive workers do not select the same effort levels. In the next section, we examine a specific form of production externalities. The argument there can be applied to more general cases in which interdependent wage schemes are optimal. We will show that a different way of improving risk sharing can lead the firm to prefer collusion in symmetric situations if workers' optimal wages are perfectly positively correlated.

(2) The proof of Proposition 2.1 heavily depends on the symmetry assumption. Without symmetry, the proof is still valid if (i) the weight on each worker's expected utility is the same; (ii) \(V^1 = V^2\); and (iii) \((a_1, a_2)\) which the firm attempts to implement satisfy \(U^1 + G^1(a_1) = \)
$U^2 + G^2(a_2)$. Conditions (i) and (ii) are utilized to show that the new interdependent wage scheme reduces the firm's expected payments by improving the risk sharing. Condition (iii) can be dropped if we weaken the participation constraints (PC) to

$$U^1(w^1, a_1, a_2) + U^2(w^2, a_2, a_1) \geq U^1 + U^2.$$  

The constraint states that the sum of workers' expected utilities must be at least as large as the sum of their reservation utility levels. This constraint may be reasonable if we assume, in addition to a binding agreement on effort choice, the feasibility of utility transfers via some binding pecuniary or nonpecuniary side contracting between workers.

3. Effort Allocation and Collusion

In this section, we introduce a specific form of production externalities as an example of a productive interpersonal behavior. We continue to assume the symmetry between two workers. Each worker is strictly risk averse, and chooses a two-dimensional effort vector $e_n = (a_n, b_n) \in A \times B \equiv [0, A] \times [0, B]$ with $A > 0$ and $B > 0$. The first element $a_n$, called his own effort, is the same as the effort in the last section: it is expended on his job. The second element $b_n$ is called his helping effort which affects the job occupied by the other worker $k$. Thus, the probability that the profits from job $n$ is $\pi_n$ is now a function of $a_n$ and $b_k$, written as $P_i(a_n, b_k)$. The expected profits from job $n$ is also written as $\Pi(a_n, b_k)$. To ease notations, let $P_{ij}(e_n, e_k) = P_i(a_n, b_k)P_j(a_k, b_n)$ be the joint probability of $\pi_n = \pi_i$ and $\pi_k = \pi_j$. The disutility of each worker is a function of his own effort and his helping effort. For simplicity, we assume that it depends on the sum of these two effort levels, written as $G(a_n + b_n)$: For example, $a_n$ and $b_n$ represent the time allocated to worker $n$'s own job and the other job, respectively, and he cares only about the total amount of time he works. This assumption makes the exposition in this section simpler, although the results do not depend on it.

For an expository reason, we assume $I = \{1, 2\}$: There are two possible outcomes of each job. Let $p(a_n, b_k)$ be the probability that job $n$ makes profit $\pi_2$. Since $\pi_2 > \pi_1$, $p(a_n, b_k)$ is the probability of "success" in job $n$ while $1 - p(a_n, b_k)$ is the probability of "failure."

We employ the following assumptions throughout this section.

Assumption 3.1. (i) $p(a, b)$ is strictly increasing and strictly concave in $a$ and $b$. (ii) $p(a, b) \in$
(0, 1) for all \((a, b)\). (iii) \(G(t)\) is strictly increasing and convex. (iv) The firm always wants to implement \(e = (a, b)\) with \(a \in (0, A)\) and \(b < B\).

Some explanation of each assumption follows. Assumption (i) implies that both own effort and helping effort are productive with decreasing returns. Under our two-outcome assumption, this also implies that the probability distribution satisfies the strict monotone likelihood ratio property. Assumption (ii) implies that there is no moving support. Assumption (iii) is a standard one implying that workers are effort averse. Finally, Assumption (iv) excludes the case in which effort levels reach upper bounds, or the case where only helping effort is positive.

This section utilizes the first-order approach. We assume that \(p(\cdot)\) and \(G(\cdot)\) are twice continuously differentiable. Itoh (1988) shows that the following assumption is sufficient for the first-order approach to be valid.

**Assumption 3.2.** Either one of the following (a) and (b) holds. (a) \(P_{11}(e_1, e_2)\) is convex in \(e_n\) for \(n = 1, 2\), and \(\phi'(v)\) is convex. (b) \(P_{22}(e_1, e_2)\) is concave in \(e_n\) for \(n = 1, 2\), and \(\phi'(v)\) is concave.

In particular, the convexity of \(\phi'\) holds when the measure of absolute risk aversion is increasing, constant, or decreasing relatively slowly in income. Utility functions which are frequently used in examples, such as exponential, logarithmic, or square root, satisfy it. Also the assumption on the joint probability implies that \(p(e)\) is concave in \(e\).

The following lemma holds under Assumptions 3.1 and 3.2. It shows that the optimal wage schedule is monotone increasing, and that it is an independent one (an interdependent one) when the firm wants workers not to help (when she wants workers to help, respectively). See Itoh (1988) for the proof.

**Lemma 3.1.** Suppose that workers play the effort choice game noncooperatively. (i) If the firm wants to implement \(e = (a, 0)\) for each worker, the optimal wage schedule is an independent one satisfying \(v_2 > v_1\). (ii) If the firm wants to implement \(e = (a, b)\) with \(b > 0\) for each worker, the optimal wage schedule is an interdependent one satisfying \(v_{i2} > v_{i1}\) and \(v_{2i} > v_{1i}\) for \(i = 1, 2\).

We call \(e = (a, b)\) effort allocation efficient (E.A.E.) if \((a, b)\) maximizes the winning probability \(p(a', b')\) on the indifferent curve \(G(a' + b') = G(a + b)\) or \(a' + b' = a + b\). It is easily
shown that \( e = (a, b) \) with \( b > 0 \) is E.A.E. if and only if \( p_a(a, b) = p_b(a, b) \) and that \( e = (a, 0) \) is E.A.E. if and only if \( p_a(a, 0) \geq p_b(a, 0) \) holds, where the subscripts represent partial derivatives. Figure 3.1 shows the path of E.A.E. pairs of effort. By Assumption 3.1 (i), each isoquant of \( p(\cdot) \) is strictly convex and decreasing. Thus, for each \( t > 0 \), there exists a unique E.A.E. effort pair, denoted by \( e(t) = (a(t), b(t)) \), such that \( a(t) + b(t) = t. \)

If \( e' = (a', b') \) with \( b' > 0 \) satisfies \( p_b < p_a \) so that \( a' < a(a' + b') \) and \( b' > b(a' + b') \) hold, we say that \( e' \) exhibits over-cooperation. By shifting helping effort to own effort along the indifference curve, the success probability could increase. Similarly, if \( e'' = (a'', b'') \) satisfies \( p_b > p_a \), we say that \( e'' \) exhibits under-cooperation. The following lemma is proved in Itoh (1988).

