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Sectoral Shift, Income Distribution, and Development

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Abstract

There are two notable phenomena widely observed when an economy departs from an underdeveloped state and starts rapid economic growth. One is the shift of production, employment, and consumption from the agriculture and unskilled-intensive service sectors to the manufacturing and skilled-intensive service sectors, and the other is a large increase in educational levels of its population. The question is why some economies have succeeded in such 'structural change', but others do not. In order to examine the question, we construct an overlapping generations model that explicitly takes into account the sectoral change and human capital accumulation as sources of growth. It is shown that a relatively equal initial wealth distribution, or to be more accurate, a sufficient size of 'middle-class' is a necessary condition for a successful sectoral shift. Once the economy initiates the 'take-off', the sectoral shift and human capital growth continue until it reaches the high-income steady state. However, when agricultural productivity is low, the economy does not succeed in the sectoral shift irrespective of the initial distribution. Thus sufficient agricultural productivity is a prerequisite for the success.

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1. Introduction

There are two notable phenomena widely observed when an economy departs from an underdeveloped state and starts rapid economic growth. One is the shift of production, employment, and consumption from the agriculture and unskilled-intensive service sectors to the manufacturing and skilled-intensive service sectors. The other is a large increase in educational levels of its population[1]. Since the manufacturing and skilled-intensive service require a larger pool of skilled labor, it is easy to see that they are related. The question is why some economies have succeeded in such 'structural change', but others do not. This is surprising considering the fact that there is the widely-held view that both industrialization and education growth are keys to higher income and many policies have been proposed and attempted. In order to tackle the question, this paper constructs an overlapping generations economy that explicitly takes into account the sectoral change and human capital accumulation as sources of growth.

The model economy has two sectors, agriculture and manufacturing, the former hiring unskilled labor and the latter employing skilled labor and physical capital[2]. The focus of the analysis is on a typical developing economy that lacks enough agricultural productivity to rely on exporting agricultural products to boost its growth. Hence it is assumed that the market of agricultural goods is closed domestically. In contrast, the market of manufacturing goods, which is also used as capital goods, is open internationally. The assumption is made for two reasons. First, such an economy tends to export manufacturing goods since when its income level is low [3]. Second, although accumulation of physical capital is an important issue for the economy too, the paper focuses exclusively on the sectoral shift and human capital accumulation[4]. In this way, the analysis is greatly simplified.

The initial situation is that people are largely engaged in agriculture and remain unskilled, while only a small portion of the population is in manufacturing and skilled. Since large room is not left for growth in the agriculture sector, the economy must accomplish the relocation of

[1]Empirical facts on the sectoral shift are summarized in Syrquin (1988) and those on the education growth are surveyed in Schultz (1988).

[2]This assumption is made for simplicity. As long as the manufacturing sector is more intensive in skilled labor and physical capital, the basic results remain unchanged.

[3]See Syrquin (1988), p.233-235 for the evidence.

[4]Earlier theoretical attempts in development economics such as Lewis (1954) concerned the problem of capital accumulation in industrialization of a low-income economy

resources from agriculture to manufacturing in order to achieve a higher standard of living.

An agent in the economy lives for two periods, the first as a child and the second as an adult. In childhood, he receives a transfer from his parent and allocates it between two investment opportunities, assets and education, in order to maximize his future income. The investment in education is required to become a skilled worker and is individually profitable, although it is costly[5]. The cost of education is the cost of hiring skilled workers as teachers. Since loan markets are nonexistent, tuition must be self-financed[6]. Consequently, poor parents cannot make investments in educating their children despite its profitability, and the investment decisions are affected by family income. In adulthood, the agent earns income from work and assets, and spends on consumption of agricultural goods, manufacturing goods, and intergenerational transfers. Preferences are such that he consumes at least the subsistence level of agricultural goods independent of its price. This formulation implies that the income (and price) elasticity of demand for agricultural goods is less than one, while those for manufacturing goods and transfers are more than one, so the share of income spent on agricultural goods decreases with income growth.

Without an increase in skilled labor, the economy stays largely agricultural and its income remains low. Hence more people must be educated, but many of them are credit constrained and cannot make optimal investments. This is the difficulty the economy is facing when it tries to change the sectoral composition and achieve a higher income level. Under what conditions can such an economy succeed?

The first point of the paper is that a sufficiently equal initial wealth distribution, or to be more precise, an adequate initial size of 'middle class' is a *necessary* condition for the sectoral change and economic growth[7]. The requirement for the 'take-off' in this economy is sizable wealth accumulation by a portion of unskilled workers so that their children can take education and get skilled jobs in manufacturing. The source of labor income of those unskilled workers is sales of agricultural goods, which is dependent on the price of agricultural goods. The price in turn depends positively on the number of skilled workers and aggregate

[5]Psacharopoulos (1994) shows that returns to education is higher in low-income nations especially at primary and secondary education levels.

[6]Although the cost considered in the model is tuition, foregone earnings are also important costs particularly in such a credit-constrained economy. The result is not affected by the inclusion of foregone earnings to the cost.

[7]Here the middle class means those who have enough wealth to take education.

assets, since more skilled labor implies higher demand and lower supply of the goods, and larger assets accumulation leads to the higher demand. Thus, for the sectoral shift to start, the proportion of people who have enough wealth to take education and aggregate assets must be above certain levels[8].

If the initial wealth distribution is relatively equal, a larger portion of the population becomes skilled workers, so the demand for agricultural goods is higher and its supply is lower, resulting in higher agricultural price and unskilled wage. If the price level is above the critical level, a richer portion of unskilled workers can send their children to school and the sectoral shift starts. Even when the price level is not high enough for the shift initially due to low wealth accumulation, the larger pool of skilled workers make rapid wealth accumulation possible, hence the price level rises to the critical level eventually and the industrialization starts. Once the sectoral change starts, it continues *autonomously*. An increase in the number of skilled workers raises the demand for agricultural goods, reduces its supply, and increases assets accumulation. All contribute to a further increase in the agricultural price and unskilled wage. This allows children of less affluent unskilled workers to access education, causing the number of skilled workers to increase further, which raises the price and the unskilled wage even more. As long as the skilled wage (net of the cost of education) is higher than the unskilled wage, this process continues. In the long run, the economy reaches the state where the return from education is equated with that from assets, thus equal opportunity is attained.

On the other hand, if the economy starts with a relatively small size of 'middle class', skilled labor is scarce and hence the agricultural price is lower. Children of unskilled workers are not financially able to obtain education and skilled labor does not increase. Still, if initial assets accumulation is low, the relative price increases over time through assets accumulation but never reaches the critical level for the sectoral shift. Since skilled labor does not increase, inequality between skilled and unskilled workers does not disappear and the investment choices are affected by family income even in the long run.

However, equal initial wealth distribution is *not* sufficient for the success. When there is the sectoral shift of consumption and agricultural productivity is very low, the economy ends up in a steady state with persistent inequality *irrespective* of the initial distribution.

[8]The required levels of 'middle class' and aggregate assets are interrelated.

Thus, sufficient agricultural productivity is a prerequisite for the sectoral shift. Since people have to consume at least subsistence levels of agricultural goods regardless of its price, when agricultural productivity is lower, its price becomes higher more than proportionally and the unskilled wage rises. The resultant lower return from education implies that the economy can sustain less skilled labor and aggregate assets. If agricultural productivity is below a certain level, the sustainable skilled labor and aggregate assets become smaller than the combination required for the 'take-off'. Therefore, in this case the industrialization never starts.

The arguments so far have assumed that productivity levels of both sectors are time-invariant. The above results are mostly *not* affected by the introduction of productivity growth, as long as the cost of education increases with skilled wage[9]. An economy starting with very low agricultural productivity still finds it very difficult to 'take-off', irrespective of its initial wealth distribution. Now, a *decreasing* relative price of agricultural goods can be consistent with sectoral shift, if productivity growth in the agriculture sector is large enough.

Empirical findings largely support the model's implications, although the model abstracts from many realistic features for simplicity. The first point of the paper, importance of income distribution, especially size of 'middle class', in economic development has been backed by many studies. Adelman and Morris (1967) pointed out that middle classes were a driving force in the economic development of Western Europe. More recently, Persson and Tabellini (1994) regressed the average GDP growth rate over 1960-1985 for a cross-section of developed and developing countries on the share of income held by the third quintile of the distribution and found a positive coefficient. They also found a positive effect of the inequality measure using time series data for nine developed countries for the years 1830-1985. Further, using cross-sectional data from 1960s to 1990s, Easterly (2001) found that a larger size of 'middle class', measured as the share of income held by the second through fourth quintiles of the distribution, is associated with more education, especially at secondary education, higher income, and higher growth[10].

The second point of the paper, sufficient agricultural productivity as a precondition for

[9]Since most of the (direct) cost of education is the cost of hiring teachers, this condition would be met in an actual economy.

[10]He also found the association with better health, better infrastructure, better economic policies, less political instability, less civil war and ethnic minorities at risk, more social "modernization" and more democracy.

successful sectoral shift, has not been formally tested, although there are several findings indirectly supporting the claim. Bairoch (1975) pointed out the large gap (about 45 percent) in agricultural productivity on average between European countries beginning their industrial revolutions and Africa and Asia in 1960s. Further, Hayami and Ruttan (1985) found a close positive association between overall output growth and agricultural productivity growth for Sub-Saharan African nations. These evidences suggest that agricultural productivity seems to be an important independent factor affecting economic development.

This paper is mainly related to two strands of literature. One is the literature that investigates mechanisms and outcome of sectoral change, such as Matsuyama (1992), Echevarria (1997), Kongsamut et. al. (2001), and Laitner (2000). Matsuyama investigates the role of agricultural productivity in economic development using a two-sector endogenous growth model and shows how openness of markets affects the relationship between productivity and growth. Using a Solow-type model with multiple consumption goods and non-homothetic preferences, Echevarria numerically shows that uneven productivity growth among sectors can lead to different aggregate growth rates at different stages of development. Kongsamut et.al investigates balanced growth paths of the model similar to Echevarria analytically. Laitner explains how an economy's measured average propensity to save rises in the course of industrialization by focusing on increasing importance of reproducible capital relative to land.

The other is the large literature that investigates the interplay between income distribution and growth theoretically, which includes Banerjee and Newman (1993), Galor and Zeira (1993), Persson and Tabellini (1994), Benabou (1996a), (1996b), Benhabib and Rustichini (1996), Durlauf (1996), Aghion and Bolton (1997), Galor and Tsiddon (1997), Aghion, Caroli, and Garcia-Penalosa (1999), and Lloyd-Ellis and Bernhardt (2000). Closely related are the papers by Galor and Zeira, and by Galor and Tsiddon, which show how credit constraints and lumpy investment in human capital can cause the interaction between initial distribution and long-run outcome of an economy in a model with one final goods sector. There is no interaction between the unskilled wage and final demand in their model.

Resting on these earlier works, this paper provides one mechanism for growth through sectoral shift focusing on human capital accumulation and credit constraints, and shows how success of an economy is related to its income distribution and agricultural productivity.

The paper is organized as follows. Section 2 presents the basic model where sectoral shift of consumption is absent. Section 3 derives and analyzes the model's dynamics and Section 4 presents and interprets the results from the basic model. In Section 5, the sectoral shift of consumption is introduced into the model and its implications for the results are examined. Section 6 concludes the paper.

2. Model

2.1. Individual decisions

Time is discrete and starts from 0. There is no uncertainty. The economy is composed of a continuum of individuals who live for two periods, the first period as children and the second period as adults.

2.1.1. Investment decisions

In childhood, an individual receives a transfer from his parent. Then he allocates the transferred money for two investment options, assets and education, in order to maximize his future income[11]. The educational investment is required to become a skilled worker and enjoy higher earnings in adulthood. The investment is a discrete choice, i.e. take education or not, incurs a fixed cost, and brings the difference between the skilled wage and unskilled wage as the gross return. Consider an individual who was born into lineage i in period $t-1$. His generation is called *generation* t. Then, his education costs e_t, and its gross return is $w_{H,t} - w_{L,t}$ in the next period, where $w_{H,t}$ and $w_{L,t}$ are the skilled and unskilled wages in period t, respectively. Assume that the education cost is the cost of hiring current skilled workers as teachers and it is proportional to $w_{H,t-1}$[12]. The investment must be self-financed because loan markets for such investment are not available in the economy. The other option, the investment in assets, is a continuous choice, and gives a gross return of $1 + r_t$ for investment of one unit of income. It is easily shown that, in an equilibrium, the return from

[11] Alternatively, one can suppose that the investment decisions are carried out by his parent in order to maximize his future income.

[12] Kendrick (1976) finds that teacher and student time constitute about 90% of all costs of education. Further, World Bank (1983) notes that about 95% of current expenses in the primary school systems of low income countries are teacher salaries.

the investment in education becomes at least as high as the return from the investment in assets, i.e. $w_{H,t} - w_{L,t} \geq (1 + r_t)e_t$.

Suppose that the individual has received b_t^i units of income as a transfer from his parent. He allocates the transfer between the investments in assets a_t^i and in education e_t^i in order to maximize his future income. If the return from investment in education is strictly higher than that from investment in assets, optimal investment choices are given by the following equations:

$$
\begin{aligned}
a_t^i &= b_t^i, \qquad\qquad\qquad\qquad (2.1)\\
e_t^i &= 0, \quad \text{if} \quad b_t^i < e_t,\\
\text{and} \quad a_t^i &= b_t^i - e_t, \qquad\qquad\qquad (2.2)\\
e_t^i &= e_t, \quad \text{if} \quad b_t^i \geq e_t.
\end{aligned}
$$

On the other hand, if the returns are equated, i.e. $w_{H,t} - w_{L,t} = (1 + r_t)e_t$, then the investment is determined by (2.1) when the transfer is less than the cost of education, while, when the transfer is large enough for taking education, both (2.1) and (2.2) are optimal choices[13]. Since innate abilities of individuals are identical, transfers *solely* determine the investment and resulting occupational choices.

