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From Urban Agglomeration To Dispersion

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Abstract:

Urban agglomeration becomes increasingly important due to globalization of world economies. This paper is a general equilibrium analysis of urban agglomeration economies due to product variety and agglomeration diseconomies due to intracity congestion in a two-city system framework. Special attention is paid to the effects of transportation costs on urban concentration and dispersion.

Our main result is that dispersion necessarily takes place when the transportation cost is very low. We also conduct a numerical calculation using specific parameter values, and depicted a structural transition from dispersion to agglomeration, and then to re-dispersion when the transportation cost decreases monotonically. Finally, we observe that agglomeration is usually bad as compared to dispersion from a welfare point of view.

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1. Introduction

The major driving force of urban agglomeration is increasing returns to scale at the urban level, which is localization economies and urbanization economies. Agglomerating in a city, firms can exchange information by face-to-face communications and reduce transaction costs between firms. In addition, consumers enjoy a variety of differentiated products. These are typical externalities favorable to urban concentration.

On the other hand, urban activities are dispersed by the existence of commuting and transporting goods. A level of urban concentration is thus determined by the balance between the agglomeration force and the dispersion force.

In general, technological progress and improvements in transportation facilities decrease the transportation costs. Such a change normally induces the agglomeration force dominant relative to the dispersion force, leading to urban concentration. This is one of Krugman's (1991) major results, and is extensively discussed and recapitulated in Fujita and Thisse (1996). Urban concentration is in accord with the emergence of large cities all over the world after the Industrial Revolution, and more recently concentration of firms and population in the Tokyo Metropolitan Area.

However, this result is not always true in somewhat different settings, such as Mun (1995) and Morisugi, Ohno, Ueda and Koike (1995). In reality, population concentration in large cities took place until around 1970 whereas the concentration ceased or dispersion has been taking place after 1970 in most of developed countries (Vining, Pallone and Plane, 1981). In fact, according to Garreau (1991), population in large cities tends to decrease while "edge cities" emerge and become gradually large especially in the United States.

Accordingly, we must conclude from these observations that some assumptions in Krugman's model is wrong. We would like to claim that what is lacking among his assumptions is land consumption for residential use. In this paper, we exactly follow Krugman's model except that *workers consume land* for residence. Incorporation of land enables us to take account of the impacts of price mechanism in land rent market on urban concentration.

We describe an overall model structure in Section 2. Analytical results derived from the model are shown in Section 3, and numerical calculations and some economic implications are given in Section 4. Section 5 concludes.

2. The Model

There are two regions, each containing a CBD (Central Business District) with negligible space. Homogeneous workers live around the CBD and commute to it. The utility function of a representative worker living in region k is expressed as

$$U_k = C_{Mk}^{\mu} C_{Sk}^{\gamma} C_{Ak}^{1-\mu-\gamma}$$

$$\text{and} \quad C_{Mk} = \left(\sum_{i=1}^{N} c_{ik}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad \text{for } k=1,2,$$

where c_{ik} is the consumption of manufacturing good i in region k, C_{Sk} is the consumption of housing space in region k, C_{Ak} is the consumption of agricultural products in region k and is a numéraire. The parameters μ, γ and σ are positive with $\mu+\gamma<1$ and $\sigma \geqq 1$. N is the total number of goods.

The income constraint for representative worker is given by

$$\sum_{i=1}^{N} p_{ik} c_{ik} + rC_{Sk} + C_{Ak} + tx = w_k ,$$

where p_{ik} is the price of good i in region k, r is the land (housing) rent, t is the commuting cost per unit distance, x is the distance from the CBD, and w_k is the wage rate in region k. Here, we assume that the commuting cost is proportional to the commuting distance.

From the first-order conditions for utility maximization, we have

$$\frac{c_{i1}}{c_{i2}} = \left(\frac{p_2}{p_1 \tau} \right)^{\sigma}, \tag{1}$$

where subscripts 1 and 2 denote regions and τ is the fraction of the good arrived to another region since we employ Samuelson's iceberg transportation cost. That is, τ is an inverse index of transportation costs.