**Lemma 3.2.** Suppose that workers play the effort choice game noncooperatively. (i) If \( e \) is E.A.E., the optimal wage schedule for \( e \) satisfies \( v_{12} = v_{21} \). (ii) If \( e \) exhibits under-cooperation (over-cooperation), the optimal wage schedule satisfies \( v_{12} < (>)v_{21} \).

The lemma shows the relation between effort allocation and risk allocation between two jobs. When the firm wants to implement an E.A.E. effort pair, she must allocate risk imposed on each worker equally between his job and the other job. This is achieved by paying the same wage \( v_{12} = v_{21} \) when one job is success and the other is failure, regardless of which job succeeds and which job fails. In other words, the firm pays wages depending only on the sum of the profits from two jobs: The wage scheme is completely team-based. To induce workers to under-cooperate, the firm imposes more risk on him through the outcome of his job than the other job \( (v_{2i} - v_{1i} > v_{i2} - v_{i1} \text{ for } i = 1, 2) \). The similar argument holds for over-cooperation as well.

Now we compare the solution as above with that under worker collusion. For the comparison, we provide the first-order conditions for workers to select \( e = (a, b) \) given a wage scheme when they play noncooperatively. When \( b = 0 \), the optimal (independent) wage scheme \( v = (v_1, v_2) \) satisfies the incentive constraints

\[
p_a(a, 0)[v_2 - v_1] = G'(a). \quad (3.1)
\]
On the other hand, if $b > 0$, the optimal wage scheme is interdependent and satisfies (NIC):

$$p_a(e)[(v_{21} - v_{11}) + p(e)\Delta] = G'(a + b)$$
$$p_b(e)[(v_{12} - v_{11}) + p(e)\Delta] = G'(a + b)$$

(3.2)

where $\Delta = v_{22} + v_{11} - v_{12} - v_{21}$. When workers collude, supposing that they maximize the sum of their expected utilities, the optimal contract $\hat{v}$ satisfies the following (CIC).

$$p_a(e)[(\hat{v}_{21} - \hat{v}_{11}) + p(e)\hat{\Delta} + (\hat{v}_{12} - \hat{v}_{11}) + p(e)\hat{\Delta}] = G'(a + b)$$
$$p_b(e)[(\hat{v}_{12} - \hat{v}_{11}) + p(e)\hat{\Delta} + (\hat{v}_{21} - \hat{v}_{11}) + p(e)\hat{\Delta}] \leq G'(a + b)$$

(3.3)

where $\hat{\Delta} = \hat{v}_{22} + \hat{v}_{11} - \hat{v}_{12} - \hat{v}_{21}$ and the inequality is strict only if $b \neq 0$. From (3.3), we immediately obtain the following result.

**Proposition 3.1.** When workers collude to play cooperatively, the firm can implement only E.A.E. pairs.

The reason is that unequal risk allocation between two jobs for a worker is canceled out by the unequal risk allocation for the other worker. Thus, their cooperative effort choice is always based on equal risk allocation between two jobs in total.

Because of Proposition 3.1, the result in the last section applies only to E.A.E. effort pairs. When $e = (a,0)$ is E.A.E., the optimal wage scheme is independent. The proof of Proposition 2.1 then applies and shows that the firm prefers workers to collude to choose $(a,0)$ because of the risk sharing among them through the new wage scheme (2.1). However, if $p_a(a,0) < p_b(a,0)$ so that $e = (a,0)$ exhibits under-cooperation, collusive workers cannot implement this because they reduce own effort and exert positive helping effort to select some E.A.E. effort pair.\(^7\)

When $e = (a, b)$ with $b > 0$ is E.A.E., the optimal wage scheme is interdependent. Then the new wage scheme defined by (2.1) gives collusive workers too much marginal utility on

\(^7\) There is one way to implement such under-cooperated $(a,0)$: physical isolation of workers. By isolation the firm can enforce zero help. Then by designing an appropriate interdependent wage scheme, the firm can induce collusive workers to select any $(a,0)$ with lower costs than when they behave noncooperatively. However, how can the firm simultaneously isolate workers and induce them to play cooperatively? These two policies do not seem to be compatible. Later in Section 4, we again consider isolation when nonproductive interpersonal activities exist. There isolation without collusion may be an attractive alternative for the firm.
income with regard to each effort: If we substitute \( \hat{v}_{ij} = \frac{1}{2}(v_{ij} + v_{ji}) = v_{ij} \) in (3.3), the left-hand side is twice as high as that in (3.2), so that collusive workers do not select the specific E.A.E. pair \( e \). The next proposition presents a different way to construct a desirable wage schedule for collusive workers. The proof is in Appendix.

**Proposition 3.2.** When the firm wants to implement an E.A.E. effort pair, she prefers workers to collude to play cooperatively because of the improvement in risk sharing.

When the E.A.E. effort pair \( e = (a, b) \) the firm wants to implement has a positive helping effort level, she designs the following new wage scheme under worker collusion.

\[
\hat{v}_{ij} = \frac{1}{2} v_{ij} + \frac{1}{2} (U + G(a + b)) \quad (3.4)
\]

The original scheme \( (v_{ij}) \) pays workers equal wages in each outcome. Thus, when workers collude under \( (v_{ij}) \), the marginal utility on income with regard to each effort is twice as high as that when they behave noncooperatively. Thus, the new wage scheme (3.4) simply reduces the variation of wages to half of that of \( (v_{ij}) \) such as to satisfy (PC). This reduction of risk does not weaken incentive effects when workers collude. And the firm's expected payments are lower because of the improvement in risk sharing. Note the difference between (2.1) and (3.4). When the optimal scheme is independent, the firm requires creating interactions between collusive workers by introducing interdependence in wage schemes. On the other hand, when the optimal scheme is interdependent such that workers' wages are perfectly positively correlated as above, no additional interaction is necessary.