2.1.2. Consumption and transfer decisions

An adult individual, who is either a skilled or unskilled worker depending on the human capital investment in the previous period, obtains income from assets and labor supply and spends the income on consumption and transfer to his child. Each adult is assumed to have a single child. There are two different consumption goods, agricultural goods and manufacturing goods in the economy, the latter also being used as capital goods. Note that the agricultural goods include unskilled-intensive service goods such as petty retail trading and domestic service, and the manufacturing goods encompass skilled-intensive service goods like financial service and health care service. Assume that an adult individual of lineage i in

[13]Actually the relative return from education is determined as the result of people's investment decisions, since it depends on the numbers of skilled and unskilled workers in the economy. More formal analysis of the investment decision is described in the next section.

generation t has the following preference:

$$U_t^i = (c_{A,t}^i)^{\gamma_A}(c_{M,t}^i)^{\gamma_M}(b_{t+1}^i)^{1-\gamma_A-\gamma_M}, \tag{2.3}$$

where $c_{A,t}^i$ and $c_{M,t}^i$ are his consumption of agricultural goods and of manufacturing goods, respectively; b_{t+1}^i is the transfer to his child (generation $t+1$); and γ_A, γ_M are preference parameters. Denote the relative price of agricultural goods to manufacturing goods in period t by P_t. Then his budget constraint takes the following form:

$$P_t c_{A,t}^i + c_{M,t}^i + b_{t+1}^i = w_t^i + (1+r_t)a_t^i, \tag{2.4}$$

where w_t^i is his earnings. Note that $w_t^i + (1+r_t)a_t^i = w_t^L + (1+r_t)b_t^i$, if $b_t^i < e_t$, and $w_t^i + (1+r_t)a_t^i = w_t^H - (1+r_t)e_t + (1+r_t)b_t^i$, if $b_t^i \geq e_t$[14].

Maximization of the utility function (2.3) subject to the budget constraint (2.4) gives the following consumption and transfer rules:

$$P_t c_{A,t}^i = \gamma_A[w_t^i + (1+r_t)a_t^i], \tag{2.5}$$

$$c_{M,t}^i = \gamma_M[w_t^i + (1+r_t)a_t^i], \tag{2.6}$$

$$\text{and} \quad b_{t+1}^i = (1-\gamma_A-\gamma_M)[w_t^i + (1+r_t)a_t^i]. \tag{2.7}$$

2.1.3. Generational structure

At the beginning of period $t+1$, current adults pass away, current children become adults, and new children are born into the economy. Since each adult has one child, the population is constant over time. The population of each generation is normalized to be one.

2.2. Production structure

There are two sectors, agriculture and manufacturing, in the economy[15]. The agriculture sector employs unskilled workers to produce agricultural goods, and the manufacturing sector employs skilled workers and physical capital to produce manufacturing goods. The manu-

[14]When the returns are equated, $w_t^H - (1+r_t)e_t + (1+r_t)b_t^i = w_t^L + (1+r_t)b_t^i$ is satisfied.

[15]As mentioned above, the agriculture sector includes unskilled-intensive service sectors and the manufacturing sector includes skilled-intensive service sectors in real economy.

facturing goods are used not only for consumption but also for investment in physical capital (capital goods).

The production functions of the two sectors are given as follows:

$$\text{Agriculture: } Y_{A,t} = A_t L_t, \tag{2.8}$$

$$\text{Manufacturing: } Y_{M,t} = M_t (H_{M,t})^{\alpha_m} (K_t)^{1-\alpha_m}. \tag{2.9}$$

In the above expressions, $Y_{A,t}$ and $Y_{M,t}$ are outputs of agricultural goods and manufacturing goods, respectively; A_t and M_t are the productivity levels of the respective sectors; L_t is the number of unskilled workers in agriculture; $H_{M,t}$ is the number of skilled workers in manufacturing; and K_t denotes physical capital employed in manufacturing. To focus on main mechanics of the model, in most parts of the paper, the productivities A_t and M_t are assumed to be constant over time, so the productivity levels are set to be equal to $A_t = A$ and $M_t = M$. As described later, the main results of the model with constant productivities go through even if exogenous productivity growth is introduced.

The assumptions that unskilled workers are employed only in agriculture, and skilled workers and physical capital are employed only in manufacturing are made for simplicity. Provided that the agriculture sector is more intensive in unskilled labor and the manufacturing sector is more intensive in skilled labor, the outcome from the model remains mostly unchanged.

2.3. Market structure and determination of prices

Suppose that the markets for agricultural goods and for labor are closed domestically and their prices are determined within the economy, while manufacturing goods and physical capital are freely mobile internationally. As explained in the introduction, this paper focuses on developing economies that do not have sufficient agricultural productivity to rely on exports of the goods for income growth. In such economies, it would be reasonable to consider the market for agricultural goods as closed, since effects of international commodity markets on domestic markets tend to be small. The assumption of open markets for manufacturing goods and for physical capital is made partly because it is more realistic than the other extreme of the closed markets for such an economy[16]. Perfect capital mobility is assumed also to focus on human capital accumulation rather than physical capital accumulation as a source of growth. These assumptions simplify the analyses significantly.

From the assumptions, the interest rate is fixed at the world interest rate $r_t = r$, so the skilled wage w_H is given by the following equation[17].

$$w_H = \alpha_m M^{\frac{1}{\alpha m}} (\frac{1-\alpha_m}{r})^{\frac{1}{\alpha m}-1}. \tag{2.10}$$

The wage rate is exogenous, and without technological growth and fluctuations of the world interest rate, it is constant over time. The wage of unskilled workers is equal to,

$$w_{L,t} = P_t A, \tag{2.11}$$

hence it depends on the relative price of agricultural goods to manufacturing goods, P_t.

The relative price is determined by the market clearing condition of agricultural goods. The demand for agricultural goods is total consumption of the goods by the adult population, which is the sum of individual consumption (2.5) over the population. So the market clearing condition becomes,

$$P_t A L_t = \gamma_A [w_{L,t} L_t + w_H H_t + (1+r) K_t^D]. \tag{2.12}$$

In the above equation, H_t is the total number of skilled workers in the economy, which is the sum of $H_{M,t}$ and $H_{E,t}$, skilled workers in the education sector[18], and K_t^D is total

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[16]As mentioned in the introduction, such an economy tends to start exporting manufacturing goods while its income level is very low. Also, compared to unskilled-intensive service goods such as domestic service, skilled-intensive service goods such as financial service are likely to be more tradable.

[17]From the first order conditions of the profit-maximizing problem of the manufacturing firm, the following two equations are derived.

$$\begin{aligned} r_t &= (1-\alpha_m) M (\frac{H_{M,t}}{K_t})^{\alpha_m}, \\ \text{and } w_{H,t} &= \alpha_m M (\frac{K_t}{H_{M,t}})^{1-\alpha_m}. \end{aligned}$$

Solving the first equation for $\frac{H_{M,t}}{K_t}$, substituting it into the second equation, and setting $r_t = r$ gives the expression in the main text.

[18]Remember that it is postulated that the cost of education is the cost of hiring skilled workers as teachers.

physical capital held domestically. Note that K_t^D can be *different* from total physical capital in the economy, K_t, since capital is freely mobile internationally. Substituting (2.11) and $H_t + L_t = 1$ [19] into the above equation and solving for P_t, the relative price of agricultural goods is given as follows:

$$P_t = \frac{\gamma_A}{1-\gamma_A} \frac{[w_H H_t + (1+r)K_t^D]}{A(1-H_t)}. \tag{2.13}$$

The relative price P_t increases with the number of skilled workers H_t and with domestically held physical capital K_t^D. Larger H_t and K_t^D imply higher total income and larger demand for agricultural goods, and larger H_t (smaller L_t) implies lower supply of the goods, hence resulting in higher P_t. Since $w_{L,t} = P_t A$, the unskilled wage is also increasing in H_t and K_t^D.

For the analyses in later sections, it is convenient to express the relative price as a function of H_t and aggregate intergenerational transfers, B_t. This is done by substituting $K_t^D = B_t - eH_t$ into the above equation (2.13)[20]:

$$P_t = \frac{\gamma_A}{1-\gamma_A} \frac{\{[w_H - (1+r)e]H_t + (1+r)B_t\}}{A(1-H_t)}. \tag{2.14}$$

The relative price and the unskilled wage are increasing in both H_t and B_t. To express the dependency of P_t and $w_{L,t}$ on H_t and B_t, they are denoted as $P(H_t, B_t)$ and $w_L(H_t, B_t)$, respectively.

The markets of manufacturing goods and of capital are not closed domestically and differences between demands and supplies of the goods are financed through net imports from foreign economies.

3. Dynamics

The previous section has described the model and derived the equilibrium conditions. In the model economy, individuals live only for two periods and participate in each market for one period alone. As a result, only one generation participate in markets each period and they

[19] This equation is satisfied since the size of each generation is normalized to be one.

[20] The relation is satisfied since current skilled workers have spent e on education out of their received transfers in the previous period. Note that the cost of education e is time-invariant because the skilled wage w_H is constant over time.

do not attend markets in past or future periods. Hence, the model can be considered as a sequence of static economies.

What connects these static economies across periods are the intergenerational transfers. Because of the credit constraint, transfers directly affect individuals' investment and occupational choices, and consequently consumption and transfer decisions. Further, the distribution of transfers over the population determines a proportion of individuals who can afford to take education, and thus it affects the relative return from education and investment decisions. The distribution also affects supplies and demands of goods, their prices and allocations. Hence, in general, the time evolution of the distribution of transfers must be examined in order to understand how the structures of the economy, that is, production and employment shares of each sector, total output, wages of skilled and unskilled workers, and income distribution, change over time.

This section first derives the dynamic equation relating current transfers to the next period's transfers within a lineage (*individual dynamics*). The dynamics depend on the time evolution of the distribution of transfers over the population. However, it turns out that the qualitative nature of dynamics of two aggregate variables, which does *not* require detailed information on the distribution, is enough to derive implications of the model. So the dynamics of the two aggregate variables are characterized next. Although the two dynamics interact, for exposition, initially the dynamics of each variable are analyzed fixing the other, then the both dynamics are analyzed together by introducing a phase diagram.

3.1. Individual dynamics

Consider an individual born into lineage i in period $t-1$, who belongs to *generation* t. He allocates the transfer b_t^i between investments in assets a_t^i and in education e_t^i so as to maximize his future income. If the transfer is less than the cost of education, i.e. $b_t^i < e$, the transfer is spent only on assets and he becomes an unskilled worker as described above. On the other hand, if the transfer is at least as large as the cost of education, i.e. $b_t^i \geq e$, the investment decision is more complicated. Since investment decisions of others affect the unskilled wage $w_L(H_t, B_t)$ and the relative return from education, he has to take into account their actions in making the decision. The key variable affecting the decision is the fraction of individuals in generation t who have received transfers b_t^i larger than e, Fr_t. In short, when only small numbers of individuals can afford education, all of them take education and become skilled; when many individuals have access to education, some of them become skilled but others not.

3.1.1. Unequal opportunity case

When the proportion of individuals who can afford to take education is small, the return from education is higher than the return from assets even if all of them actually take education, i.e. $w_H - (1+r)e > w_L(Fr_t, B_t)$. In this case, the individual allocates the transfer b_t^i between investments in assets a_t^i and in education e_t^i in the following manner:

$$\begin{aligned} a_t^i &= b_t^i, \\ e_t^i &= 0, \quad \text{if} \quad b_t^i < e, \end{aligned} \tag{3.1}$$

$$\text{and} \quad \begin{aligned} a_t^i &= b_t^i - e\ , \\ e_t^i &= e, \quad \text{if} \quad b_t^i \geq e. \end{aligned} \tag{3.2}$$

From the above equations, it is clear that all young individuals who are financially eligible for education become skilled workers, hence $H_t = Fr_t$ is satisfied. Since transfers from parents constrain available investment opportunities and the return from the investments, this case is called the *unequal opportunity case.*

In the next period, the individual, given asset a_t^i and acquired ability (skilled or unskilled), determines the amount of transfers to *his child* b_{t+1}^i according to (2.7). By substituting the above investment rules into the transfer rule (2.7), the dynamic equation linking the received transfer b_t^i to the transfer given to the next generation b_{t+1}^i is derived.

If he is a skilled worker, i.e. $b_t^i \geq e$, the equation takes the following form, which is obtained by substituting (3.2) into (2.7):

$$b_{t+1}^i = b_s(b_t^i) \equiv (1 - \gamma_A - \gamma_M)\{w_H + (1+r)(b_t^i - e)\}. \tag{3.3}$$

The assumption $(1 - \gamma_A - \gamma_M)(1 + r_t) < 1$ is made so that the fixed point for the equation $(b_s)^* \equiv \frac{1-\gamma_A-\gamma_M}{1-(1-\gamma_A-\gamma_M)(1+r)}[w_H - (1+r)e]$ exists.

For an unskilled worker, i.e. when $b_t^i < e$, the equation is obtained by substituting (3.1)

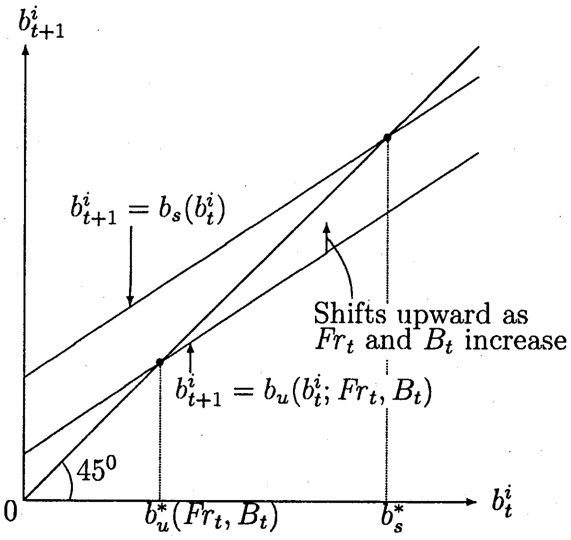

Figure 3.1: Individual dynamics of intergenerational transfers

and $H_t = Fr_t$ into (2.7):

$$b^i_{t+1} = b_u(b^i_t; Fr_t, B_t) \equiv (1 - \gamma_A - \gamma_M)\{w_L(Fr_t, B_t) + (1+r)b^i_t\}, \tag{3.4}$$

where

$$w_L(Fr_t, B_t) = P(Fr_t, B_t)A, \tag{3.5}$$

$$\equiv \frac{\gamma_A}{1-\gamma_A}\frac{\{[w_H - (1+r)e]Fr_t + (1+r)B_t\}}{(1 - Fr_t)}. \tag{3.6}$$

The dynamic equation for an unskilled worker does depend on aggregate variables $H_t = Fr_t$ and B_t, since they affect the relative price of agricultural goods, thus the unskilled wage. The fixed point of the equation for *given* Fr_t and B_t is denoted by, $b^*_u(Fr_t, B_t) \equiv \frac{1-\gamma_A-\gamma_M}{1-(1-\gamma_A-\gamma_M)(1+r)} w_L(Fr_t, B_t)$ [21].