The total number of population is normalized to 1, and the number of peasants in each region is fixed and given by $(1-\mu)/2$. Denote the number of manufacturing workers in region k by L_k, then $L_1+L_2=\mu$ holds. While the peasants are completely immobile, manufacturing workers are freely migrate according to the utility difference between two regions in the long run. So as to simplify the notations, we drop subscript k hereafter unless necessary.

The production of an individual manufacturing good i involves a fixed cost and a constant marginal cost:

$$l_i = \alpha + \beta c_i,$$

where l_i is the labor input for good i and c_i is the output of good i. Each firm maximizes its net profit $p_i c_i - w(\alpha + \beta c_i)$ with respect to p_i given the constant elasticity of substitution σ in a monopolistic-competition market. The first-order conditions yield

$$\frac{p_2}{p_1} = \frac{w_2}{w_1}, \tag{2}$$

where subscripts 1 and 2 denote regions.

Manipulating the above equations, we have the ratio of the utility functions:

$$\frac{U_1}{U_2} = \frac{w_1 - tx_1}{w_2 - tx_2}\left[\frac{fw_1^{-\sigma+1} + (1-f)\left(\frac{w_2}{\tau}\right)^{-\sigma+1}}{f\left(\frac{w_1}{\tau}\right)^{-\sigma+1} + (1-f)w_2^{-\sigma+1}}\right]^{\frac{\mu}{\sigma-1}}, \tag{3}$$

where $f \equiv \frac{L_1}{L_1 + L_2} \in [0,1]$. Notice that a long-run equilibrium is attained when (3) is equal to 1.

Calculating the land market equilibrium in a monocentric urban model as shown in the Appendix, we can express the number of manufacturing workers in each region as:

$$L_k = \frac{2\pi r_A \int_0^{x_k} x(1 - tx / w_k)^{1/\gamma - 1} dx}{\gamma w_k (1 - tx_k / w_k)^{1/\gamma}} \quad \text{for } k=1,2, \tag{4}$$

where r_A is the agricultural rent and x_k is the distance between the CBD and the city border.

Following Krugman (1991), define z_{1k} as the ratio of region k expenditure on region 1 products to that on region 2 products for k=1,2. Using (1) and (2), z_{1k} can be written as follows:

$$z_{11} = \frac{L_1}{L_2}\left(\frac{w_1 \tau}{w_2}\right)^{-\sigma+1}, \tag{5a}$$

$$z_{12} = \frac{L_1}{L_2}\left(\frac{w_1}{w_2\tau}\right)^{-\sigma+1}. \tag{5b}$$

In each region, the total income of workers is equal to the total spending of workers, i.e.,

$$w_1 L_1 = \mu\left[\left(\frac{z_{11}}{1+z_{11}}\right)Y_1 + \left(\frac{z_{12}}{1+z_{12}}\right)Y_2\right], \tag{6a}$$

$$w_2 L_2 = \mu\left[\left(\frac{1}{1+z_{11}}\right)Y_1 + \left(\frac{1}{1+z_{12}}\right)Y_2\right]. \tag{6b}$$

The total income in each region is given by:

$$Y_k = \frac{1-\mu}{2} + \varphi_k w_k L_k \qquad \text{for } k=1,2, \tag{7}$$

where
$$\varphi_k = \frac{\int_0^{x_k} (1 - tx / w_k)^{1/\gamma} dx}{2\pi \int_0^{x_k} x(1 - tx / w_k)^{1/\gamma - 1} dx} \qquad \text{for } k=1,2. \tag{8}$$

Setting (3) equal to 1, the nine equations of (3)-(7) determine the nine variables of f, x_1, x_2, w_1, w_2, Y_1, Y_2, z_{11} and z_{12} in general equilibrium. Since the system of nine equations are highly nonlinear, we cannot obtain analytical solutions in explicit forms.