The foregoing argument can be extended to more general cases in which both correlation in noise terms and any form of production externalities exist. In particular, if team production exists so that only the sum of the profits from two jobs can be monitored, the wage scheme similar to the one defined by (3.4) can implement the effort pair given with lower costs by improving risk sharing. When the profit from each job can be monitored separately, the firm can achieve technologically efficient effort combinations by wage schemes which make workers' wages perfectly positively correlated with each other.\(^8\) Then the firm who pursues such efficiency prefers worker collusion as shown above. The risk sharing advantage of worker

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\( ^8 \) In the two-outcome model, only perfectly positively correlated wage schemes can achieve allocation efficiency. When each job has more than two outcomes, team-based linear wage schedules attain efficient effort combination. See Drago and Turnbull (1988) who show that,
collusion cannot be applied to other kinds of wage schemes. In particular, if workers' wages are negatively correlated with each other as in the case of a tournament, we cannot reduce the risk imposed on each worker without altering their incentives to select effort. In the symmetric situation, collusion under tournaments only lead workers to select least costly effort levels, so that worker collusion is detrimental.

4. "Human Relations" through Socialization

In this section, we introduce a nonproductive interpersonal activities called socialization into the basic model in Section 2. Worker $n$ chooses his own effort level and a level of socialization $(a_n, s_n) \in A \times S$. The level of socialization does not affect the probability distributions on outcomes of workers' jobs. Instead, it comes into their disutility functions. Throughout this section, we assume that workers have an identical disutility function $G(\cdot)$: Worker $n$'s disutility is written as $G(a_n, s_n, s_k)$.

Because socialization does not affect the probability distribution of the outcome of either job, the optimal wage schedule to worker $n$ is independent of the outcome of job $k$: If $(v^n_{ij})$ implements $(a_n, s_n)_{n=1,2}$, then the new wage schedule $(\hat{v}^n_{ij})$ defined by $\hat{v}^n_{ij} = \sum_j P^n(a_n) v^n_{ij}$ for each $i$ can implement the same effort and socialization levels with lower costs by improving risk sharing between the firm and worker $n$. Thus, hereafter in this section, we can consider wage schedules to worker $n$ which are independent of the outcome of job $k$. This excludes the risk sharing advantage of collusion analyzed in the previous two sections, and so we can focus on a different aspect of worker collusion.

This observation implies that the firm cannot directly control the levels of socialization workers choose by designing wage schemes: The firm selects wage schedules only to implement particular effort levels, taking into consideration the effect of workers' interaction through socialization on effort levels they choose. The objective of this section is to examine how the firm can affect workers' choice of socialization levels in order to reduce the cost of implementing a particular pair of effort $(a_n, a_k)$. We hence assume for simplicity that each worker chooses either a high level of effort or a low level. We let $A = \{h, l\}$ with $h > l$. We assume that for each $n$, $\Pi^n(h) \gg \Pi^n(l)$: the expected profits under the high level of effort are much in the similar model of production externalities, where the profits have a continuous probability distribution, group-based piece rate schemes elicit E.A.E. effort pairs. However, other nonlinear wage schemes may possibly achieve such efficiency.
larger than those under the low level of effort, so that the firm always wants to implement 
\((a_1, a_2) = (h, h)\).

Let the set of feasible socialization levels \(S\) be an interval \([0, S]\) with \(S > 0\). Let 
\(G_h(s_n, s_k) = G(h, s_n, s_k)\) and \(G_l(s_n, s_k) = G(l, s_n, s_k)\). We introduce several assumptions 
on these disutility functions. First, since workers are effort averse, we have the following 
assumption.

**Assumption 4.1.** For all \((s_n, s_k)\), \(G_h(s_n, s_k) > G_l(s_n, s_k)\).

We next assume that the disutility of effort of a worker decreases at a decreasing rate as 
a level of socialization from his co-worker increases.

**Assumption 4.2.** \(G_h(s_n, s_k)\) and \(G_l(s_n, s_k)\) are strictly decreasing and convex in \(s_k\).

Are workers unwilling to increase socialization for co-workers? I believe that this is true 
for sufficiently high levels of socialization. For low levels of socialization, however, worker 
\(n\)'s increasing his socialization level may not raise his disutility: He may be indifferent in 
his socialization level or he may even derive some "social pleasure" from reducing his co-
worker's disutility. Thus, we state the following two assumptions separately: the assumption 
of no social pleasure where the disutility of a worker is nondecreasing in his socialization; and 
the assumption of social pleasure in which his disutility decreases at a decreasing rate in his 
socialization for small socialization levels. However, most results that follow hold under either 
assumption.\(^9\)

**Assumption 4.3a.** (No social pleasure.) \(G_h(s_n, s_k)\) and \(G_l(s_n, s_k)\) are nondecreasing in \(s_n\).

**Assumption 4.3b.** (Existence of social pleasure.) For each \(s_k\), there exist \(\bar{s}\) and \(\underline{s}\) in the in-
terval \((0, S)\) such that \(G_h(s_n, s_k)\) is strictly decreasing and convex in \(s_n \in [0, \bar{s}]\) and \(G_l(s_n, s_k)\) 
is strictly decreasing and convex in \(s_n \in [0, \underline{s}]\).

Whether or not social pleasure exists, we assume that for sufficiently high levels of social-
ization, workers dislike giving more socialization to co-workers. In addition, we assume that 
the disutility increases at an increasing rate for such socialization levels.

\(^9\) These two assumptions have a different effect on the comparison between isolation and 
noncooperative behavior, as we will see later in an example.
Assumption 4.4. For each $s_k$, there exist $s$ and $s'$ in $[0, S)$ such that $G_h(s_n, s_k)$ is strictly increasing and strictly convex in $s_n > s$ and $G_l(s_n, s_k)$ is strictly increasing and strictly convex in $s_n > s'$.

We draw typical graphs of the disutility function without and with social pleasure in Figures 4.1a and 4.1b, respectively.

We next assume that both $G_h$ and $G_l$ are continuously differentiable in $s_n$ and $s_k$ and define

$$
\sigma_h(s_k) = \max\{s \in S \mid s \in \arg \min_{s'} G_h(s', s_k)\}
$$

$$
\sigma_l(s_k) = \max\{s \in S \mid s \in \arg \min_{s'} G_l(s', s_k)\}.
$$

These functions are well defined and continuous by assumptions. The definition implies that $\sigma_h$ and $\sigma_l$ are the worker's best responses to socialization $s_k$ from the other worker when the former worker's effort level is $h$ and $l$, respectively. When several values are possible, we assume that a worker chooses the highest level of socialization that minimizes the disutility of the other worker.

