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Vapor flows condensing at incidence onto a plane condensed phase in the presence of a noncondensible gas. I. Subsonic condensation

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A steady flow of a vapor in a half space condensing onto a plane condensed phase of the vapor at incidence is considered in the case where another gas that neither evaporates nor condenses (the noncondensable gas) is present near the condensed phase. The behavior of the vapor and noncondensable gas is investigated on the basis of kinetic theory under the assumption that the molecules of the noncondensable gas are mechanically identical with those of the vapor. In particular, the relation, among the parameters of the vapor at infinity (the pressure, temperature, and flow velocity of the vapor), those related to the condensed phase (the temperature of the condensed phase and the corresponding saturation pressure of the vapor), and the amount of the noncondensable gas, that admits a steady solution is obtained numerically by the use of a model Boltzmann equation proposed by Garzo et al. [Phys. Fluids A 1, 380 (1989)]. The present analysis is the continuation of an earlier work by Sone et al. [Transp. Theory Stat. Phys. 21, 297 (1992)], where the case in which the vapor flow is condensing perpendicularly onto the condensed phase is investigated exclusively. The case with subsonic condensation is discussed in the present paper (the case with supersonic condensation is left to the subsequent paper). © 2003 American Institute of Physics. [DOI: 10.1063/1.1539476]

I. INTRODUCTION

The half-space problem of strong evaporation and condensation, more specifically, steady flows of a vapor condensing onto or evaporating from a plane condensed phase of the vapor with a high evaporation or condensation rate, has been one of the important subjects in kinetic theory of gases in the following aspects.

(i) In spite of the fact that it appears to be the simplest boundary-value problem of the full Boltzmann equation, the behavior of the solution is not obvious at all. For example, there is a steady solution only when the parameters of the vapor at infinity (the pressure, temperature, and flow velocity of the vapor) and those associated with the condensed phase (the temperature of the condensed phase and the corresponding saturation pressure of the vapor) satisfy certain relations. Furthermore, the relations are qualitatively different depending on whether the vapor is evaporating or condensing, and furthermore, whether it is condensing with a supersonic speed or a subsonic speed. (See Refs. 1 and 2.)

(ii) The half-space problem also plays an important role in the continuum limit (i.e., the limit where the mean free path of the vapor molecules or the Knudsen number of the system goes to zero) for vapor flows around arbitrarily shaped boundaries, consisting of the condensed phase of the vapor, on which strong evaporation or condensation is taking place. To be more specific, the half-space problem corresponds to the Knudsen-layer problem in this case, and the relations among the parameters mentioned above provide the boundary conditions for the fluid-dynamic equations (the Euler set of equations for a perfect gas). (See Refs. 2 and 3.)

The fact mentioned in (i) was clarified and the parameter relations, together with the behavior of the physical quantities, were obtained by means of intensive numerical analysis (Refs. 4 and 5 for evaporation and Refs. 6, 7, and 5 for condensation) based on the Bhatnagar–Gross–Krook (BGK) model8–10 of the Boltzmann equation. Some analytical results based on the Boltzmann equation as well as the BGK model are also available for slow evaporation and condensation11 and for transonic condensation.12 We refer to Ref. 13 as a pioneering work and note that numerical methods other than using the BGK model have also been employed to obtain the aforementioned relations (e.g., Refs. 14–17). In addition, it should be mentioned that the entropy inequality (or the H-theorem) was used recently to estimate the relations rigorously.18,19 The reader is referred to Refs. 1, 2, and 18 for the review on this problem. These numerical and analytical results have induced mathematicians’ interest in the rigorous mathematical treatment of the problem. In fact, several successful results on the half-space problem corresponding to the case of evaporation and condensation have been reported so far (e.g., Refs. 20–23).
The numerical analysis of the half-space problem of strong condensation was extended to the case where another component that does not participate in evaporation or condensation (say, noncondensable gas) is present near the condensed phase.\cite{24,25} The effect of the noncondensable gas on the vapor flows, especially on the relation among the parameters allowing a steady solution, is clarified in these references.

The half-space problem of strong condensation in the presence of a noncondensable gas was revisited recently in connection with the continuum limit of the vapor flows around the boundary, consisting of the condensed phase of the vapor, in the case where a small amount of the noncondensable gas is contained in the system. In Ref. 26, a simple one-dimensional problem, i.e., a vapor flow caused by evaporation and condensation between two parallel plane condensed phases was investigated, and it was pointed out that the noncondensable gas with an infinitesimal average concentration has a significant effect on the vapor flow in the continuum limit. The physical reasoning of this seemingly paradoxical effect is as follows. The infinitesimal amount of the noncondensable gas is concentrated in the thickless Knudsen layer on the condensing surface by the vapor flow, so that its local number density on the surface becomes high enough (comparable to that of the vapor) to affect the vapor flow. The method of analysis employed in Ref. 26 is the systematic asymptotic analysis of the Boltzmann equation for small Knudsen numbers developed by Sone (e.g., Refs. 27–31, and 3; see Refs. 2 and 32 for the summary of the asymptotic theory). Recently, the analysis of Ref. 26 was extended to the case of general geometry.\cite{33} The continuum limit in this situation is outlined as follows. The vapor flow is free from the noncondensable gas except in the Knudsen layer on the boundary where condensation is taking place. Therefore, the fluid-dynamic equations and their boundary condition on the boundary where evaporation is taking place are the same as those in the case without the noncondensable gas,\cite{3} more specifically, the equations are the Euler set of equations for a perfect gas. The thickless Knudsen layer on the condensing boundary may contain the noncondensable gas, as mentioned above. Such Knudsen layer is described by the half-space problem under consideration, i.e., that of strong condensation in the presence of the noncondensable gas. Then, the relation among the parameters allowing the steady solution in the half-space problem, together with the continuity equation of the flow of the noncondensable gas along the boundary (inside the thickless Knudsen layer), gives the boundary condition for the Euler set on the condensing boundary. Therefore, the analysis of the half-space problem in Refs. 24 and 25 gives important information also in practical point of view. However, these references deal exclusively with the case where the vapor flow is condensing on the condensed phase perpendicularly. In order to obtain the boundary condition for the Euler set for the general geometry, we need to extend the results of Refs. 24 and 25 to the case where the vapor is condensing onto the condensed phase at incidence.