3. Some Results

Our main focus is the impacts of the interregional transportation cost τ on the city-system structure. Due to the complexity of the general equilibrium model, however, analytical results are limited to cases of the infinite (τ =0) and zero (τ =+∞) transportation costs.

Proposition 1

When τ approaches 0, urban concentration occurs if

$$\mu > \frac{\sigma - 1}{\sigma}. \tag{9}$$

Proof

Setting f=1 in (3), we have

$$\frac{U_1}{U_2} = \left(\frac{w_1 - tx_1}{w_2}\right)\tau^{-\mu}, \tag{10}$$

where $w_1 = \dfrac{1-\mu}{1-\varphi_1\mu}$ and $w_2 = \left(\dfrac{1-\mu}{1-\varphi_1\mu}\right)^{\frac{\sigma-1}{\sigma}}\left[\tau^{\sigma-1}\left(\dfrac{1-\mu}{2}+\dfrac{1-\mu}{1-\varphi_1\mu}\varphi_1\mu\right)+\tau^{-\sigma+1}\left(\dfrac{1-\mu}{2}\right)\right]^{\frac{1}{\sigma}}$.

It is easily shown that if τ approaches 0 and (9) holds, then (10) becomes $+\infty$. This implies that manufacturing concentration is stable. ■

Equation (9) shows that in the case of prohibitively high transportation costs of manufacturing goods, firms concentrate when (i) the elasticity of substitution σ is low, and (ii) the ratio of manufacturing employment (and that of manufacturing expenditure) μ is high. The former implies that when the substitutability between manufacturing goods is low under very high transportation costs, it is not attractive for firms and workers to locate in a desert. Implications of the latter are straightforward. Agglomeration takes place when the manufacturing is important relative to the agriculture and residence.

The case of low τ would correspond to ancient times while the case of high τ modern times. Proposition 2 is applied to the latter.

Proposition 2

When τ approaches 1, even dispersion is the unique stable equilibrium for any value of the parameters.

Proof

Setting $\tau=1$ in (3), we have $U_1/U_2=(w_1-tx_1)/(w_2-tx_2)$. From (5), $z_{11}=z_{12}$. From (5) and (6), we obtain $w_1=w_2$. Using these results, we can derive $\partial(U_1/U_2)/\partial f<0$ for all $f\in[0,1]$. That means $(L_1, L_2)=(\mu/2, \mu/2)$ is globally stable. ■

Proposition 2 is a very strong result. It implies that dispersion is the ultimate state of the city system for any initial condition and for any parameter value. When the interregional transportation costs become negligible due to the technological progress, firms and workers will be dispersed. It is thus inferred that urban agglomeration will cease in the far future.

It should be noticed that the same is true for information service industries. In this case, interregional transmission of information is comparable to interregional transportation of commodities. Technical progress in telecommunications works similar to that in transportation.

4. Illustration

So far, we confined our analysis to the extreme cases. In order to understand the model further, let us examine it using specific values of parameters.

Consider the case of $\sigma=4$, $\mu=0.3$, $\gamma=0.5$, $t=1$ and $r_A=10$. We know from Proposition 1 that when $\tau=0$, concentration is not an equilibrium. Since the elasticity of substitution σ is high and the manufacturing share μ is low, dispersion of firms and workers is an equilibrium instead.

Numerical calculation is conducted as follows. Given the above parameter values, we fix the value of f, and set initial values of x_k's. Then, w_k's are determined by (4), φ_k's are by (8), z_{lk}'s are by (5), and Y_k are by (7). By putting these values into (6), we evaluate the differences between the RHS's and LHS's. If they are large, we change the values of x_k's and repeat the same calculation procedure until they become small enough.