The next two assumptions characterize the workers' best responses. We assume that a worker is more unwilling to increase socialization the harder he works, or the lower his co-worker's socialization level is. These assumptions seem to be quite reasonable.

Assumption 4.5. For each $s_k$ and $s > s'$, $G_h(s, s_k) - G_h(s', s_k) > G_l(s, s_k) - G_l(s', s_k)$. The inequality is strict over the range where $G_h(\cdot, s_k)$ is strictly increasing.

Assumption 4.6. For $s_n \geq s'_n$ and $s_k \geq s'_k$, $G_h(s_n, s_k) - G_h(s'_n, s'_k) \geq G_h(s_n, s_k) - G_h(s'_n, s_k)$ holds. The same inequality holds for $G_l(\cdot)$ as well.

By Assumptions 4.5 and 4.6, we can show that a worker increases his socialization when he works less hard or when his co-worker provides more socialization. The proofs are in Appendix.

Lemma 4.1. (i) Assumption 4.5 implies that for each $s_k$, $\sigma_h(s_k) \leq \sigma_l(s_k)$; (ii) Assumption 4.6 implies that both $\sigma_h(s_k)$ and $\sigma_l(s_k)$ are nondecreasing in $s_k$.

We assume that for each $(a_n, a_h) \in A^2$, there exists a unique Nash equilibrium of socialization.\footnote{See Friedman (1986) for some sufficient conditions for this to be true.} Let $s_{xy} (x, y = h, l)$ be a worker's equilibrium socialization when his effort level is $x$.\footnote{See Friedman (1986) for some sufficient conditions for this to be true.}
and his co-worker's level is $y$. By definition, $s_{hh} = \sigma_h(s_{hh})$, $s_{hl} = \sigma_h(s_{lh})$, $s_{lh} = \sigma_l(s_{hl})$, and $s_{ll} = \sigma_l(s_{ll})$. The next lemma provides some partial order of these four levels of equilibrium socialization. See Appendix for the proof.

**Lemma 4.2.** $s_{hh} < s_{hl} < s_{lh}$ and $s_{hh} < s_{lh} < s_{ll}$ hold. Also $s_{ll} < S$ holds.

This lemma shows that a worker's equilibrium socialization is higher the less hard he or his co-worker works. For example, consider the equilibrium $(s_1, s_2) = (s_{hh}, s_{hh})$. When worker 2 decreases his effort level from $h$ to $l$, then with $s_1 = s_{hh}$ fixed, worker 2 increases his socialization from $s_{hh}$ by Lemma 4.1 (i), which change leads to an increase in worker 1's socialization from $s_{hh}$ by Lemma 4.1 (ii). This process converges to the new equilibrium $(s_{lh}, s_{hh})$ which has higher levels of socialization of both workers than those in the original equilibrium.\(^{11}\)

These equilibrium socialization levels are clearly inefficient: The workers could have achieved Pareto superior socialization given effort levels. To see this, note that both $G_h(s_n, s_k)$ and $G_l(s_n, s_k)$ are smooth and convex in $s_n$ and $s_k$. Thus, for each $x, y = h, l$, at the equilibrium $(s_n, s_k) = (s_{xy}, s_{xy})$, the partial derivatives of $G_h(s_n, s_k)$ and $G_l(s_n, s_k)$ with regard to $s_n$ are zero. The marginal increase from $s_{xy}$ hence does not raise worker $n$'s disutility while it strictly decreases the other worker's disutility. This is true for worker $k$ as well. Thus, workers could have achieved lower disutility levels by simultaneously increasing their socialization levels. This under-socialization is certainly costly to the firm as well. If workers' disutility levels were lower, the expected payments in utility units which implement given effort levels would also be lower.

In particular, this implies that the first-best solution (which is the solution when the firm can monitor each agent's choice of effort and socialization level perfectly) cannot be achieved even when workers are risk neutral. This is in contrast with the standard models of principal-agent relationships without precontractual private information where the principal can achieve the first-best solution by paying the whole marginal profits to each agent. This is also true in the model in Section 3.

The first-best solution is defined as follows. Suppose that effort and socialization levels

\(^{11}\) See de Groote (1988) and Lippman et al. (1987) for more general results.
are publicly observable, and define for \( x = h, l \),

\[
C_{FB}^n(x, s_n, s_k) = \phi^n \left( U^n + G_x(s_n, s_k) \right).
\]

Then the firm can implement \((h, s_n)_{n=1,2}\) by paying a fixed wage \(C_{FB}^n(h, s_n, s_k)\) to worker \( n \) if workers choose \((h, s_n)_{n=1,2}\), and by paying a sufficiently small wage otherwise.\(^{12}\) Let \((s^*_1, s^*_2)\) be the first-best socialization. Since we assume that \((h, h)\) is the first-best effort,

\[
(h, s^*_n)_{n=1,2} \in \arg \max \sum_{a_1, a_2, s_1, s_2} \{ \Pi^n(a_n) - C_{FB}^n(a_n, s_n, s_k) \}. \tag{4.1}
\]

In particular, the first-best social transfers \((s^*_1, s^*_2)\) solve

\[
\min_{s_1, s_2} C_{FB}^1(h, s_1, s_2) + C_{FB}^2(h, s_2, s_1).
\]

When workers are risk neutral, this is equivalent to minimizing \(G_h(s, s)\). Suppose that the solution is unique and denote it by \(s^*_h\). Then by the previous argument, we have \(s^*_h > s_{hh}\).

In summary, we have the following proposition.

**Proposition 4.1.** The first-best solution cannot be achieved when risk neutral workers play noncooperatively.

On the other hand, when workers collude to choose their levels of socialization cooperatively, the firm can achieve the first-best solution. Given \((h, h)\), cooperative workers select

\[
(s_1, s_2) \in \arg \min_{s'_1, s'_2} G_h(s'_1, s'_2) + G_h(s'_2, s'_1)
\]

which yields \((s^*_h, s^*_h)\). Assume without loss of generality \(V^n(w) = w\) and define the wage contract to worker \( n \) by

\[
\pi_n^{w^*} = \pi_n^w - F_n
\]

where \(F_n = \Pi^n(h) - U^n - G_h(s^*_h, s^*_h)\). Then the firm obtains the fixed residual profits equal to the net profits under the first-best solution. And the expected utility of worker \( n \) is given by

\[
U^n(a_n, s_n, s_k) = \Pi^n(a_n) - G(a_n, s_n, s_k) - F_n. \tag{4.2}
\]

\(^{12}\) Some additional assumptions are generally required to ensure that the firm can implement \((h, s_n)_{n=1,2}\) by the payment which guarantees workers exactly their reservation utility level. See Grossman and Hart (1983).
Since \((h, h)\) is the first-best effort levels, by (4.1), given \((s_n, s_k) = (s_{hh}, s_{hh})\), each worker chooses \(a_n = h\) in their self-interests. And given \((h, h)\), workers select \((s_{hh})\) cooperatively. Other possible choice of workers, if any, cannot increase (4.2) because of the definition of the first-best solution in (4.1). Finally, when each worker chooses \((h, s_{hh})\), his expected utility is exactly his reservation utility level. Thus, we have the following.