The dynamics of intergenerational transfers of a currently skilled worker $b^i_{t+1} = b_s(b^i_t)$ and of a currently unskilled worker $b^i_{t+1} = b_u(b^i_t; Fr_t, B_t)$ for *given* Fr_t and B_t are depicted in Figure 3.1. As long as $w_H - (1+r)e > w_L(Fr_t, B_t)$ is satisfied, $b^i_{t+1} = b_u(b^i_t; Fr_t, B_t)$ is located below $b^i_{t+1} = b_s(b^i_t)$, but as Fr_t and B_t, hence $P(Fr_t, B_t)$ and $w_L(Fr_t, B_t)$ increase, it shifts upward.

[21] In general, this is *not* the long-run transfer level of a lineage of a currently unskilled worker, since his descendants may become skilled workers *and* Fr_t and B_t may change over time. Although this fixed point seems to have no economic importance, it turns out that the level of $b^*_u(Fr_t, B_t)$ is crucial for aggregate dynamics (detailed later).

3.1.2. Equal opportunity case

Next consider the case where many individuals can afford education so that the return from education *fails* to be higher than the return from assets, *if* all of them invest in education, i.e. $w_H - (1+r)e \leq w_L(Fr_t, B_t)$. In this situation, the number of skilled workers H_t is determined at the point where the two returns are equated, that is, H_t is the solution of $w_H - (1+r)e = w_L(H_t, B_t)$. Now *not* all of financially eligible individuals take education and become skilled workers, i.e. $H_t \leq Fr_t$. Since the return from the investments does *not* depend on transfers from parents, this case is named the *equal opportunity case.* Dynamics of transfers of both skilled and unskilled workers are described by the single equation $b^i_{t+1} = b_s(b^i_t)$:

$$b^i_{t+1} = b_s(b^i_t) \equiv (1-\gamma_A-\gamma_M)\{w_H+(1+r)(b^i_t-e)\}. \tag{3.7}$$

In Figure 3.1, this is the situation where $P(Fr_t, B_t)$ is large enough that $b^i_{t+1} = b_u(b^i_t; Fr_t, B_t)$ coincides with $b^i_{t+1} = b_s(b^i_t)$.

3.1.3. Dividing line

The economy belongs to either of the two cases depending on Fr_t and B_t. The combination of Fr_t and B_t satisfying $w_H - (1+r)e = w_L(Fr_t, B_t)$ is the dividing line, and it is obtained by substituting $P_t = [w_H - (1+r)e]/A$ and $H_t = Fr_t$ into (2.13) and solving the equation for Fr_t:

$$Fr_t = H^e(B_t) \equiv (1-\gamma_A) - \frac{\gamma_A(1+r)B_t}{[w_H-(1+r)e]}. \tag{3.8}$$

The unequal opportunity case corresponds to $Fr_t < H^e(B_t)$, while the equal opportunity case amounts to $Fr_t \geq H^e(B_t)$.

3.1.4. Summary

The analysis of this subsection can be summarized as follows. When a relatively small number of individuals have enough wealth to take education, i.e. $Fr_t < H^e(B_t)$, the return from education is higher than the return from assets even though all of them take education.

Then, all those who have received $b_t^i \geq e$ become skilled workers, i.e. $H_t = Fr_t$, and their transfers to the next generation are determined by $b_{t+1}^i = b_s(b_t^i)$. Those who have received $b_t^i < e$ become unskilled workers and the dynamics of transfers are governed by $b_{t+1}^i = b_u(b_t^i; Fr_t, B_t)$. This case has been named the unequal opportunity case since $w_H - (1+r)e > w_L(H_t, B_t)$ is satisfied.

In contrast, when many individuals can afford education, i.e. $Fr_t \geq H^e(B_t)$, the number of skilled workers H_t is determined so that the returns from the two investment opportunities are equated, i.e. $w_H - (1+r)e = w_L(H_t, B_t)$. Hence this case has been called the equal opportunity case. In this case, those who have received $b_t^i \geq e$ become either skilled *or* unskilled workers, i.e. $H_t = H^e(B_t) \leq Fr_t$, those who have received $b_t^i < e$ become unskilled workers, and transfers of both workers follow $b_{t+1}^i = b_s(b_t^i)$.

3.2. Aggregate dynamics

What has become clear now is that the individual dynamics depend on dynamics of two aggregate variables, aggregate transfers B_t and the fraction of individuals who have received transfers b_t^i larger than the cost of education e, Fr_t. These dynamics in turn depend on the time evolution of the distribution of transfers, which is quite difficult to characterize. However, in order to derive the main implications of the model, the information on the distributive dynamics is *not* needed and the *qualitative* nature of the dynamics of the two aggregate variables is enough. Thus this subsection analyzes the dynamics of the two aggregate variables qualitatively. For exposition, first each of them is examined separately fixing the other's dynamics, then their interaction is taken into account at the end.

3.2.1. Dynamics of aggregate transfers

Here the dynamics of aggregate intergenerational transfers B_t are examined for *given* Fr_t. First, consider the unequal opportunity case, i.e. $Fr_t < H^e(B_t)$. As seen in the previous subsection, in this case, $w_H - (1+r)e > w_L(H_t, B_t)$ and $H_t = Fr_t$ hold. The dynamic equation of aggregate transfers B_{t+1} is given by the following equation, which is obtained by aggregating individual dynamics of skilled (3.3) and of unskilled workers (3.4) over the population and using $H_t = Fr_t$:

$$B_{t+1} \ = B(Fr_t, B_t) \equiv \frac{1-\gamma_A-\gamma_M}{1-\gamma_A}\{[w_H-(1+r)e]Fr_t+(1+r)B_t\}. \qquad (3.9)$$

The dynamics of B_{t+1} are dependent on Fr_t. The assumption $\frac{1-\gamma_A-\gamma_M}{1-\gamma_A}(1+r) < 1$ is made so that there exists a fixed point for the equation given Fr_t [22], where the fixed point $B^*(Fr_t)$ is defined as,

$$B^*(Fr_t) \equiv \frac{1}{1-\frac{1-\gamma_A-\gamma_M}{1-\gamma_A}(1+r)}\frac{1-\gamma_A-\gamma_M}{1-\gamma_A}[w_H-(1+r)e]Fr_t. \qquad (3.10)$$

Higher $H_t = Fr_t$ implies higher total income (measured in terms of manufacturing goods), since (i) skilled wage is higher than unskilled wage, and (ii) an increase in total labor income of skilled workers raises the demand for agricultural goods, the relative price of agricultural goods, and the relative wage of unskilled workers. The higher total income then increases the demand for assets, resulting in higher $B_{t+1} \ = B(Fr_t, B_t)$ and $B^*(Fr_t)$.

Alternatively, in the equal opportunity case where $w_H-(1+r)e \leq w_L(Fr_t, B_t)$ is satisfied, $w_H-(1+r)e = w_L(H_t, B_t)$, i.e. $H_t = H^e(B_t)$, holds in an equilibrium. In this case the dynamics of B_{t+1} are described by $B_{t+1} = B(H^e(B_t), B_t)$, which is obtained by substituting $H_t = H^e(B_t)$ into $B_{t+1} = B(H_t, B_t)$:

$$B_{t+1} = B(H^e(B_t), B_t) \equiv (1-\gamma_A-\gamma_M)\{[w_H-(1+r)e]+(1+r)B_t\}. \qquad (3.11)$$

The fixed point of this equation B^{**} is,

$$B^{**} = \frac{1-\gamma_A-\gamma_M}{1-(1-\gamma_A-\gamma_M)(1+r)}[w_H-(1+r)e], \qquad (3.12)$$

which is equal to $(b_s)^*$. The number of skilled workers at B^{**}, $H^{**} \equiv H(B^{**})$ is given by,

$$\begin{aligned} H^{**} &\equiv H^e(B^{**}) = (1-\gamma_A) - \frac{\gamma_A(1+r)B^{**}}{[w_H-(1+r)e]} \\ &= [1-\frac{\gamma_A}{1-(1-\gamma_A-\gamma_M)(1+r)}]. \end{aligned} \qquad (3.13)$$

[22] Again, this fixed point is usually not the long-run aggregate transfers level, since Fr_t may change over time.

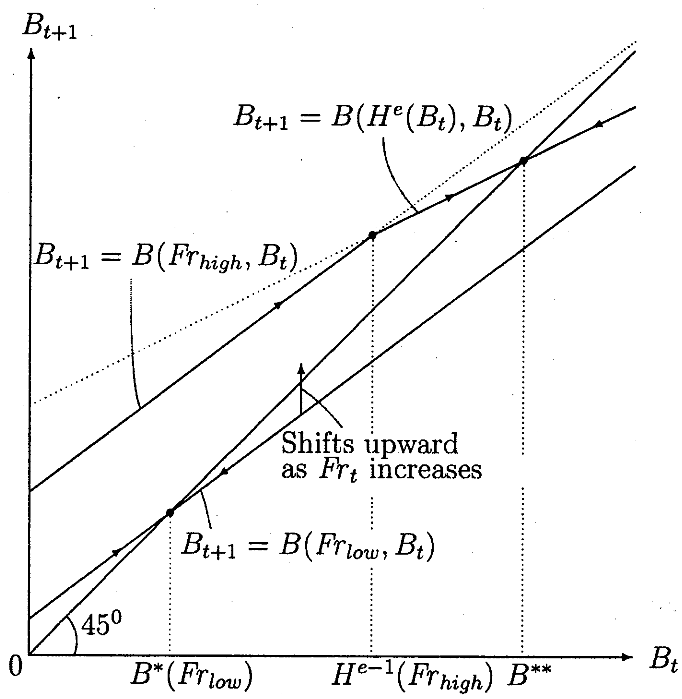

Figure 3.2: Dynamics of aggregate transfers when $Fr_t = Fr_{low}$ and Fr_{high}

Figure 3.2 displays the dynamics of aggregate transfers when Fr_t is equal to Fr_{low} and Fr_{high}. In the former case, only small numbers of individuals can financially afford education, accordingly all of them take education and become skilled, $H_t = Fr_{low}$, and the dynamics of aggregate transfers follow $B_{t+1} = B(Fr_{low}, B_t)$ for any B_t in the figure. If Fr_t remains unchanged, the transfers converge to $B^*(Fr_{low})$ in the long run. In contrast, when $Fr_t = Fr_{high}$, many people can take education. Consequently, *if* B_t is sufficiently large, i.e. $B_t \geq H^{e-1}(Fr_{high}) \equiv \{B_t : w_H - (1+r)e = w_L(Fr_{high}, B_t)\}$, not all of them become skilled $H_t = H^e(B_t) \leq Fr_t$, and the dynamics follow $B_{t+1} = B(H^e(B_t), B_t)$. In this case, the transfers converge to B^{**} if Fr_t does not change. As Fr_t becomes larger, $B_{t+1} = B(Fr_t, B_t)$ shifts upward, and the range of B_t where the dynamics are governed by $B_{t+1} = B(H^e(B_t), B_t)$ expands.

3.2.2. Dynamics of Fr_t

Next the dynamics of the proportion of people who can afford to take education, Fr_t, are examined for *given* B_t. Actually, the dynamic equation relating Fr_t to Fr_{t+1} depends on the distribution of transfers over the population, so the exact form of the equation cannot be

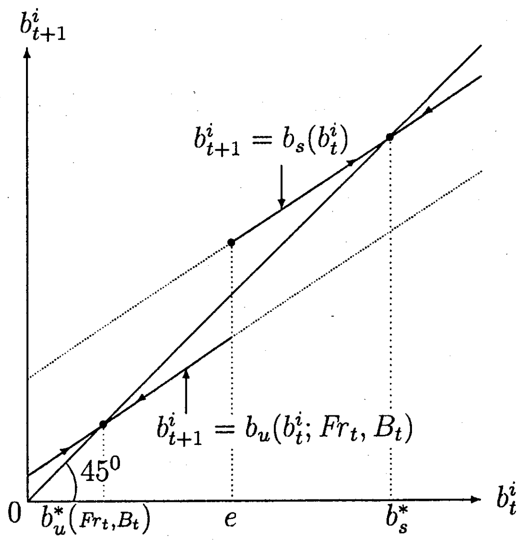

Figure 3.3: Unequal opportunity case when $b^*_u(Fr_t, B_t) < e$

known without the complete information on the distribution. However, whether Fr_t changes or not and whether it increases or decreases *can* be discerned only with the information of the two aggregate variables B_t and Fr_t, and this qualitative information on the dynamics is enough to obtain the main results of the model.

Assume that $(1 - \gamma_A - \gamma_M)w_H \geq e$, i.e. $B^{**} = b^*_s \geq e$, is satisfied[23]. Then, Fr_t is *non-decreasing* over time, since $b^i_{t+1} \geq e$ is satisfied whenever $b^i_t \geq e$ is true (See Figure 3.3).

First, consider the unequal opportunity case. Remember that $H_t = Fr_t$ is satisfied in this case. Whether Fr_t is constant or increases over time is determined by the relative position of $b^*_u(Fr_t, B_t)$ to e. When $b^*_u(:) < e$, any children of unskilled workers do not receive transfers larger than e, so Fr_t is constant (See Figure 3.3). In contrast, when $b^*_u(:) \geq e$ is satisfied, $Fr_{t+1} \geq Fr_t$ holds, since education becomes affordable to children of unskilled workers over time (See Figure 3.4). Of course, how much Fr_{t+1} increases depends on the exact distribution of transfers over the population and is impossible to know only with the information on Fr_t and B_t.