Now, setting (3) equal to 1, we have a collection of equilibria. Eliminating unstable ones, we obtain the stable equilibrium distribution of manufacturing workers according to the transportation cost parameter τ as follows:

$$
\begin{aligned}
(L_1, L_2) &= (\mu/2,\ \mu/2) && \text{for } 0 \leqq \tau < 0.358 && \text{[Figure 1a]},\\
&= (\mu/2,\ \mu/2), (0,\ \mu), (\mu, 0) && \text{for } 0.358 < \tau < 0.651 && \text{[Figure 1b]},\\
&= (0,\ \mu), (\mu, 0) && \text{for } 0.651 < \tau < 0.913 && \text{[Figure 1c]},\\
&= (\mu^*,\ \mu-\mu^*), (\mu-\mu^*,\ \mu^*) && \text{for } 0.913 < \tau < 1 && \text{[Figure 1d]},\\
&= (\mu/2,\ \mu/2) && \text{for } \tau = 1,
\end{aligned}
$$

where $\mu^* \in (0,\ \mu/2)$.

The existence of multiple equilibria means indeterminacy of the state. So as to avoid such multiple equilibria, we start from the case of very high transportation cost, which then decreases monotonically. That is, we start from $\tau = 0$ and increase it monotonically until $\tau = 1$.

In doing so, the equilibrium distribution of manufacturing firms/workers is given by

$$
\begin{aligned}
(L_1, L_2) &= (\mu/2,\ \mu/2) && \text{for } 0 < \tau < 0.651, \\
&= (0,\ \mu), (\mu, 0) && \text{for } 0.651 < \tau < 0.913, \\
&= (\mu^*,\ \mu - \mu^*), (\mu - \mu^*,\ \mu^*) && \text{for } 0.913 < \tau < 1, \\
&= (\mu/2,\ \mu/2) && \text{for } \tau = 1,
\end{aligned}
$$

To visualize the transition of stable equilibria due to the change in τ, we depict the utility in each region with respect to the employment distribution f in Figure 1. The big dots are stable equilibria.

Figure 1 shows that *dispersion of firms and workers takes place for the small or large transportation cost whereas concentration occurs for the intermediate transportation cost.* We may interpret this finding in the following manner.

Cases (a) and (b): When the cost of transporting goods is sufficiently high, interregional trade seldom takes place, and each region is nearly self-sufficing. In such a case, the utility level in each region is determined mainly by the amount of housing space and the variety of manufacturing goods within the region. In the region with the smaller number of firms and workers ($\equiv$ small city), local monopoly prevails in manufacturing industry leading to higher prices of manufacturing goods and a higher wage rate. So, workers will consume more agricultural products and more housing space there. Because the prices and the wage rate rise proportionally according to equation (2), workers must be better off. The reverse is true in the region with many firms and workers ($\equiv$ large city). Since there are a variety of manufacturing goods, their prices are lower and the wage rate is lower, and so workers consume less agricultural products and less space leading to a lower utility level.

It should be noted that in these circumstances the bigness of export industry does not enhance the regional welfare level. Instead, workers in the small city are well off (and peasants in the large city are well off). The above is a discussion of short-run equilibrium: it is valid only when the workers and firms cannot migrate between regions. However, since they migrate costlessly from the large region with the lower utility level in the long run, both

Case (c): When the transportation cost decreases such that the parameter τ exceeds a critical value (0.651), sudden agglomeration takes place, i.e., every manufacturing firms and workers migrate to one region. A decrease in the transportation cost encourages firms in the large city to export their manufacturing goods, and tends to diminish the price and wage differentials, which destroys the local monopoly in the small city. In the large city, variety in manufacturing goods (agglomeration force) becomes more important than scarcity in residential land (dispersion force) leading to an increase in the utility level. Thus, agglomeration becomes stable. Note that while the change in τ is continuous, catastrophic agglomeration takes place at the critical value.