**Proposition 4.2.** Suppose that workers are risk neutral. If they choose their levels of socialization cooperatively, then the firm can achieve the first-best solution by paying each worker the whole marginal profits from his job.

When workers are risk averse, however, whether worker collusion is better or not is not clear. To see this, we turn to the firm’s problem. Assuming that the firm wants to implement \((a_1, a_2) = (h, h)\) and workers do not collude, she solves the following implementation problem.

\[
\min_{v_1, v_2} \sum_{i \in I_1} P_i^1(h)\phi^1(v_i^1) + \sum_{j \in I_2} P_j^2(h)\phi^2(v_j^2)
\]

subject to, for each \(n = 1, 2,\)

(NIC) \[
\sum_{i \in I_n} P_i^n(h)v_i^n - G_h(s_{hh}, s_{hh}) \geq \sum_{i \in I_n} P_i^n(l)v_i^n - G_l(s_{lh}, s_{hh})
\]

(PC) \[
\sum_{i \in I_n} P_i^n(h)v_i^n - G_h(s_{hh}, s_{hh}) \geq U_n.
\]

The Nash incentive constraint for worker \(n\) states that given worker \(k\)'s choice \((h, s_{hh})\), worker \(n\) has no incentive to reduce effort from \(h\) to \(l\) and accordingly to change his socialization from \(s_{hh}\) to \(s_{lh}\). Since all the constraints hold with equality at the optimum, the optimal solution \((v_i^n)\) satisfies the following two equations for each worker:

\[
\sum_{i \in I_n} (P_i^n(h) - P_i^n(l))v_i^n = G_h(s_{hh}, s_{hh}) - G_l(s_{lh}, s_{hh}); \quad (4.3)
\]

\[
\sum_{i \in I_n} P_i^n(h)v_i^n = U_n + G_h(s_{hh}, s_{hh}). \quad (4.4)
\]

Equation (4.4) shows that the smaller the disutility level of worker \(n\) at the optimum is, the smaller is the firm’s expected payments in utility units. Equation (4.3) states that, assuming
that the right-hand side is positive, at the optimum the increase in worker n’s expected utility on income by raising his effort from l to h is equal to the increase in his disutility by the same change of his effort level and the corresponding change in his socialization, given his co-worker’s equilibrium choice fixed.\textsuperscript{13}

Let \( s^*_{xy} \) be defined by

\[
(s^*_{xy}, s^*_y) = \arg \min_{s_n, s_k} G_x(s_n, s_k) + G_y(s_k, s_n)
\]

for \( x, y = h, l \). We assume the uniqueness of each \( s^*_{xy} \). When workers collude to choose socialization levels, given their effort choice \((x, y)\), they select the socialization \((s^*_{xy}, s^*_y)\). To exclude the possibility of risk sharing among workers, suppose that wage schemes are independent so that collusive workers select their effort levels independently. Then by the argument similar to the case of noncooperative behavior, the optimal solution \( (\nu_i^n) \) for collusive workers satisfies

\[
\sum_{i \in J_n} (P_i^n(h) - P_i^n(l)) \nu_i^n = G_h(s^*_{hh}, s^*_{hh}) - G_i(s^*_{ih}, s^*_{hh}); 
\tag{4.5}
\]

\[
\sum_{i \in J_n} P_i^n(h) \nu_i^n = \overline{U}^n + G_h(s^*_{hh}, s^*_{hh}). 
\tag{4.6}
\]

Since \( G_h(s^*_{hh}, s^*_{hh}) < G_h(s_{hh}, s_{hh}) \), the firm pays smaller expected payments in utility units when workers collude than not. Thus, if the marginal disutility of effort in (4.5) is smaller than that in (4.3), then we can conclude that the firm prefers worker collusion. However, this is not necessarily true. Smaller disutility and smaller marginal disutility have no logical connection here.\textsuperscript{14}

We analyze the case of risk averse workers by an example. The example shows that how a worker’s marginal disutility of effort is affected by his co-worker’s socialization level is important for collusion to be preferred by the firm. In the example, we assume that the harder a worker works, the larger the effect of socialization from his co-worker is. In other words, workers are less effort averse when they receive more socialization from co-workers.

\textsuperscript{13} Here we ignore the problem associated with multiple Nash equilibria. If \((a_n, s_n) = (l, s_{ll})\) for each \( n \) is also an equilibrium given \((\nu^i, \nu^o)\), then this equilibrium yields higher expected utilities to workers. We consider this problem explicitly in the example analyzed below.

\textsuperscript{14} When we assume that collusive workers choose their effort levels as well as their socialization levels cooperatively, the risk sharing argument comes in. Collusion is hence more likely to be preferred by the firm. However, the same problem remains.
Assumption 4.7. For each $s_n$ and $s > s'$, $G_h(s_n, s') - G_h(s_n, s) > G_i(s_n, s') - G_i(s_n, s)$.

We assume that the disutility functions of workers are given by the following quadratic functions: For $s_n, s_k \in S = [0,1],
\begin{align*}
G_h(s_n, s_k) &= \frac{1}{2} \theta (1 - s_k)^2 - \frac{1}{3}s_n s_k + \frac{1}{3}s_n^2 \\
G_i(s_n, s_k) &= \frac{1}{2} \lambda (1 - s_k)^2 - \frac{1}{3}s_n (s_k + \gamma) + \frac{1}{3}s_n^2.
\end{align*}
We assume $\theta > \lambda$ and $0 < \gamma < \frac{1}{4}$. Then these disutility functions satisfy the assumptions stated above. In particular, workers derive social pleasure for low levels of socialization. (Assumption 4.3b is adopted.)