Alternatively, in the equal opportunity case, the return from education is equated with the return from assets, i.e. $H_t = H^e(B_t) \leq Fr_t$. Hence, individual transfers of both skilled

[23]Note that this assumption does *not* place any restrictions on the manufacturing productivity. Since e is proportional to w_H, it can be expressed as $e = s_e w_H$, where s_e is a constant. Hence the assumption can be rewritten as $1 - \gamma_A - \gamma_M \geq s_e$. What this condition states is that people are altruistic enough towards their children.

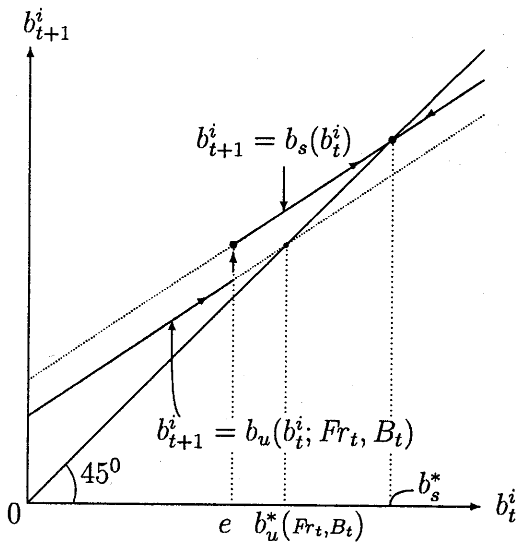

Figure 3.4: Unequal opportunity case when $b^*_u(Fr_t, B_t) \geq e$

and unskilled workers change over time according to $b^i_{t+1} = b_s(b^i_t)$, and $Fr_{t+1} \geq Fr_t$ is satisfied.

3.2.3. Joint dynamics of Fr_t and B_t

Now the dynamics of Fr_t and B_t are analyzed together by introducing the phase diagram (Figure 3.5), in which the $x-$axis takes Fr and the $y-$axis takes B. The feasible combinations of Fr and B are equal to the area bound by $Fr = 0$, $Fr = 1$, and $B = eFr_t$. The economy must satisfy $B \geq eFr$ because Fr is defined as the fraction of individuals who have received transfers b^i larger than the cost of education e.

The diagram is divided into two regions, one corresponding to the unequal opportunity case and the other region for the equal opportunity case. The locus dividing the two cases is $Fr = H^e(B)$. The region below the locus is the unequal opportunity case, where all the individuals who can afford education take education and become skilled workers, i.e. $H = Fr$. The region above it is the equal opportunity case, where the number of skilled workers is determined so that the return from education is equated with the return from assets, i.e. $H = H^e(B) \leq Fr$.

In the unequal opportunity case, the direction of motion of B is determined by the position of current (Fr_t, B_t) relative to $B = B^*(Fr)$. When the current economy is located on the line, B_t is constant, while when located above, it decreases, and when located below,

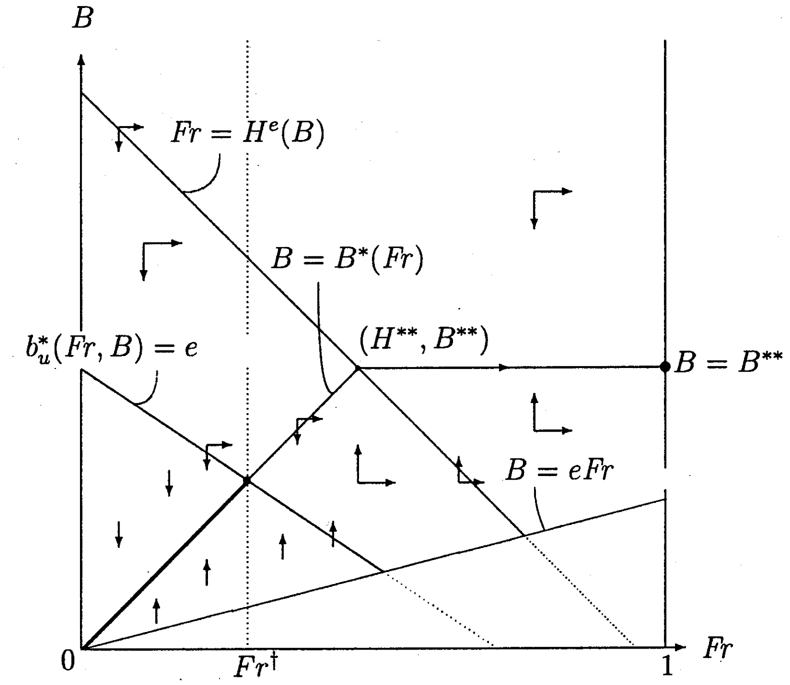

Figure 3.5: Phase Diagram

it increases over time[24]. The direction of the change of B in each region is expressed with vertical arrows. As for the direction of change of Fr, it is determined by the current location of (Fr_t, B_t) relative to $b_u^*(Fr_t, B) = e$. In the region below the line, $b_u^*(:) < e$ is satisfied, accordingly $Fr_{t+1} = Fr_t$ holds, while in the region above and on the line, $b_u^*(:) \geq e$, hence $Fr_{t+1} \geq Fr_t$ is satisfied[25].

Alternatively, in the equal opportunity case, the direction of motion of B is determined by the relative location to $B = B^{**}$. In the region above $B = B^{**}$, B_t decreases, and in the region below, it increases[26]. The direction of change of Fr is simple in this case, and $Fr_{t+1} \geq Fr_t$ is always satisfied[27].

With this diagram, qualitative properties of transitional dynamics and long-run outcome of the aggregate variables (Fr, B) are transparent. Except for a few unimportant cases, they are completely known only with the knowledge of current position of (Fr_t, B_t) on the

[24]The dynamics are governed by $B_{t+1} = B(Fr_t, B_t)$, (3.9).

[25]The exact dynamics are known only if the complete information of the distribution of b_t^i over the population is available, so as the dynamics of H_t.

[26]In this case, the dynamics are determined by $B_{t+1} = B(H^e(B_t), B_t)$, (3.11).

[27]Again the characterization of the exact dynamics requires the complete information on the wealth distribution, but now the dynamics of H_t can be known completely since $H_t = H^e(B_t)$ is satisfied.

diagram. The detailed analyses of the dynamics are given in Appendix I.

4. Analyses

4.1. Initial distribution and long-run economic structure

Using this diagram, the relationship between the initial wealth distribution and the long-run structure of the economy along with its transition can be easily investigated.

First, consider an economy that attains equal opportunity from the beginning, whose initial position is in the region on and above $Fr = H^e(B)$ on the diagram. The number of skilled workers is determined so that the returns from the two investment opportunities are equated, that is, $w_H - (1+r)e = w_L(H^e(B), B)$ and $H = H^e(B)$ hold. Both skilled and unskilled workers earn the same level of earnings (*net* of the cost of education), so education become affordable to children of unskilled workers over time (Fr increases). The result is that, unless the economy starts with $Fr_0 < Fr^{\dagger}$, it converges to $(Fr, B) = (1, B^{**})$ for certain, where not only the net earnings but also net income and wealth are equalized and $b^i = b_s^*$ holds. Thus *perfect equality* is attained. The long-run outcome of the economy with the initial condition $Fr_0 < Fr^{\dagger}$ depends on the exact initial distribution. If wealth is highly concentrated in the rich, then Fr would increase only slightly and the economy would regress to the unequal opportunity case (crosses the line $Fr = H^e(B)$). Otherwise, it would converge to $(1, B^{**})$ in the long run.

Next examine the case where the economy starts from the state where the return from education is higher than the return from assets (unequal opportunity case), but children of unskilled workers gain access to education over time, i.e. $b_u^*(Fr_0, B_0) \geq e$. As is clear from the diagram, if the initial condition satisfies $Fr_0 \geq Fr^{\dagger}$, more individuals obtain education and hence become skilled workers over time. As the number of skilled workers increases wage inequality between skilled and unskilled workers diminishes, so does wealth inequality, as long as B_t does not decrease greatly[28]. And the production and employment shares of manufacturing rise, while those of agriculture fall. Eventually the economy reaches the equal opportunity case and perfect equality is attained in the long run. When initial inequality is relatively high, i.e. $Fr_0 < Fr^{\dagger}$, the long-run outcome depends on the exact information

[28]When the economy is in the region below $B = B^*(Fr)$, aggregate transfers increase over time, so inequality always falls.

on the initial distribution. If the distribution is too concentrated in the few rich, then the number of skilled workers increases only slightly over time, and it is possible that the economy crosses $b_u^*(:) = e$.

The unequal opportunity case where children of unskilled workers cannot have access to education, i.e. $b_u^*(:) < e$, is a stagnant state. Since the number of skilled workers does not increase, the production and employment shares of the sectors remain constant. In particular, if the initial condition is $Fr_0 < Fr^\dagger$, the economy ends up with the same number of skilled workers Fr_0 and aggregate transfers levels $B^*(Fr_0)$. *Unequal opportunity and inequality persist* in the long run. If the economy begins with $Fr_0 \geq Fr^\dagger$, its long-run prospect is much brighter. While its assets accumulation is low, poor people cannot afford education and hence the number of skilled workers remains constant; but after a certain amount of assets accumulation, the economy transits to $b_u^*(:) \geq e$ and the sectoral shift starts. In the long run, it attains equal opportunity and perfect equality of earnings and wealth.

The analysis has shown that there is a clear-cut negative relationship between inequality of initial wealth distribution and long-run growth and inequality. Here the measure of inequality is a fraction of individuals who have enough resources to take education, Fr[29]. Thus, a size of 'middle class' of an economy matters for growth through sectoral change. If initial (aggregate) wealth is very low, too much equality is *not* good for long-run economic outcome. Further, the minimum level of wealth accumulation ($B_0 \geq Fr^\dagger e$) is needed for the sectoral shift to happen. The economy starting below this minimum level remains stagnant with any initial distribution.

4.2. Comparisons among long-run equilibria

There are two kinds of steady state equilibria, $(Fr_{ss}, B_{ss}) = (1, B^{**})$ and $(Fr, B^*(Fr))$, where $Fr < Fr^\dagger$. In the former equilibrium, the number of skilled workers is H^{**}, skilled wage (net of the cost of education) is equal to unskilled wage, and all the individuals hold the same level of wealth, $\frac{1}{1-\gamma_A-\gamma_M}(b_s)^*$[30]. On the other hand, in the latter type of equilibria, the number of skilled workers is Fr ($< Fr^\dagger < H^{**}$), skilled wage is higher than unskilled wage, and the wealth of skilled workers $\frac{1}{1-\gamma_A-\gamma_M}(b_s)^*$ is larger than that of unskilled workers

[29] When the economy starts with the conditions $Fr_0 < Fr^\dagger$ and $b_u^*(Fr_0, B_0) \geq e$, the more detailed information on the initial wealth distribution is required to find out its long-run outcome.

[30] Wealth is defined as $w_H - (1+r)e + (1+r)(b_s)^*$.

$\frac{1}{1-\gamma_A-\gamma_M} b_u^*(Fr, B^*(Fr))$. The relative price of agricultural goods $P(H, B^*(H))$ is increasing in H, so as $w_L(H, B^*(H))$ and $b_u^*(H, B^*(H))$. Hence, these steady state equilibria can be ranked in terms of the wage and wealth of unskilled workers, which are also measures of inequality between skilled and unskilled workers[31], and the total wealth of the economy[32]. The best equilibrium is $(1, B^{**})$, then among equilibria $(Fr, B^*(Fr))$, $Fr < Fr^\dagger$, one with higher Fr is better.

Note that wages and wealth are measured in terms of manufacturing goods, and the relative price of agricultural goods is different across equilibria, so they are not best measures of people's welfare. For accurate welfare comparisons, the utility of each type of individuals is computed. At equilibrium $(1, B^{**})$, both skilled and unskilled workers have the same utility level, which is:

$$U(1, B^{**}) = \frac{\gamma_A^{\gamma_A}\gamma_M^{\gamma_M}(1-\gamma_A-\gamma_M)^{1-\gamma_A-\gamma_M}}{1-(1-\gamma_A-\gamma_M)(1+r)}(A)^{\gamma_A}[w_H-(1+r)e]^{1-\gamma_A}. \tag{4.1}$$

At equilibria $(Fr, B^*(Fr))$, $Fr < Fr^\dagger$, the utility of skilled workers is given by:

$$U_s(Fr, B^*(Fr)) = U(1, B^{**}) \cdot (\frac{1-H^{**}}{H^{**}}\frac{Fr}{1-Fr})^{-\gamma_A}. \tag{4.2}$$

$U_s(Fr, B^*(Fr)) > U(1, B^{**})$ is satisfied since $Fr < H^{**}$. Skilled workers have higher utilities than at equilibrium $(1, B^{**})$ because agricultural goods are cheaper. In contrast, unskilled workers have lower utilities than at $(1, B^{**})$:

$$U_u(Fr, B^*(Fr)) = U(1, B^{**}) \cdot (\frac{1-H^{**}}{H^{**}}\frac{Fr}{1-Fr})^{1-\gamma_A}. \tag{4.3}$$

Note that $U_s(Fr, B^*(Fr))$ is decreasing and $U_u(Fr, B^*(Fr))$ is increasing in Fr, so inequality in welfare decreases with Fr. The average utility, which can be interpreted as the expected utility of an individual before birth, is given by the following expression:

$$E[U(H, B^*(H))] = U(1, B^{**}) \cdot (\frac{H}{H^{**}})^{1-\gamma_A}(\frac{1-H}{1-H^{**}})^{1-\gamma_A}. \tag{4.4}$$

[31]Remember that the wage and wealth of skilled workers are the same in all the steady state equilibria.

[32]The total wealth at equilibrium $(1, B^{**})$ is equal to $\frac{1}{1-\gamma_A-\gamma_M}(b_s)^*$, while at equilibria $(Fr, B^*(Fr))$, $Fr < Fr^\dagger$, it is equal to $\frac{1}{1-\gamma_A-\gamma_M}(b_s)^*\frac{Fr}{H^{**}}$.

The average utility is strictly increasing in H and attains the highest value at $H = H^{**}$. Hence the ranking among the steady state equilibria remains unchanged if the average utility is used for the comparison.