Case (d): When the transportation cost gets sufficiently high, however, the constraint of residential land outweighs the benefits of urban agglomeration, therefore leading to re-dispersion of firms and workers. That is, when the transportation cost parameter τ approaches one, which is equivalent to zero transportation cost, the location of production and consumption of manufacturing goods does not matter any longer. The only concern in location decision is the space for housing. Thus, as shown in Proposition 2, firms and workers will be re-dispersed in the far future.

So far, we considered the change in the *interregional* transportation cost τ while the *intraregional* transportation cost t remained constant. But, it may be natural to think that technological progress reduces the intraregional transportation cost too, which might hinder the future re-dispersion.

However, the reduction in the intraregional transportation cost is limited since it is commuting. It would be true that technological progress decreases the interregional transportation costs of commodities substantially, but decreases the intraregional transportation costs of commuters little. This is due to the existence of rush-hour congestion of roads and trains, which cannot be substantially reduced. In other words, compared to pecuniary costs of commodity flows, time costs of commuting are difficult to overcome. Hence, Proposition 2 would be valid and important.

Finally, we would like to mention on welfare considerations. Since the profits of the firms are zero, we pay attention to the utility level of workers. As the transportation cost decreases, the equilibrium utility level increases monotonically. However, there is an exception at the critical value (τ =0.651) when the sudden agglomeration occurs. In this instance, the equilibrium utility level declines sharply. This implies that agglomeration is worse than dispersion from a social welfare point of view. In fact, casual observations of Figure 1 indicate that agglomeration is not optimum for all four values of τ while dispersion attains a high utility level. In other words, urban agglomeration policies are by no means justified in such cases. Needless to say, the discrepancy between equilibrium and optimum stems from urban externalities: product variety and congestion.

5. Conclusion

We have presented a general equilibrium model of agglomeration and dispersion of firms and workers in a two-city system setting. We obtained a condition of agglomeration in the case of very high transportation cost in Proposition 1. We then showed that dispersion necessarily occurs in the case of very low transportation cost in Proposition 2.

We also conducted a numerical analysis by using particular parameter values, and illustrated a transition from dispersion to agglomeration, and then from agglomeration to re-dispersion when the transportation cost decreases monotonically. Finally, we found that welfare level in the agglomerated state is usually lower than that in the dispersed state.

Appendix: Equilibrium in an Urban Land Market

Let us calculate the land market equilibrium as is done in monocentric urban economic models. Each consumer maximizes the utility with respect to the agricultural and manufacturing goods, housing space and location subject to the income constraint. From the first-order conditions, we obtain the well-known location equilibrium condition:

$$r'(x)C_S(x)+t=0.$$

Eliminating the variables c_i and C_A by the first-order conditions, we have

$$r(x)C_S(x)/\gamma + tx = w.$$

These two equations yield

$$-t/(w-tx) = \gamma\, r'(x) / r(x),$$

or

$$\log(w-tx) = \gamma \log(r(x)) + \text{Const.}$$

The rent curve is therefore given by

$$r(x) = r_o(1\text{-}tx/w)^{1/\gamma},$$

where r_o is the rent at the CBD. At the city border, this rent is equated to the agricultural rent r_A:

$$r_A = r_o(1\text{-}tx_k/w)^{1/\gamma},$$

where x_k is the city border.

The population density is given by

$$1/C_S = r_o\,(1\text{-}tx/w)^{1/\gamma\text{-}1}/(\gamma\, w).$$

Since the number of urban residents is equal to the number of manufacturing workers in each region k, we obtain

$$L_k = \int_o^{x_k} \frac{2\pi x}{C_S(x)} dx = \frac{2\pi r_A \int_0^{x_k} x(1 - tx / w_k)^{1/\gamma - 1} dx}{\gamma w_k (1 - tx_k / w_k)^{1/\gamma}} \qquad \text{for } k\text{=1,2,}$$

which is equation (4).

References

Fujita, M. and J.-F. Thisse (1996) "Economics of agglomeration," mimeo., forthcoming in *Journal of the Japanese and International Economies.*

Garreau, J. (1991) *Edge city: Life on the new frontier*, Doubleday, New York.