The best response functions are calculated as $a_h(s_k) = \frac{1}{2}s_k$ and $a_i(s_k) = \frac{1}{2}(s_k + \gamma)$. These functions provide the equilibrium socialization as follows: $s_{hh} = 0; s_{il} = 2\gamma; s_{lh} = \frac{1}{3}\gamma$; and $s_{th} = \frac{3}{4}\gamma$. The marginal disutility of effort in (4.3) is then calculated as
\begin{equation}
G_h(s_{hh}, s_{hh}) - G_i(s_{th}, s_{hh}) = \frac{1}{6}(\theta - \lambda) + \frac{1}{2}\gamma^2.
\end{equation}

It is increasing in $\theta - \lambda$ and $\gamma$ because higher values of these parameters lead to a larger difference in disutility between $a = l$ and $a = h$.

We have to exclude the case where given the optimal wage scheme for $(h, h)$, there exists the other equilibrium $(l, l)$ with $(s_{ll}, s_{ll})$. Otherwise, workers select more preferable $(l, l)$. The sufficient condition is
\begin{align*}
\sum_{i \in I_n} P_i^n(l)v^n_i - G_i(s_{ll}, s_{ll}) < \sum_{i \in I_n} P_i^n(h)v^n_i - G_h(s_{lh}, s_{ll})
\end{align*}
or by (4.3),
\begin{equation}
G_h(s_{hh}, s_{hh}) - G_h(s_{ll}, s_{ll}) > G_i(s_{lh}, s_{hh}) - G_i(s_{ll}, s_{ll}).
\end{equation}
Loosely speaking, the condition holds when a worker’s degree of effort aversion is affected sufficiently more by socialization from his co-worker (via Assumption 4.7) than by his own socialization level (Assumption 4.5). In the example, this means that $\theta - \lambda$ is sufficiently larger than $\gamma$. A simple calculation shows that (4.8) is equivalent to
\begin{equation}
\theta - \lambda > \frac{11\gamma}{36(1 - \gamma)}.
\end{equation}
Next we turn to collusive choice of socialization. A calculation shows that in this example, \( s^*_z = 1 \) holds for \( z, y = h, l \): For each effort pair, the level of socialization selected by collusive workers is exactly the upper bound. Under independent wage schemes, the marginal disutility of effort in (4.5) is given by

\[
G_h(1,1) - G_i(1,1) = \frac{1}{4} \gamma. \tag{4.10}
\]

It is increasing in \( \gamma \) with the rate \( \frac{1}{4} \), which is higher than the rate \( \frac{1}{2} \) in (4.7). On the other hand, (4.7) is more affected by \( \theta - \lambda \) than (4.10). (That (4.10) does not depend on \( \theta - \lambda \) is an artifact of the example.) This is because levels of socialization are higher under collusion than under noncooperative behavior.

Since \( G_h(1,1) < G_h(s_{hh}, s_{hh}) \), if the value in (4.10) is smaller than that in (4.7), collusion is better than noncooperative behavior. The condition is given by

\[
\theta - \lambda \geq \gamma(1 - \frac{1}{4} \gamma). \tag{4.11}
\]

However, this condition is not sufficient for collusion to be the best: The firm can also control workers’ socialization by isolating them. Isolation can enforce zero socialization and may reduce the marginal disutility of effort.

In fact, in this example, isolation is better than noncooperative behavior: The marginal disutility of effort under isolation is given by

\[
G_h(0,0) - G_i(0,0) = \frac{1}{4} (\theta - \lambda). \tag{4.12}
\]

In addition, Since \( s_{hh} = 0 \), \( G_h(s_{hh}, s_{hh}) = G_h(0,0) \). Isolation hence leads to the same level of the disutility and the lower marginal disutility of effort. The superiority of isolation results from two properties of the example: \( s_{hh} = 0 \) and the existence of social pleasure. Although the level of disutility is generally higher under isolation than noncooperative behavior, the first property equates these disutility levels. When social pleasure exists, deviation from \( h \) to \( l \) under isolation becomes less attractive to workers since \( G_i(0,0) \) is higher than when there is no social pleasure. In the example, no social pleasure means that \( G_i(s_n, s_h) \) is equal to \( G_i(\sigma_i(s_h), s_h) \) for \( s_n \leq \sigma_i(s_h) \). Then

\[
G_h(0,0) - G_i(\frac{1}{2} \gamma, 0) = \frac{1}{4} (\theta - \lambda) + \frac{1}{4} \gamma^2. \tag{4.13}
\]
Equilibrium socialization levels do not change under no social pleasure. Thus, by (4.7), when social pleasure does not exist, isolation is worse than noncooperative behavior. In fact, this is a general result: If there exists no social pleasure (Assumption 4.3a), isolation cannot be better than noncooperative behavior.\(^{15}\)

Under the existence of social pleasure, collusion is hence the best if (4.10) is not larger than (4.12), that is,

\[ \theta - \lambda \geq \gamma. \tag{4.14} \]

On the other hand, collusion is best under no social pleasure if (4.11) holds. Both conditions (4.11) and (4.14) satisfy (4.9) so that there is no problem associated with multiple Nash equilibria. In either case, collusion is more likely to be optimal the higher \( \theta - \lambda \) or the lower \( \gamma \) is. A sufficiently high \( \theta - \lambda \) or low \( \gamma \) implies that the increase of socialization by collusion (\( s_{hh}^* > s_{hh} \)) reduces the other's marginal disutility of effort (via Assumption 4.7) sufficiently more than it raises the own marginal disutility (via Assumption 4.5). Whether such a condition is satisfied or not will depend on job characteristics and workers' personalities to some extent. Besides them, however, it seems more likely to be satisfied when workers share their work floors and so their friendly relationship is important than when their tasks are more individualistic as professional ones.

5. Discussions

The previous sections identify cases where the firm prefers worker collusion in the symmetric situation as follows:

1. When workers have no interaction and an independent wage contract is employed, the firm is better off by enforcing worker collusion and designing an interdependent wage scheme, which creates interaction among workers and enables them to share risks among themselves.

\(^{15}\) There is another case in which isolation may be better than noncooperative behavior. Suppose that workers behave noncooperatively as follows: workers first select their effort levels simultaneously, and after observing each other's effort choice they choose their socialization levels. Then the marginal disutility of effort changes to \( G_h(s_{hh}, s_{hh}) - G_i(s_{hh}, s_{ih}) \), which is larger than (4.9): Since the effort choice of a worker can affect the other worker's choice of socialization, the deviation from \( h \) to \( l \) becomes easier. Then it turns out that in our example isolation is better than noncooperative behavior even under the assumption of no social pleasure.