4.3. Mechanism behind the model

The mechanism of the model yielding the above results can be explained intuitively by the following illustration. Consider an economy in which skilled workers are scarce ($H_0 = Fr_0 < H^{**}$) and assets accumulation is low ($B_0 < B^*(H_0)$). How can this economy increase its income level? One way is through assets accumulation. Since the economy starts with a low assets level, people increase assets holdings over time. The increased assets raise the demand for agricultural goods, the price of the goods, and the unskilled wage. The higher unskilled wage increases savings by unskilled workers, which further increases assets accumulation. This process continues until aggregate assets reach the steady state level. However, with the number of skilled workers remain constant, the income increase is moderate since the ultimate source of assets accumulation is labor income. Further, inequality between skilled and unskilled workers remains large because the investment opportunities are constrained by family income.

Thus, in order to attain a large income growth, an *increase in the number of skilled workers and the sectoral shift* of production and employment from agriculture to manufacturing is crucial. The sectoral shift starts if some children of unskilled workers receive enough resources to take education. Since unskilled workers are in the agriculture sector, the unskilled wage and hence the relative price of agricultural goods must be above critical levels for the sectoral change to start. The agricultural price positively depends on the number of skilled workers and aggregate assets, since the more skilled worker imply higher demand and lower supply of the goods and larger assets accumulation means higher demand. So the number of skilled workers and/or aggregate assets must be above certain levels for the 'take-off'.

When the economy begins with a relatively equal initial wealth distribution ($Fr_0 \geq Fr^{\dagger}$), there are more skilled workers, thus higher total labor income and larger assets accumulation for the next period. As a result, the relative price of agricultural goods is higher. If the combination of Fr_0 and B_0 satisfies $b_u^*(Fr_0, B_0) \geq e$, then a relatively wealthier portion of unskilled workers can get their children to take education and the sectoral shift starts immediately. If not, initially children of unskilled workers cannot access education initially, but as asset accumulation continues, the relative price eventually reaches the crucial level and the sectoral change starts. Once the sectoral shift starts, it continues autonomously. An increase in the number of skilled workers raises the demand for agricultural goods, reduces its supply, and also raises assets accumulation. All contribute to a further rise in the price of agricultural goods. Then children of less affluent unskilled workers gain access to education and the number of skilled workers increases further, which raises the price of the goods more and even a poorer portion of unskilled workers can send their children to school. As long as the skilled wage (net of the cost of education) is higher than the unskilled wage, financially eligible children take education and become skilled workers. Consequently this sectoral shift continues until the relative price of agricultural goods increases to a point where net wages of skilled and unskilled workers are equated. Once equal opportunity is attained, the unskilled wage does not change, but as far as aggregate assets increases, the demand for agricultural goods increases and the sectoral shift continues. The sectoral shift ends after aggregate transfers reach the steady state level B^{**}.

In contrast, when the economy starts with a relatively unequal assets distribution ($Fr_0 < Fr^{\dagger}$), the initial number of skilled workers is small and hence the relative price of agricultural goods is low. Children of unskilled workers are not financially eligible for education and cannot become skilled workers. Still the relative price increases over time through assets accumulation but never reaches the critical level for the sectoral shift. With the number of skilled workers constant, the inequality between skilled and unskilled workers does not disappear, and the economy ends up with the equilibria with the lower output and less equal distribution $(Fr, B^*(Fr))$, $Fr < Fr^{\dagger}$.

When initial assets accumulation is relatively high ($B_0 > B^*(H_0)$), the relative price of agricultural goods and the unskilled wage is higher for given Fr_0, so the sectoral shift occurs more easily. However, unless the initial wealth inequality is small ($Fr_0 \geq Fr^{\dagger}$), the convergence to the best steady state is not assured. This is because the ultimate source of assets accumulation is labor income. Since the initial assets accumulation is more than labor income can support in the long run, there is a tendency for assets to decrease over time. As a consequence, if wealth inequality is high, and as a result, the number of skilled workers increases only moderately, assets accumulation decreases over time and the economy converges to an unequal steady state.

4.4. Productivity growth

Now productivity growth is introduced into the model to see if the results are robust to this modification. Assume that agricultural productivity grows at a constant rate of g_A and manufacturing productivity grows steadily at g_M. With the given constant interest rate r, the wage of skilled workers now grows at $(g_M)^{\frac{1}{\alpha_m}}$ and the unskilled wage grows at $g_A + g_{P,t}$, where $g_{P,t}$ is the time-variant growth rate of the relative price at period t.

Individual's investment decisions follow the same rules as before, but now that the wages are growing over time, cost of education also should grow. Since the cost of education of generation t, e_t is assumed to be proportional to the skilled wage in period $t-1$, $w_{H,t-1}$, it is given by $e_t = s_e w_{H,t-1}$. For simplicity, assume that s_e is constant over time[33].

All the equations describing the individual and aggregate dynamics are the same as before. The difference is that there are no fixed points for the dynamics because of the productivity growth. However, by dividing both sides of these dynamics by $\{(g_M)^{\frac{1}{\alpha_m}}\}^t$ and redefining the variables accordingly, the modified dynamics can be analyzed in a similar way as before[34]. For the modified dynamics, the qualitatively same phase diagram can be drawn, hence all the results go through with this change[35].

[33]The evidence shows that, although teacher-student ratios increase with per capita income, the increase is much smaller compared to the increase in enrollment rates, especially at the secondary level (Schultz, 1989).

[34]The variables are redefined as follows:

$$\begin{aligned} w_H &\equiv w_{H,t}/\{(g_M)^{\frac{1}{\alpha_m}}\}^t, \\ e &\equiv e_t/\{(g_M)^{\frac{1}{\alpha_m}}\}^{t-1}, \\ b_t^i &\equiv b_t^i/\{(g_M)^{\frac{1}{\alpha_m}}\}^{t-1}, \\ B_t &\equiv B_t/\{(g_M)^{\frac{1}{\alpha_m}}\}^{t-1}, \\ \text{and } P(H_t, B_t) &\equiv P(H_t, B_t)/\{(g_M)^{\frac{1}{\alpha_m}}\}^t. \end{aligned}$$

With this redefinition of the variables, the modified dynamics for the economy with productivity growth are different from the respective dynamics for the economy with constant productivity only in one respect. That is, now $1+r$ is replaced by $(1+r)/(g_M)^{\frac{1}{\alpha_m}}$.

[35]Of course, the steady state number of skilled workers in the equal opportunity case, H^{**}, and the critical size of 'middle class' for the 'take-off', $Fr^{\dagger}$, increase with the growth rate of manufacturing productivity, g_M. That is, with the higher manufacturing productivity growth, an economy suceeding in the sectoral shift can have more skilled workers in the long run, but it becomes more difficult for an economy with a small size of 'middle class' to initiate the 'take-off'.

5. Introducing sectoral shift of consumption

The analysis so far has employed the Cobb-Douglas utility function, and hence has abstracted from the sectoral shift of consumption from agriculture (and unskilled-intensive service) to manufacturing (and skilled-intensive service). It is the stylized fact that consumers spend more of their incomes on manufacturing goods and transfers as they become richer[36]. In this section, the utility function is modified so that this feature can be observed in the model. After this modification, *agricultural productivity* as well as initial wealth distribution become determinants of the long-run structure of the economy. In particular, it is shown that if an economy's agricultural productivity is sufficiently low, it ends up in a steady state with low output and high inequality *regardless of* its initial wealth distribution.

5.1. Model

5.1.1. Individual decisions and price determination

The modified utility function takes the following form:

$$U_t^i = (c_{A,t}^i - c_A^0)^{\gamma_A}(c_{M,t}^i)^{\gamma_M}(b_{t+1}^i)^{1-\gamma_A-\gamma_M}, \tag{5.1}$$

where c_A^0 is a constant. c_A^0 can be interpreted as the minimum consumption level of agricultural goods needed for subsistence. A consumer maximizes his new utility subject to the budget constraint,

$$P_t c_{A,t}^i + c_{M,t}^i + b_{t+1}^i = w_t^i + (1+r)a_t^i, \tag{5.2}$$

where a_t^i is investment in assets made in the previous period (childhood) and $w_t^i + (1+r)a_t^i$ is given by[37]:

$$\begin{aligned} w_t^i + (1+r)a_t^i &= w_{L,t} + (1+r)b_t^i, \text{ if } b_t^i < e_t, & (5.3)\\ &= w_{H,t} - (1+r)e_t + (1+r)b_t^i, \text{ if } b_t^i \geq e_t. & (5.4) \end{aligned}$$

[36]See, for example, Syrquin (1988), p.231.

[37]Further, when the returns from the two investment opportunities are equated, $w_{H,t} - (1+r)e_t^0 + (1+r)b_t^i = w_{L,t} + (1+r)b_t^i$ is satisfied.

By solving the maximization problem, the following consumption and transfer rules are obtained:

$$
\begin{aligned}
P_t c^i_{A,t} &= \gamma_A[w^i_t + (1+r)a^i_t] + (1-\gamma_A)P_t c^0_A, && (5.5)\\
c^i_{M,t} &= \gamma_M[w^i_t + (1+r)a^i_t - P_t c^0_A], && (5.6)\\
\text{and } \ b^i_{t+1} &= (1-\gamma_A-\gamma_M)[w^i_t + (1+r)a^i_t - P_t c^0_A]. && (5.7)
\end{aligned}
$$

The consumer first spends his income to purchase the subsistence level of agricultural goods $P_t c^0_A$, then allocates the rest of the income to the goods and transfers in fixed proportions. As income grows, the share of wealth $w^i_t + (1+r)a^i_t$ spent on agricultural goods decreases and the shares spent on manufacturing goods and transfers increase, if the agricultural price P_t does not grow as fast as the income. Also the price elasticity of agricultural goods is less than one, while those of manufacturing goods and transfers are larger than one.

Initially the model is presented assuming no productivity growth. Then, since the market structure is the same as before, wages of skilled workers and of unskilled workers are given by the same equations, and the skilled wage is time-invariant:

$$
\begin{aligned}
w_H &= \alpha_m M^{\frac{1}{\alpha_m}}\left(\frac{1-\alpha_m}{r}\right)^{\frac{1}{\alpha_m}-1}, && (5.8)\\
\text{and } \ w_{L,t} &= P_t A. && (5.9)
\end{aligned}
$$

The market clearing condition of agricultural goods is different from the one in the previous model because of the term with $P_t c^0_A$:

$$P_t A L_t = \gamma_A[w_{L,t}L_t + w_H H_t + (1+r)K^D_t] + (1-\gamma_A)P_t c^0_A. \qquad (5.10)$$

Remember that L_t is the number of unskilled workers, H_t is the number of skilled workers, and K^D_t is total physical capital held domestically. Substituting (5.9), $L_t = 1 - H_t$, and $K^D_t = B_t - eH_t$ into the above equation, and solving it for P_t, the relative price of agricultural goods is given by,

$$P_t = P(H_t, B_t) \equiv \frac{\gamma_A}{1-\gamma_A}\frac{[w_H - (1+r)e]H_t + (1+r)B_t}{A(1-H_t) - c^0_A}. \qquad (5.11)$$

It is assumed that agricultural productivity is high enough ($A > c_A^0$) so that the economy can support at least the subsistence level of consumption c_A^0, if the whole population is engaged in agriculture[38]. With the presence of c_A^0, higher agricultural productivity A decreases P_t *more than proportionally*, because, as income grows, people start spending less portions of their incomes on agricultural goods.

5.1.2. Individual dynamics

In the unequal opportunity case, i.e. when $w_H - (1+r)e > w_L(Fr_t, B_t)$ is satisfied, transfers of skilled workers are determined by the following equation, which is obtained by substituting (5.4) and $H_t = Fr_t$ into (5.7),

$$\begin{aligned} b_{t+1}^i &= b_s(b_t^i; Fr_t, B_t) \\ &\equiv (1-\gamma_A-\gamma_M)\{[w_H-(1+r)e]+(1+r)b_t^i - P(Fr_t, B_t)c_A^0\}. \end{aligned} \tag{5.12}$$

Now the equation depends *negatively* on the size of 'middle class' Fr_t and aggregate transfers B_t through P_t. Increases in Fr_t and B_t raise the price of agricultural goods, so more income must be spent to purchase the subsistence level of the goods, resulting in lower spending on transfers. The fixed point of the equation for given $P(Fr_t, B_t)$, $b_s^*(Fr_t, B_t)$, is equal to,

$$b_s^*(Fr_t, B_t) \equiv \frac{1-\gamma_A-\gamma_M}{1-(1-\gamma_A-\gamma_M)(1+r_t)}\{[w_H-(1+r)e] - P(Fr_t, B_t)c_A^0\}. \tag{5.13}$$

As for unskilled workers, dynamics of transfers are governed by the following equation, which is obtained by substituting (5.3) and $H_t = Fr_t$ into (5.7),

$$b_{t+1}^i = b_u^i(b_t^i; Fr_t, B_t) \equiv (1-\gamma_A-\gamma_M)\{(1+r)b_t^i + (A-c_A^0)P(Fr_t, B_t)\}. \tag{5.14}$$

The dynamic equation for unskilled workers depends *positively* on Fr_t and B_t through P_t, since the positive effect of the higher agricultural price on their earnings exceeds the negative effect on spending on the subsistence level of agricultural goods. The fixed point for given

[38]With this assumption $A(1-H_t) - c_A^0 > 0$ is satisfied in an equilibrium.