Krugman, P. (1991) "Increasing returns and economic geography," *Journal of Political Economy,* Vol.99, 483-499.

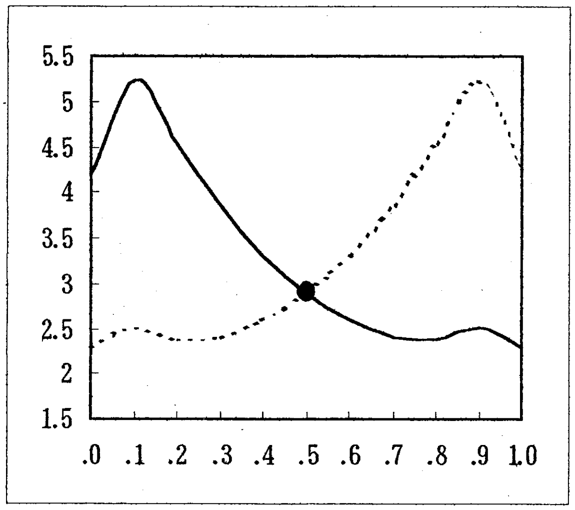
Morisugi, H., E. Ohno, T. Ueda and A. Koike (1995)"Impact of transport investment under mono-polar concentration," paper presented at WCTR, Sydney.

Mun, S. (1995) "Kotsu Network to Toshi Kibo Bunpu," paper presented at ARSC, Tokyo (in Japanese).

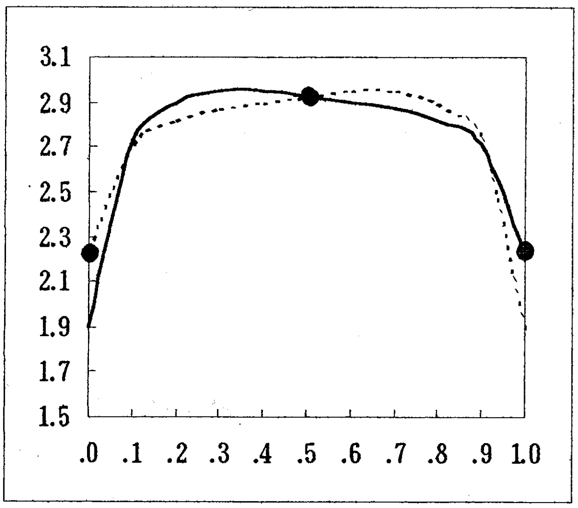
Vining, D.R., R. Pallone and D. Plane (1981) "Recent migration pattern in the developed world: A classification of some differences between our and IIASA's findings," *Environment and Planning A,* Vol.13, 243-250.

Figure 1 Change in the transportation cost parameter

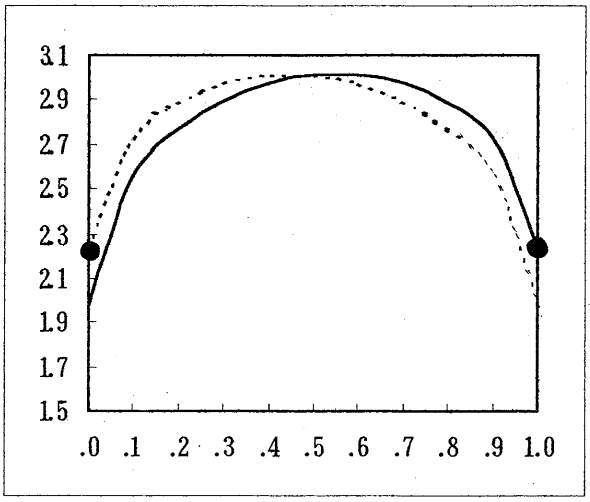
(a) τ =0.1



(b) τ =0.5



(c) τ =0.8



(d) τ =0.95