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2. When workers' relationship is interrelated through production externalities, the firm can attain technologically efficient effort combination by designing team-based wage schemes. Then worker collusion allows her to improve risk sharing by reducing the risk imposed on each worker.

3. If team production exists so that only the sum of the profits from all the jobs is observable, the firm prefers worker collusion by the same reason as 2.

4. When workers interact with each other through nonproductive interpersonal activities such as socialization, worker collusion results in higher and efficient levels of socialization than noncooperative behavior does. When workers are risk neutral, this fact allows the firm to achieve the first-best solution only under worker collusion.

Some characteristics of the stylized Japanese firm are similar to those summarized above: Many observers find that in the Japanese firm, responsibility for decisions is shared among team members and employees are engaged in much higher levels of socialization. (See Lincoln and McBride (1987) for a survey and references of qualitative and quantitative comparative researches. The evidences for most of the characteristics of the Japanese firm that we discuss in this section have been found in this survey.) Since neither such risk sharing nor high levels of socialization seems to lead Japanese workers to exert low effort levels, worker collusion is likely to be prevailing in Japanese organizations.

Then the question is: Why do Japanese workers collude to make decisions cooperatively? Culturalists' answer will be group-oriented attributes of Japanese. I do not intend to reject this view. However, I would like to argue that organization structures and personnel policies of the stylized Japanese firm are also designed so as to elicit worker collusion.

Economists' explanation of cooperation by self-interested agents involves repetition of a one-shot decision making situation. Loosely speaking, the literature on repeated games shows that cooperation can be achieved as a perfect Nash equilibrium of the repeated game if the one-shot game is repeated sufficiently many times and players discount their future payoffs sufficiently small. See, for example, Fudenberg and Maskin (1986a) and Kreps et al. (1982). Green and Porter (1984) and Abreu et al. (1986) remind us that perfect monitoring is important for the result above. (See also Kreps (1984).) If neither worker can observe his co-workers' choice after each stage game, then cooperation must sometimes break down. Fudenberg and Maskin (1986c) and Radner, Myerson and Maskin (1986) in fact show that in such a situa-
tion, equilibrium average payoffs are generally bounded away from the Pareto frontier of the one-shot game.\textsuperscript{16}

The practice of the "lifetime employment" in Japan will guarantee long-term relationships among regular workers. It is also the case that there are far more opportunities for interpersonal contacts in Japanese firms than its Western counterparts. These features enable workers to reciprocate cooperative levels of effort, help, or socialization more easily. In addition, fluid job classifications and the practice of frequent job rotations will facilitate the mutual monitoring of workers. Kagono et al. (1985) conclude from their survey research that information sharing and interpersonal interactions are important organizing methods of Japanese firms.

The literature on human cooperation identifies other factors to derive cooperation. (See Dawes and Thaler (1988) for a survey and references.) It is typical that cooperation can be observed even in one-shot experiments. And some experiments find that cooperation rates are high when discussion is allowed before the decision and each subject is told that his/her cooperation improves the welfare of the other members in the team to which he/she belongs, rather than those in the other team. The importance of this "group identity" is particularly interesting since it is now well known that work groups are more important bases of Japanese organizations than occupational positions. (See Aoki (1989) as well as those cited above.) Though economics cannot provide any explanation for this experimental result, the emphasis on work groups as functional units seems to be the most important device to elicit worker collusion.

Though we focused on the benefits of Japanese practices as devices to encourage cooperation, the policies discussed above may introduce costs as well. Tirole (1986) fully discusses the costs of long-term relationships (the possibility of detrimental collusion). Fluid job demarcation and teamwork are likely to create team production, which in turn leads the firm to prefer worker collusion. Team production however makes it hard to identify the contribution of each worker. Thus, whether to introduce teamwork or an unambiguous division of labor must be examined. (See Itoh (1988) for a preliminary work.) Finally, the emphasis on group identity may cause a new problem of inter-group rivalry. The cooperation within each group does not necessarily result in the cooperation of the organization as a whole. Both Aoki

\textsuperscript{16} In a game with two workers, if one of them can monitor the other's choice, then cooperation can be approximately attained (Fudenberg and Maskin, 1986b; Radner, 1986).
(1989) and Lincoln and McBride (1987) mention the possibility of such rivalry in Japanese firms. The comparative analysis of organizations will not complete without examining the tradeoff between these costs and benefits.
References


Appendix

Proof of Proposition 2.1.

In the proof, we allow the case in which the noise terms are not independent but the optimal wage scheme is still an independent one. Let $P_{ij}(a_n, a_k)$ be the joint probability of $\pi^n = \pi_i$ and $\pi^k = \pi_j$. In particular, when the noise terms are stochastically independent, $P_{ij}(a_n, a_k) = P_i(a_n)P_j(a_k)$. Because of no production externality, the marginal probability of $\pi^n = \pi_i$ does not depend on $a_k$ even in the non-independent case where it is defined as $P_i(a_n) = \sum_j P_{ij}(a_n, a_k)$.

Suppose that $(v^1)$ and $(v^2)$ are the optimal wage scheme for $(a, a)$. We find a new wage schedule, which is interdependent, such that under the new schedule collusive workers choose $(a, a)$, achieve their reservation utility levels, and that the firm prefers the new ones to the original independent ones. Define the new wage schedule $(\tilde{v}^n_i)$ by $\tilde{v}^n_i = \frac{1}{2}v^1_i + \frac{1}{2}v^2_i$. Then since $\tilde{v}^n_i = \tilde{v}^n_j$ for $i, j \in I$, $\frac{1}{2}v^1_i + \frac{1}{2}v^2_i = \frac{1}{2}v^1_j + \frac{1}{2}v^2_j$ holds. Thus, the sum of the workers' expected utilities under the new wage scheme is equal to

$$\left( \sum_{i \in I} P_i(a_n)v^1_i - G(a_1) \right) + \left( \sum_{j \in I} P_j(a_n)v^2_j - G(a_2) \right), \quad (A1)$$

which is the sum of their expected utilities under the original one. Since $a$ maximizes each worker's expected utility under the original scheme, $(A1)$ implies that cooperative workers also choose $(a, a)$ under the new schedule. Also a simple calculation shows that