$P(Fr_t, B_t)$, $b_u^*(Fr_t, B_t)$, is given by,

$$b_u^*(Fr_t, B_t) \equiv \frac{1-\gamma_A-\gamma_M}{1-(1-\gamma_A-\gamma_M)(1+r)}[(A-c_A^0)P(Fr_t, B_t)]. \tag{5.15}$$

In the equal opportunity case, i.e. when $w_H - (1+r)e \leq w_L(Fr_t, B_t)$ holds, transfers of both types of workers follow $b_{t+1}^i = b(b_t^i)$:

$$b_{t+1}^i = b(b_t^i) \equiv (1-\gamma_A-\gamma_M)\{(1+r)b_t^i + [w_H-(1+r)e](1-\frac{c_A^0}{A})\}. \tag{5.16}$$

The equation is obtained by substituting $P(H_t, B_t) = [w_H - (1+r)e]/A$ into the transfer dynamics for the unequal opportunity case. The fixed point of the equation b^* is equal to,

$$b^* \equiv \frac{1-\gamma_A-\gamma_M}{1-(1-\gamma_A-\gamma_M)(1+r)}[w_H-(1+r)e](1-\frac{c_A^0}{A}). \tag{5.17}$$

The combinations of Fr_t and B_t satisfying $w_H - (1+r)e = w_L(Fr_t, B_t)$ separate the two cases. The dividing line is obtained by substituting $P_t = [w_H - (1+r)e]/A$ and $H_t = Fr_t$ into (5.11) and solving the equation for Fr_t:

$$Fr_t = H^e(B_t) \equiv (1-\gamma_A)(1-c_A^0/A) - \frac{\gamma_A(1+r)B_t}{[w_H-(1+r)e]}. \tag{5.18}$$

5.1.3. Dynamics of aggregate transfers

In the unequal opportunity case, dynamics of aggregate transfers B_t follow the same equation as before,

$$B_{t+1} = B(Fr_t, B_t) \equiv \frac{1-\gamma_A-\gamma_M}{1-\gamma_A}\{[w_H-(1+r)e]Fr_t + (1+r)B_t\}. \tag{5.19}$$

The fixed point of the equation for given Fr_t is equal to $B^*(Fr_t) \equiv [1 - \frac{1-\gamma_A-\gamma_M}{1-\gamma_A}(1+r)]^{-1} \times \frac{1-\gamma_A-\gamma_M}{1-\gamma_A}[w_H - (1+r)e]Fr_t$ as before.

In the equal opportunity case, the dynamics are described by $B_{t+1} = B(H^e(B_t), B_t)$, which is obtained by substituting $H_t = H^e(B_t)$ into $B_t = B(H_t, B_t)$:

$$B_{t+1} = B(H^e(B_t), B_t) \equiv (1-\gamma_A-\gamma_M)\{[w_H-(1+r)e](1-\frac{c_A^0}{A}) + (1+r)B_t\}. \tag{5.20}$$

The fixed point of this equation B^{**} is given by,

$$B^{**} \equiv \frac{1-\gamma_A-\gamma_M}{1-(1-\gamma_A-\gamma_M)(1+r)}[w_H-(1+r)e](1-\frac{c_A^0}{A}), \tag{5.21}$$

which is equal to b^*. The number of skilled workers at B^{**}, $H^{**} \equiv H(B^{**})$ is:

$$H^{**} \equiv H(B^{**}) = [1-\frac{\gamma_A}{1-(1-\gamma_A-\gamma_M)(1+r)}](1-\frac{c_A^0}{A}). \tag{5.22}$$

5.1.4. Joint dynamics of Fr_t and B_t

As before, joint dynamics of Fr_t and B_t, and relationship between initial wealth distribution and long-run performance of an economy are investigated using a phase diagram. Unlike the previous model, now the qualitative nature of the dynamics depends on agricultural productivity.

Case of high agricultural productivity: When agricultural productivity is large enough that $A[(1-\gamma_A-\gamma_M)w_H-e] > (1-\gamma_A-\gamma_M)[w_H-(1+r)e]c_A^0$ is satisfied, the phase diagram looks like the one in Section 3 (Figure 3.5)[39]. Remember that, on the diagram, the feasible combinations of Fr and B are equal to the area bound by $Fr = 0$, $Fr = 1$, and $B = eFr$. The region below $Fr = H^e(B)$ is the unequal opportunity case and $H = Fr$ is satisfied, while the region on and above the locus is the equal opportunity case and $H = H^e(B) \leq Fr$ holds. The direction of change of B is determined by the current location of (Fr_t, B_t) relative to $B = B^*(Fr)$ in the unequal opportunity case, and the location relative to $B = B^{**}$ in the equal opportunity case. The direction of change of Fr is determined by the current position of (Fr_t, B_t) relative to $b_u^*(Fr, B) = e$. As before, $b_u^*(:) = e$ is located below $Fr = H^e(B)$, and $B^{**} = b^* > e$ is satisfied[40].

The qualitative nature of the dynamics is the same as the original economy. That is, when the economy starts with a relatively equal initial wealth distribution, it converges to $(Fr, B) = (1, B^{**})$, where perfect equality is achieved, and when the economy begins with

[39]There are only two differences from the economy without sectoral shift of consumption. First, $Fr = H^e(B)$ shifts inwards by factor $(1-\frac{c_A^0}{A})$. Second, the slope of $b_u^*(Fr, B) = e$ becomes steeper by the presence of $(1-\frac{c_A^0}{A})$. The intercept of the line is the same as before.

[40]This condition is equivalent to the above condition on agricultural productivity and the condition that $b_s^*(Fr, B) = e$ is located above $Fr = H^e(B)$.

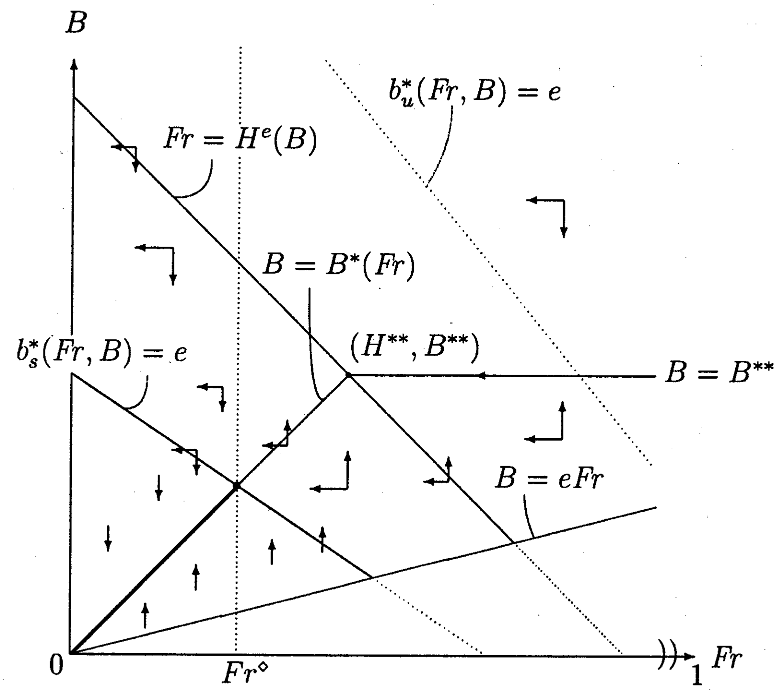

Figure 5.1: Economy with low agricultural productivity

an unequal wealth distribution and/or too small aggregate wealth, it converges to one of $(Fr, B) = (Fr, B^*(Fr))$ $(Fr < Fr^\dagger)$, where the return from education is higher than that from assets, and consequently inequality between skilled and unskilled workers is persistent.

Case of low agricultural productivity: On the other hand, when $A[(1-\gamma_A-\gamma_M)w_H - e] < (1-\gamma_A-\gamma_M)[w_H-(1+r)e]c_A^0$ is satisfied, that is, when agricultural productivity is relatively small, the phase diagram looks like Figure 5.1. Unlike the previous case, $b_u^*(Fr,B) = e$ is located *above* $Fr = H^e(B)$, hence $b_u^*(H,B) < e$ is satisfied for all the feasible combinations of (H,B)[41], while $b_s^*(Fr,B) = e$ is located *below* $Fr = H^e(B)$[42]. Consequently, descendants of unskilled workers *never* have enough assets to become skilled workers. Since $b_s^*(Fr,B)$ is decreasing in Fr and B, in the region *above* $b_s^*(Fr,B) = e$, $b_s^*(:) < e$ is satisfied and $Fr_{t+1} \leq Fr_t$ holds, while in the region *below* the line, $b_s^*(:) \geq e$ and $Fr_t = Fr_t$ hold. On the diagram, the value of Fr at the intersection of $B = B^*(Fr)$ and $b_s^*(:) = e$ is denoted by $Fr^\diamond$.

[41]The condition on agricultural productivity is equivalent to $b^* < e$. As a result, in the equal opportunity case, $b_u^*(H,B) = b_u^*(H^e(B),B) = b^* < e$ is satisfied. Then, $b_u^*(H,B) < e$ is satisfied for the unequal opportunity case as well, since $b_u^*(H,B) = b_u^*(Fr,B) < b_u^*(H^e(B),B)$.

[42]This is equivalent to $B^{**} = b^* < e$. So $B = eFr$ intersects with $B = B^{**}$ at $Fr < 1$ in this case.

The detailed analysis of the dynamics is in Appendix I.

Using this diagram, relationship between initial wealth distribution and an economy's long-run outcome is examined. The biggest difference from the results of the economy with high agricultural productivity or without sectoral shift of consumption is that equal opportunity and perfect equality are *not* sustained in the long run. In the equal opportunity case, that is, in the region on and above $Fr = H^e(B)$ on the diagram, $b^* < e$ holds, so a proportion of individuals financially qualified for education decreases over time. Consequently the economy eventually moves to the unequal opportunity case. In the unequal opportunity case, as long as the economy is located above $b_s^*(Fr, B) = e$, the number of skilled workers decreases over time. Thus, the long-run equilibria of the economy are $(Fr, B) = (Fr, B^*(Fr)), Fr \leq Fr^\diamond$, where (net) skilled wage is higher than unskilled wage and unequal opportunity persists. Among the steady state equilibria, one with higher Fr achieves higher equality between skilled and unskilled workers and higher total income.

In this economy, there is *no* clear relationship between initial distribution and long-run economic structure. Among the economies whose initial positions are on or below $b_s^*(Fr, B) = e$ and $Fr_0 \leq Fr^\diamond$, one with higher Fr_0 achieves higher income and lower inequality in the long run. However, if the economy starts from the other regions, there is no such clear relationship. To which point the economy converges is dependent on the exact initial wealth distribution. Comparing two economies starting with different Fr_0, it is possible that the economy starting with higher Fr_0 can end up with lower Fr in the long run.

5.2. Analyses

With the introduction of the sectoral shift of consumption into the model, the long-run outcome of the economy becomes affected by agricultural productivity. When the productivity is high, the economy behaves qualitatively in the same manner as before. In contrast, when the productivity is low, *regardless* of initial wealth distribution, the economy ends up with $(Fr, B) = (Fr, B^*(Fr))$, $Fr \leq Fr^\diamond$, where inequality between skilled and unskilled workers persists. Even if starting with a relatively equal distribution $Fr_0 > Fr^\diamond$, the economy does not succeed in shifting employment and production from agriculture to manufacturing and converging to $(1, B^{**})$ in the long run.

Why does not sectoral shift happen when agricultural productivity is low? The requirement for the industrialization is sizable wealth accumulation by a portion of unskilled workers so that they can send their children to school. How does the lower productivity affect their transfers? With the introduction of the sectoral shift of consumption, now people have to consume at least c_A^0 units of agricultural goods *irrespective* of its price, and as a result, the price elasticity of demand for the goods is less than one. Consequently, when agricultural productivity is lower, its price becomes higher more than proportionally and the unskilled wage is higher, other things being equal. This makes it easier for unskilled workers to spend on transfers. On the other hand, this price increase forces them to spend more on agricultural goods. The net effect is in favor of more transfers to children: now smaller 'middle class' and assets accumulation are sufficient for the sectoral shift to happen, i.e. $b_u^*(Fr, B) = e$ is located closer to the origin on the phase diagram. However, there is the other effect associated with the lower productivity and the resultant higher unskilled wage: because of the fall of return to education for given Fr and B, the economy can sustain less skilled labor and aggregate assets under equal opportunity, i.e. $Fr = H^e(B)$ is closer to the origin. If agricultural productivity is low enough, as in Figure 5.1, the second effect dominates the first effect, i.e. $b_u^*(Fr, B) = e$ is located above $Fr = H^e(B)$, hence the sectoral shift never starts.

Further, the economy cannot keep an initial condition with sizable skilled labor ($Fr_0 > Fr^{\diamond}$), because the agricultural price is too high for some of skilled workers to keep transfers sufficient for education. The number of skilled workers must decline until the price level falls enough that they can sustain the needed transfers.

The result shows that, when the sectoral shift of consumption is introduced, a certain level of agricultural productivity is a *prerequisite* for sectoral shift, and without such productivity, the economy is destined to converge to a lower output steady state where inequality between skilled and unskilled workers persists. When there is no sectoral shift of consumption, i.e. $c_A^0 = 0$, industrialization happens even when agricultural productivity is low, as long as the economy starts with a reasonably equal wealth distribution. This is because people always spend a fixed portion of their incomes on each of the goods and transfers, hence the rising agricultural price does not become an obstacle for industrialization.

5.3. Productivity growth

Introduction of productivity growth does not change the main results from this section, but unlike the economy without the sectoral shift of consumption, now the phase diagram is not qualitatively the same as the one without productivity growth. The reason is that $Fr = H^e(B; A_t)$, $b_s^*(Fr, B; A_t) = e$, and $b_u^*(Fr, B; A_t) = e$ shift upward over time as A_t increases. This complication brings some new phenomena to the dynamics. The detailed analysis is contained in Appendix II.

First consider the case where initial agricultural productivity is high so that the phase diagram at the initial period looks like the one in Section 3 (Figure 3.5). When the initial size of 'middle class' is sufficiently large or small, the results of the model without productivity growth go through. A new possibility arises when a proportion of individuals who can afford education initially is of a middle size. In this intermediate range, an economy initially experiencing sectoral shift may end up in a low-output and high-inequality steady state. This can happen if wealth is relatively concentrated in the rich, and consequently an increase in skilled labor cannot keep up with the outward shift of $b_u^*(:; A_t) = e$ resulting from the productivity growth. Unskilled workers can increase transfers rapidly enough to keep up with a rising cost of education, if their income growth or the price fall of agricultural goods are fast enough. Given other things equal, the productivity growth makes the goods cheaper but *decreases* the relative wage of unskilled workers. It turns out that its net effect on their assets accumulation and transfers is *negative*, that is, growth of their transfers lag that of the cost of education. Thus the sectoral shift is halted if the number of skilled workers and aggregate assets do not increase rapidly enough to support the unskilled wage.

When initial agricultural productivity is low, the phase diagram at the initial period looks like the one presented earlier in this section (Figure 5.1). Obviously, the economy does not remain in this state forever, since the productivity growth eventually brings it to the high agricultural productivity case. However, it is very unlikely that the economy starting from this case can succeed in the sectoral shift and reaches the equal-opportunity steady state in the long run. While the economy is in this low productivity state, a size of 'middle class' is constant or decreases over time. If the economy enters the high productivity state with small *Fr*, industrialization never starts. But entering the high productivity state with sizable *Fr* requires large initial 'middle class' and rapid productivity growth in agriculture, those seem to be very unlikely for the very poor economy.

6. Conclusion

There are two notable phenomena widely observed when an economy departs from an underdeveloped state and starts rapid economic growth. One is the shift of production, employment, and consumption from agriculture and unskilled-intensive service to manufacturing and skilled-intensive service, and the other is a large increase in educational levels of its population. The question is why some economies have succeeded in such 'structural change', but others do not. In order to examine the question, this paper has constructed an overlapping generations model that explicitly takes into account the sectoral change and human capital accumulation as sources of growth.

It has been shown that a relatively equal initial wealth distribution, or to be more accurate, a sufficient size of 'middle-class' is a necessary condition for a successful sectoral shift. Once the economy initiates the 'take-off', the sectoral shift and human capital growth continue until it reaches the steady state, where equal opportunity is attained. However, when agricultural productivity is low, the economy does not succeed in the sectoral shift irrespective of the initial distribution. Thus sufficient agricultural productivity is a prerequisite for the success.

The main points of the paper, (i)importance of inequality, especially size of 'middle class', in economic development, and (ii)sufficient agricultural productivity as a precondition for successful sectoral shift, are largely supported by empirical studies, although supportive evidences on the second point are more indirect. It may be worthwhile to test this point in a more formal manner in future.

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7. Appendix I: Detailed analysis of aggregate dynamics

7.1. Dynamics of aggregate variables when there is no sectoral shift of consumption or agricultural productivity is high

This subsection describes the detailed dynamics of Fr_t and B_t, when there is no sectoral shift of consumption ($c_A^0 = 0$) or agricultural productivity is high, i.e. $A[(1-\gamma_A-\gamma_M)w_H - e] \geq (1-\gamma_A-\gamma_M)[w_H-(1+r)e]c_A^0$. As explained in the main text, both cases follow the same dynamics qualitatively. The phase diagram (Figure 3.5) is used for the explanation.

(I) *Equal opportunity case* (the region where (Fr, B) satisfies $Fr \geq H^e(B)$)

(i) The region where $B \leq B^{**}$ is satisfied.

Since $b_t^i \geq e$ implies $b_{t+1}^i \geq e$, $Fr_{t+1} \geq Fr_t$ is satisfied. The dynamics of aggregate transfers follow $B_{t+1} = B(H^e(B_t), B_t)$, so if $B_t = B^{**}$, $B_{t+1} = B_t = B^{**}$, and if $B_t < B^{**}$, $B^{**} > B_{t+1} > B_t$ hold. Hence when $(Fr_t, B_t) = (1, B^{**})$, the economy stays at this point forever. Otherwise, the economy continues to stay in this region and eventually converges to $(1, B^{**})$. As is clear from the phase diagram, the economy never moves to the unequal opportunity case, where $Fr < H^e(B)$ is satisfied. It does not move to the region $B > B^{**}$ either, since $B_{t+1} \leq B^{**}$ is true.

(ii) The region with $B > B^{**}$.

In this region $Fr_{t+1} \geq Fr_t$ and $B^{**} < B_{t+1} < B_t$ are satisfied. There are two possibilities. If the distribution is relatively equal, the economy continues to stay in this region and converges to $(1, B^{**})$ for the same reasons mentioned in (i). Especially when the economy is in the region $Fr_t \geq H^{**}$, the convergence to $(1, B^{**})$ is assured, which is straightforward from the diagram. If the distribution is relatively skewed to the right tail, the increase of Fr_t is slow, so it moves to the unequal opportunity case (crosses $Fr = H^e(B)$) with $B > B^{**}$.

(II) *Unequal opportunity case* (the region where $Fr < H^e(B)$ is satisfied)

(i) The region where $b_u^*(Fr, B) \geq e$ is satisfied.

In this region Fr_t is non-decreasing over time, as is clear from Figure 3.4.

a) Case $B \leq B^*(Fr)$

Dynamics of aggregate transfers follow $B_{t+1} = B(Fr_t, B_t)$, so when $B_t = B^*(Fr_t)$, $B_{t+1} = B_t = B^*(Fr_t)$, and when $B_t < B^*(Fr_t)$, $B^*(Fr_t) > B_{t+1} > B_t$ hold. Eventually the economy shifts to the equal opportunity case with $Fr \geq H^{**}$. Since $B^*(Fr_t) \geq B_{t+1}$ and $Fr_{t+1} \geq Fr_t$ are satisfied, $B^*(Fr_{t+1}) \geq B^*(Fr_t) \geq B_{t+1}$ holds. So the economy does not move to the unequal opportunity case with $b_u^*(Fr, B) \geq e$ and $B > B^*(Fr)$, or the equal opportunity case with $Fr < H^{**}$. It does not stay in this region forever because then $b_t^i \geq e$ becomes satisfied for all the individuals eventually.

b) Case $B > B^*(Fr)$

In this case $B^*(Fr_t) < B_{t+1} < B_t$ holds. When $Fr_t \geq Fr^\dagger$, the economy moves to a) or the equal opportunity case eventually[43]. When $Fr_t < Fr^\dagger$, depending on the wealth distribution, the economy transits to a), the equal opportunity case, or the region with $b_u^*(Fr, B) < e$. The last scenario happens if the distribution is unequal and Fr_t increases only slightly.

(ii) The region satisfying $b_u^*(Fr, B) < e$.

Fr_t remains constant in this region. As is clear from the phase diagram, in the region $Fr < Fr^\dagger$, the economy converges to $(Fr_t, B^*(Fr_t))$. When $B \leq B^*(Fr)$, there is also the region with $Fr \geq Fr^\dagger$. The economy in this region eventually moves to the region satisfying $b_u^*(Fr, B) \geq e$ and $B \leq B^*(Fr)$.

7.2. Dynamics of aggregate variables when agricultural productivity is low

This subsection describes the detailed dynamics of Fr_t and B_t, when agricultural productivity is low, i.e. $A[(1 - \gamma_A - \gamma_M)w_H - e] < (1 - \gamma_A - \gamma_M)[w_H - (1 + r)e]c_A^0$ in an economy with sectoral shift of consumption. The phase diagram (Figure 5.1) is used for the explanation.

(I) *Equal opportunity case* (the region where $Fr \geq H^e(B)$ is satisfied)

[43]The transition to a) is possible since $B^*(Fr_{t+1}) > B_{t+1} > B^*(Fr_t)$ can happen.

Since $b_s^*(Fr_t, B_t) < e$ is satisfied, Fr_t decreases over time. As for aggregate transfers, $[B^{**} \gtreqqless B_{t+1} \gtreqqless B_t] \Leftrightarrow B_t \lesseqqgtr B^{**}$is satisfied. As a result, the economy transits to the unequal opportunity case eventually.

(II) *Unequal opportunity case* (the region where $Fr < H^e(B)$ is satisfied)

(i) The region above $b_s^*(Fr, B) = e$.

Since $b_s^*(Fr_t, B_t) < e$ is satisfied, Fr_t decreases over time.

a) Case $B \leq B^*(Fr)$

Aggregate transfers satisfy $[B^*(Fr_t) \geq B_{t+1} \geq B_t] \Leftrightarrow B_t \leq B^*(Fr_t)$. Eventually the economy transits to one of the three regions, the equal opportunity case, the unequal opportunity case with $b_s^*(Fr_t, B_t) < e$ and $B_t > B^*(Fr_t)$, or the unequal opportunity case with $b_s^*(Fr_t, B_t) \geq e$. It does not stay in this region forever because if it continues to stay $b_t^i < e$ is satisfied for all individuals eventually.

b) Case $B > B^*(Fr)$

In this case $B^*(Fr_t) < B_{t+1} < B_t$ holds. So the economy unambiguously moves to the unequal opportunity case with $b_s^*(Fr, B) \geq e$ and $B > B^*(Fr)$ eventually.

(ii) The region on or below $b_s^*(Fr, B) = e$.

Since $b_s^*(Fr_t, B_t) \geq e$ is satisfied, Fr_t is constant. When $B_t > B^*(Fr_t)$, B_t decreases over time and the economy converges to one of $(Fr, B) = (Fr, B^*(Fr))$, $Fr \leq Fr^\diamond$ in the long run. When $B_t < B^*(Fr_t)$, B_t increases over time. If $Fr_t \leq Fr^\diamond$, the economy converges to one of $(Fr, B) = (Fr, B^*(Fr))$, $Fr \leq Fr^\diamond$ in the long run. In contrast, if $Fr_t > Fr^\diamond$, it transits to case (i), b).

8. Appendix II: Economy with sectoral shift of consumption and productivity growth

This appendix analyzes the economy with sectoral shift of consumption *and* productivity growth in details. As in the main text, assume that the productivity of the agriculture sector grows at a constant rate of g_A and that of the manufacturing sector grows steadily at g_M. Individual's investment decisions follow the same rules as before, but now that the wages are growing over time, education cost also should grow. Since the cost of education of generation

t, e_t is assumed to be proportional to the skilled wage in period $t-1$, $w_{H,t-1}$, it is given by $e_t = s_e w_{H,t-1}$. For simplicity, assume that s_e is constant over time.

8.1. Dynamics

8.1.1. Aggregate dynamics

All the equations describing the individual and aggregate dynamics are the same as the economy without productivity growth, but they cannot be analyzed in the same way because of the technological change. For example, consider the dynamic equation of aggregate transfers in the unequal opportunity case:

$$B_{t+1} = B(Fr_t, B_t) = \frac{1-\gamma_A-\gamma_M}{1-\gamma_A}\{[w_{H,t}-(1+r)e_t]Fr_t + (1+r)B_t\}. \tag{8.1}$$

Since $w_{H,t}$ and e_t grow over time, the equation does not have a fixed point for given Fr_t.

However, by dividing both sides of the above equation by $\{(g_M)^{\frac{1}{\alpha m}}\}^t$ and redefining the variables, the modified equation can be analyzed in a similar way as before. The variables are redefined as follows:

$$\begin{aligned} w_H &\equiv w_{H,t}/\{(g_M)^{\frac{1}{\alpha m}}\}^t, \\ e &\equiv e_t/\{(g_M)^{\frac{1}{\alpha m}}\}^{t-1}, \\ B_t &\equiv B_t/\{(g_M)^{\frac{1}{\alpha m}}\}^{t-1}. \end{aligned}$$

Then the *modified* equation is expressed as,

$$\begin{aligned} B_{t+1} &= B(Fr_t, B_t) \\ &\equiv \frac{1-\gamma_A-\gamma_M}{1-\gamma_A}\{[w_H - R(g_M)e]Fr_t + R(g_M)B_t\}, && (8.2) \\ \text{where} \quad R(g_M) &\equiv \frac{1+r}{(g_M)^{\frac{1}{\alpha m}}}. && (8.3) \end{aligned}$$

where w_H and e are now *constants*. There exists a fixed point for the equation given Fr_t,

$B^*(Fr_t)$, which is defined as,

$$B^*(Fr_t) \equiv \frac{\frac{1-\gamma_A-\gamma_M}{1-\gamma_A}}{1-\frac{1-\gamma_A-\gamma_M}{1-\gamma_A}R(g_M)}[w_H - R(g_M)e]Fr_t. \tag{8.4}$$

The unequal opportunity case is separated from the equal opportunity case by the combinations of Fr_t and B_t satisfying $w_{H,t} - (1+r)e_t = w_{L,t}(Fr_t, B_t)$. Remember that Fr_t is a proportion of individuals who have received transfers greater than the cost of education. Redefining the variables in the same manner, the modified equation satisfying this relation is given by,

$$Fr_t = H^e(B_t; A_t) \equiv (1-\gamma_A)(1-\frac{c_A^0}{A_t}) - \frac{\gamma_A R(g_M)B_t}{[w_H - R(g_M)e]}. \tag{8.5}$$

$H^e(B_t; A_t)$ is positively dependent on A_t, because productivity growth in the agriculture sector reduces the price of agricultural goods more than proportionally, resulting in a lower unskilled wage and a higher return to education. Notice that when $c_A^0 = 0$, that is, when spending shares of consumers do not change with income growth, the dependence on A_t disappears.

The modified dynamic equation of B_{t+1} in the equal opportunity case ($Fr_t \geq H^e(B_t; A_t)$) becomes,

$$\begin{aligned} B_{t+1} &= B(H^e(B_t; A_t), B_t) \\ &\equiv (1-\gamma_A-\gamma_M)\{[w_H - R(g_M)e](1-\frac{c_A^0}{A_t}) + R(g_M)B_t\}. \end{aligned} \tag{8.6}$$

Since A_t is increasing over time, $B_{t+1} = B(H^e(B_t; A_t), B_t)$ *shifts upward*, but eventually it converges to $B_{t+1} = B(H^e(B_t; A_t), B_t)$ when $c_A^0 = 0$, whose fixed point is B^{**} :

$$B^{**} \equiv \frac{1-\gamma_A-\gamma_M}{1-(1-\gamma_A-\gamma_M)R(g_M)}[w_H - R(g_M)e]. \tag{8.7}$$

8.1.2. Individual dynamics

Individual dynamics too are modified by dividing both sides of the dynamics by $\{(g_M)^{\frac{1}{\alpha m}}\}^t$ and redefining the variables[44]. In the unequal opportunity case, the modified dynamic equation of transfers for skilled workers is given by,

$$\begin{aligned} b^i_{t+1} &= b_s(b^i_t, Fr_t, B_t; A_t) \\ &\equiv (1-\gamma_A-\gamma_M)\{[w-R(g_M)e] + R(g_M)b^i_t - P(Fr_t, B_t; A_t)c^0_A\}, \end{aligned} \tag{8.8}$$

where

$$P(Fr_t, B_t; A_t) \equiv \frac{\gamma_A}{1-\gamma_A} \frac{[w_H - R(g_M)e]Fr_t + R(g_M)B_t}{A_t(1-Fr_t) - c^0_A}. \tag{8.9}$$

Given Fr_t and B_t, it shifts upward over time with growth of A_t, but eventually converges to $b^i_{t+1} = b_s(b^i_t)$ when $c^0_A = 0$. The modified dynamic equation for unskilled workers is,

$$\begin{aligned} b^i_{t+1} &= b_u(b^i_t, Fr_t, B_t; A_t) \\ &\equiv (1-\gamma_A-\gamma_M)\{R(g_M)b^i_t + (A_t - c^0_A)P(Fr_t, B_t; A_t)\}. \end{aligned} \tag{8.10}$$

Given Fr_t and B_t, it shifts upward over time and converges to $b^i_{t+1} = b_s(b^i_t; Fr_t, B_t)$ when $c^0_A = 0$.