$$\sum_{i \in I} \sum_{j \in I} P_{ij}(a, a)\tilde{v}^n_i = \frac{1}{2} \sum_{i \in I} P_i(a)v^1_i + \frac{1}{2} \sum_{j \in I} P_j(a)v^2_j \geq \bar{U} + G(a)$$

since $(v^1, v^2)$ satisfies (PC). As the last step, the expected wage payments of the firm are calculated as follows:

$$\sum_{i \in I} \sum_{j \in I} P_{ij}(a, a)(\phi(\tilde{v}^1_i) + \phi(\tilde{v}^2_j)) = \sum_{i \in I} \sum_{j \in I} P_{ij}(a, a)(\phi(\frac{1}{2}v^1_i + \frac{1}{2}v^2_j) + \phi(\frac{1}{2}v^2_j + \frac{1}{2}v^1_i))$$

$$\leq \sum_{i \in I} \sum_{j \in I} P_{ij}(a, a)(\phi(v^1_i) + \phi(v^2_j))$$

$$= \sum_{i \in I} P_i(a)\phi(v^1_i) + \sum_{j \in I} P_j(a)\phi(v^2_j).$$

The last expression is the expected wage payments under the original wage scheme. The inequality is strict when $(a, a)$ are not least costly effort. Thus, the firm can implement $(a, a)$ with lower costs by the interdependent wage schedule when workers collude than not.
Proof of Proposition 3.2.

Let \( e = (a, b) \) be an E.A.E. effort pair the firm wants to implement. If \( b = 0 \), the wage scheme defined in the proof of Proposition 2.1 can implement \( e \) with lower costs. Thus, suppose \( b > 0 \). Define the new wage scheme for collusive workers by

\[
\hat{v}_{ij} = \frac{1}{4} v_{ij} + \frac{1}{4} (\bar{U} + G(a + b)).
\]

Since \((v_{ij})\) satisfies (3.2), \((\hat{v}_{ij})\) satisfies (3.3) so that collusive workers select \( e \). The expected utility of worker \( n \) is calculated as

\[
\sum_{i \in I} \sum_{j \in I} P_{ij}(e, e) v_{ij} = \frac{1}{4} \sum_{i \in I} \sum_{j \in I} P_{ij}(e, e) v_{ij} + \frac{1}{4} (\bar{U} + G(a + b)) \geq \bar{U} + G(a + b)
\]

so that \((\hat{v}_{ij})\) satisfies (PC). Finally, the firm's expected payments are calculated as follows:

\[
\sum_{i \in I} \sum_{j \in I} P_{ij}(e, e) (\phi(\hat{v}_{ij}) + \phi(\hat{v}_{ji}))
\]

The last inequality holds because of the property that each worker receives exactly his reservation utility level (Grossman and Hart, 1983, Proposition 2) and of Jensen's inequality. Thus, the firm's expected payments are lower under \((\hat{v}_{ij})\), which completes the proof.

Proof of Lemma 4.1.

Proof of (i): Suppose instead \( \sigma_h(s_k) > \sigma_i(s_k) \). Then by Assumption 4.5, \( G_h(\sigma_h(s_k), s_k) - G_h(\sigma_i(s_k), s_k) \geq G_i(\sigma_h(s_k), s_k) - G_i(\sigma_i(s_k), s_k) \) holds. The left-hand side is at most as small as zero by the definition of \( \sigma_h(\cdot) \). On the other hand, the right-hand side is strictly positive since \( G_i(\cdot, s_k) \) must be strictly increasing for \( s > \sigma_i(s_k) \) by the definition of \( \sigma_i(\cdot) \). A contradiction.

Proof of (ii): Let \( s_k > s'_k \) and suppose \( \sigma_h(s_k) < \sigma_h(s'_k) \). Then by Assumption 4.6, we have \( G_h(\sigma_h(s'_k), s'_k) - G_h(\sigma_h(s_k), s_k) \geq G_h(\sigma_h(s'_k), s_k) - G_h(\sigma_h(s_k), s_k) \). The left-hand side is at most as small as zero by the definition of \( \sigma_h(\cdot) \). On the other hand, the right-hand side is strictly positive since \( G_h(\cdot, s_k) \) must be strictly increasing for \( s > \sigma_h(s_k) \). A contradiction.
Proof of Lemma 4.2

This proposition is a special case of more general results by de Groote (1988) and Lippman et al. (1987). The proof here follows de Groote (1988).

We compare two equilibria \((s_n, s_k) = (s_{hh}, s_{hh})\) and \((s_n, s_k) = (s_{hl}, s_{lh})\) here. The other comparisons proceed similarly. When worker \(k\) decreases his effort from \(h\) to \(l\), given worker \(n\)'s social transfer \(s_{hh}\), worker \(k\) chooses \(s_{k}^1 = \sigma_l(s_{hh}) \geq \sigma_h(s_{hh}) = s_{hh}\) by Lemma 4.1 (i). Then, given \(s_{k}^1\), worker \(n\) chooses \(s_{n}^1 = \sigma_h(s_{k}^1) \geq \sigma_h(s_{hh}) = s_{hh}\) by Lemma 4.1 (ii). Worker \(k\) then chooses \(s_{k}^2 = \sigma_l(s_{k}^1) \geq \sigma_l(s_{hh}) = s_{hh}\). By continuing this process, we obtain nondecreasing sequences \(s_{hh} \leq s_{n}^1 \leq s_{n}^2 \cdots \) and \(s_{hh} \leq s_{k}^1 \leq s_{k}^2 \cdots\). Since \(S\) is compact, these sequences converge to \((s_n, s_k)\) in \(S\), which is \((s_{hl}, s_{lh})\) by the continuity of \(\sigma_h(\cdot)\) and \(\sigma_l(\cdot)\). Thus, \(s_{hh} \leq s_{hl}\) and \(s_{hh} \leq s_{lh}\). Finally, \(s_{ll} < S\) follows from Assumption 4.4.
Isoquants of \( p(a, b) \)

The path of E.A.E. effort pairs

Indifference curves (\( a+b= \text{const.} \))

FIGURE 3.1.
Disutility of workers

$G_h(\cdot, s_k')$

$G_h(\cdot, s_k)$

$s_k > s_k'$

$G_1(\cdot, s_k')$

$G_1(\cdot, s_k)$

$\sigma_h(s_k'), \sigma_h(s_k)$

$\sigma_1(s_k'), \sigma_1(s_k)$

$s_n$

FIGURE 4.1a.
Disutility of workers

FIGURE 4.1b.