The dynamic equation for the equal opportunity case is modified as follows:

$$b^i_{t+1} = b(b^i_t; A_t) \equiv (1-\gamma_A-\gamma_M)\{R(g_M)b^i_t + [w_H - R(g_M)e](1-\frac{c^0_A}{A})\}. \tag{8.11}$$

In sum, when the appropriate modifications are made, the dynamics can be described in a similar way as the case without technological change. All the modified dynamics except $B_{t+1} = B(Fr_t, B_t)$ shift upward over time because of productivity growth in agriculture, and converge to the respective dynamics when $c^0_A = 0$ in the long run.

[44]The variables are redefined as follows.

$$\begin{aligned} b^i_t &\equiv b^i_t/\{(g_M)^{\frac{1}{\alpha m}}\}^{t-1}, \\ P(H_t, B_t; A_t) &\equiv P(H_t, B_t; A_t)/\{(g_M)^{\frac{1}{\alpha m}}\}^t. \end{aligned}$$

8.2. Results

Joint dynamics of Fr_t and B_t, and relationship between initial wealth distribution and long-run performance of an economy are investigated using a phase diagram as before. Additional complication arises since $Fr = H^e(B; A_t)$, $b_s^*(Fr, B; A_t) = e$, and $b_u^*(Fr, B; A_t) = e$ shift upward over time as A_t grows. The growth of agricultural productivity decreases the price of agricultural goods more than proportionally[45], hence the unskilled wage declines for given (Fr, B) and $Fr = H^e(B; A_t)$ shifts outward. Locus $b_s^*(:; A_t) = e$ shifts outward, because cheaper agricultural goods make it possible for skilled workers to allocate more income on transfers. On the other hand, unskilled workers are affected both positively (by the price decline) and negatively (by the wage decrease), but the net effect is *negative* and $b_u^*(:; A_t) = e$ shifts upward [46]. Denote the intersection of $Fr = H^e(B; A_t)$ and $B = B^*(Fr)$ by $(Fr^{**}(A_t), B^{**}(A_t))$[47].

8.2.1. Economy with high agricultural productivity

First, consider the case where an economy's initial agricultural productivity is large enough that $A_0[(1 - \gamma_A - \gamma_M)w_H - e] > (1 - \gamma_A - \gamma_M)[w_H - R(g_M)e]c_A^0$ is satisfied (Figure 8.1)[48]. In this case, the phase diagram for *given* agricultural productivity looks like the diagram for the same case without productivity growth (Figure 3.5). The difference is that, over time, $Fr = H^e(B; A_t)$ shifts outward parallelly, while $b_u^*(:; A_t) = e$ shifts outward with its intercept constant, and the both converge to the respective equations when $c_A^0 = 0$.The phase diagram displays the both loci when agricultural productivity is at the initial level and at the long run level ($t \longrightarrow \infty$).

The relationship between initial distribution and long-run outcome is very similar to the case without productivity growth. When the initial condition satisfies $Fr_0 \geq Fr^\dagger$, sectoral shift happens and the economy approaches $(Fr, B) = (1, B^{**}(A_t))$ in the middle run and

[45]As explained in the previous subsection, productivity growth in the agriculture sector and the resulting income growth cause the shift of consumption from agricultural goods to manufacturing goods, resulting in the larger price decline than the case with $c_A^0 = 0$.

[46]Remember that (Fr, B) located above $b_u^*(Fr, B; A_t) = e$ satisfies $b_u^*(:) > e$, while the region *below* $b_s^*(Fr, B; A_t) = e$ satisfies $b_s^*(:) > e$.

[47]$B^{**}(A_t)$ is the fixed point of $B_{t+1} = B(H^e(B_t; A_t), B_t)$ for *given* A_t.

[48]This condition is equivalent to the condition that the fixed point of $b_{t+1}^i = b(b_t^i; A_t)$, $b^*(A_t)$ is greater than e. It is also equivalent to the condition that $b_s^*(Fr, B; A_t) = e$ is located above $Fr = H^e(B; A_t)$.

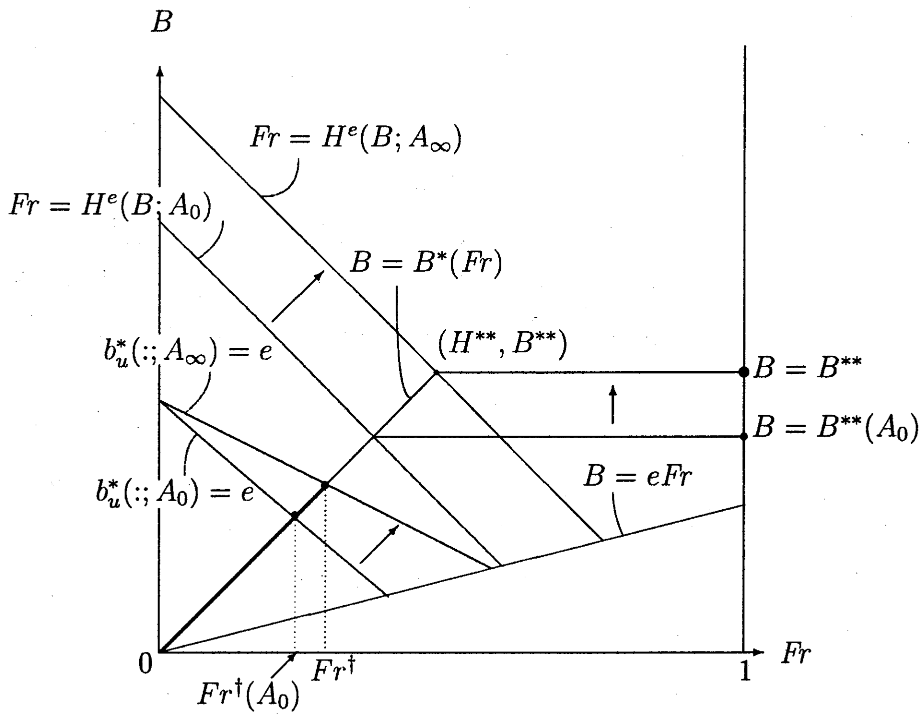

Figure 8.1: Economy with Productivity Growth: High Agricultural Productivity Case

converges to $(1, B^{**})$ in the long run. Accordingly the number of skilled workers approaches $H^{**}(A_t)$ in the middle run and converges to H^{**} in the long run. In contrast, when the initial distribution satisfies $Fr_0 < Fr^\dagger(A_0)$ and $b_u^*(:; A_0) \leq e$, sectoral shift never happens and the economy converges to $(Fr, B) = (Fr, B^*(Fr))$, $Fr \leq Fr^\dagger(A_0)$ in the long run.

What is new is the case where the economy starts with $Fr_0 \in [Fr^\dagger(A_0), Fr^\dagger)$. In this case whether the convergence to $(1, B^{**})$ succeeds or not is dependent on the detailed information on the initial distribution. Even if the economy starts from the region $b_u^*(:; A_0) \geq e$ and its skilled labor increases initially, it may end up in a steady state $(Fr, B^*(Fr))$, $Fr \leq Fr^\dagger$. This can happen if wealth is concentrated in the few rich, and consequently, an increase in skilled labor cannot keep up with the outward shift of $b_u^*(:; A_t) = e$ resulting from the productivity growth. Unskilled workers can increase transfers rapidly enough to keep up with a rising cost of education, if their income growth or the price fall of agricultural goods are fast enough. Given other things equal, the productivity growth makes the goods cheaper but *decreases* the relative wage of unskilled wage. It turns out that its net effect on their assets accumulation and transfers is *negative*, that is, growth of their transfers lag that of the cost of education. Thus the sectoral shift is halted if the number of skilled workers and

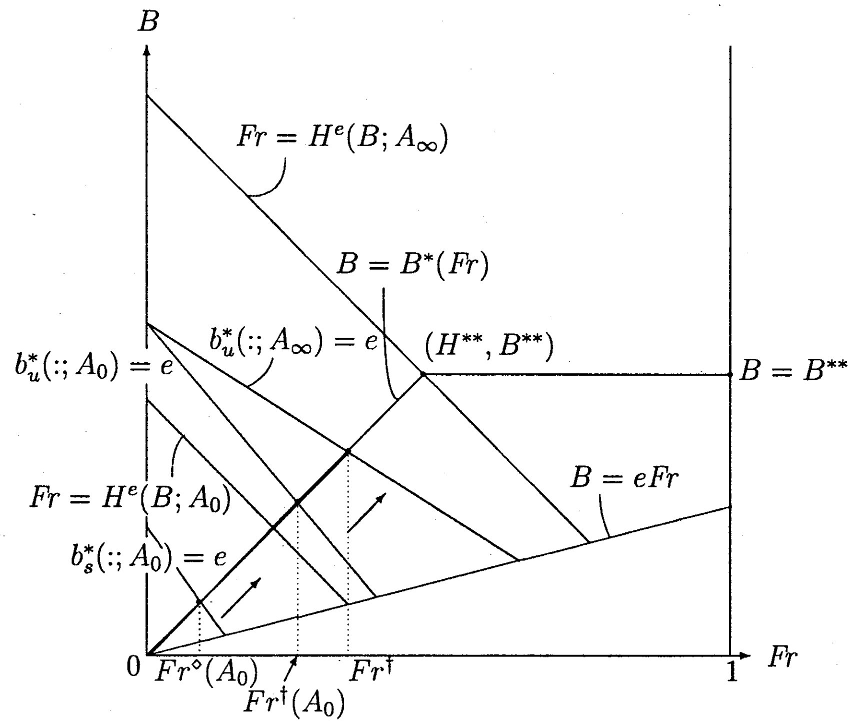

Figure 8.2: Economy with Productivity Growth: Low Agricultural Productivity Case

aggregate assets do not increase rapidly enough to support the unskilled wage.

Growth of agricultural productivity increases income and output, but for the successful sectoral shift, initial wealth distribution is critical as before. Because of the productivity growth, all the (original) variables except Fr grow over time in the long run; the wages, the individual and aggregate assets all grow at rate $(g_M)^{\frac{1}{\alpha_m}}$ and the relative price of agricultural goods grows at $(g_M)^{\frac{1}{\alpha_m}}/g_A$. Without the productivity growth, a rise of the relative price of agricultural goods is necessarily associated with sectoral shift, but now the relative price can *decrease* if g_A is large enough.

8.2.2. Economy with low agricultural productivity

When an economy's initial agricultural productivity is small so that $A_0[(1-\gamma_A-\gamma_M)w_H-e] < (1-\gamma_A-\gamma_M)[w_H - R(g_M)e]c_A^0$ is satisfied, the phase diagram looks like Figure 8.2. The phase diagram with agricultural productivity *fixed* at the initial level is qualitatively the same as the diagram for the same case without productivity growth (Figure 5.1). That is, $b_u^*(Fr, B; A_0) = e$ is located *above* $Fr = H^e(B; A_0)$, and $b_s^*(Fr, B; A_0) = e$ is located *below*

$Fr = H^e(B; A_0)$[49]. If the productivity remains at this level, the economy would converge to $(Fr, B) = (Fr, B^*(Fr)), Fr \leq Fr^\diamond(A_0)$ eventually.

Now, because of the productivity growth, as time passes, $Fr = H^e(B; A_t)$ shifts outward parallelly, while $b_s^*(:; A_t) = e$ shifts outward with its slope increasing. Eventually $b_s^*(:; A_t) = e$ moves *above* $Fr = H^e(B; A_t)$, and $b_u^*(:; A_t) = e$ moves *below* $Fr = H^e(B; A_t)$. That is, the economy transits to the high productivity case examined above.

Is there any chance for this economy to converge to the steady state with equal opportunity? There is, but it is likely to be very small. Consider an economy that starts with small assets accumulation, i.e. $B_0 < B^*(Fr_0)$. Suppose that the economy shifts to the high productivity case at period s. From this period, sectoral shift from agriculture to manufacturing becomes possible[50]. For such sectoral shift to actually happen, the economy must be located at least above $Fr^\dagger(A_s)$ at period s. If the economy initiates with $Fr_0 < Fr^\dagger(A_s)$, this condition is impossible to meet and hence it will end up with $(Fr, B_t) = (Fr, B^*(Fr)), Fr \leq Fr^\dagger(A_s)$ for certain. Even if the initial condition satisfies $Fr_0 \geq Fr^\dagger(A_s)$, skilled labor decreases until period s, so large initial Fr_0 or high productivity growth is necessary at least. Further, for the sectoral shift to *continue*, skilled labor must increase rapidly enough over time to keep up with the outward shift of $b_u^*(:; A_t) = e$ resulting from the productivity growth.

In sum, the results obtained for the economy with fixed productivity mostly remain intact with the introduction of productivity growth. So if the initial agricultural productivity is low, the economy is highly likely to end up with a lower income steady state regardless of the initial distribution. In the model economy, tuition is assumed to be proportional to the skilled wage. If tuition becomes cheaper relative to the skilled wage as income grows, then the economy would converge to the steady state with equal opportunity in the long run. Still, if the reduction of the relative cost of education is not large, the economy starting with low agricultural productivity would have to spend many years until it reaches the best steady state.

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[49]Thus $b_u^*(H, B; A_0) < e$ is satisfied for any H.

[50]Before this period the shift *from manufacturing to agriculture* happens if the economy is located above $b_s^*(:; A_t) = e$.