For this reason, in the present study, we consider a uniform vapor flow in a half-space condensing onto a plane condensed phase at incidence in the presence of a noncondensable gas. We are going to investigate the problem numerically on the basis of kinetic theory, following Ref. 24. In this reference, the case where the molecules of the noncondensable gas are mechanically identical with those of the vapor is considered. In this case, as discussed in Ref. 24, we can successfully decompose the problem into two problems: one is a nonlinear problem for the total mixture and the other is a linear and homogeneous problem for the noncondensable gas. In particular, since the former problem is equivalent to the half-space problem of strong condensation for a pure vapor, we can exploit the rich knowledge and resources accumulated so far. In addition, this decomposition not only reduces the necessary amount of computation dramatically, but also provides a clear understanding of the features of the solution. This situation is the same in the present problem where the vapor is condensing obliquely.

In the present paper, therefore, we consider the same situation in which the molecules of the noncondensable gas are mechanically identical with those of the vapor. Furthermore, as in Refs. 24 and 25, we employ a model Boltzmann equation for a gas mixture proposed by Garzó, Santos, and Brey [the Garzó–Santos–Brey (GSB) model]\cite{34} rather than the original Boltzmann equation, in the actual numerical computation. In this paper (I), we consider the case in which the speed of condensation is subsonic, more precisely, the case where the Mach number based on the component of the flow velocity of the vapor at infinity normal to the boundary is less than unity. The case in which the speed of condensation is supersonic will be considered in the forthcoming paper.

II. FORMULATION OF THE PROBLEM

A. Problem

Consider a vapor in a half space \( X_1 > 0 \) bounded by a stationary plane condensed phase of the vapor located at \( X_1 = 0 \), where \( X_1 \) is a rectangular coordinate system. There is a uniform vapor flow at infinity toward the condensed phase with velocity \( (v_{x1}, v_{x2}, 0) \) \( (v_{x1} < 0, v_{x2} = 0) \), temperature \( T_\infty \), and pressure \( p_\infty \) (or molecular number density \( n_\infty = p_\infty / kT_\infty \), where \( k \) is the Boltzmann constant). The condensed phase is kept at a constant and uniform temperature \( T_w \). Steady condensation of the vapor is taking place on the condensed phase, and another gas neither condensing nor evaporating on the condensed phase, which we call the noncondensable gas, is confined near the condensed phase by the condensing vapor flow. (See Fig. 1.) We investigate the steady behavior of the vapor and the noncondensable gas on the basis of kinetic theory, under the following assumptions: (i) the behavior of the vapor and the noncondensable gas is described by the Boltzmann equation for a binary mixture (the GSB model\cite{34} will be employed for numerical computation); (ii) the vapor molecules leaving the condensed phase are distributed according to (the part corresponding to the leaving molecules of) the Maxwellian distribution describing the saturated equilibrium state at rest at temperature \( T_w \); (iii) the noncondensable-gas molecules leaving the condensed phase are distributed according to (the part corresponding to
the leaving molecules of) the Maxwellian distribution with
temperature $T_w$ and flow velocity 0, and there is no net particle
flow across the condensed phase (diffuse reflection); (iv) the molecules of the noncondensable gas are mechan-
ically identical with those of the vapor (this assumption,
which will be introduced in Sec. III, is not used in the for-
mulation of the problem in Sec. II).

For later use, we introduce the following notation: $n_w$ is
the saturated pressure of the vapor at temperature $T_w$, and
$n_{\infty}$ is the corresponding molecular number density ($n_{\infty}
= p_w/kT_w$). In the following, we assign the label A to the
vapor (it will also be called A component) and B to the
noncondensable gas (it will also be called B component).

**B. Basic equation**

We first introduce the basic notations: $\xi_i$ is the molecular
velocity, $F^a$ the velocity distribution function of the $a$
component ($a=A$ corresponds to the vapor and $a=B$
to the noncondensable gas); $n^a$ is the molecular number density, $\rho^a$
the mass density, $T^a$ the temperature, $p^a$ the pressure, and
$\mathbf{v}^a=(v^a_1, v^a_2, 0)$ the flow velocity of the $a$
component; $n$ is the molecular number density, $\rho$, the mass density, $T$
the temperature, $p$ the pressure, and $\mathbf{v}=(v_1, v_2, 0)$
the flow velocity of the total mixture; $m^a$ is the mass of a molecule of the
$\alpha$ component. Then we introduce the dimensionless variables
$(x_i, \xi_i, \mathbf{F}^a, \mathbf{n}^a, \mathbf{\rho}^a, \mathbf{\hat{n}}^a, \mathbf{\hat{\rho}}^a, \mathbf{\hat{v}}^a, \mathbf{\hat{n}}, \mathbf{\hat{\rho}}, \mathbf{\hat{\mathbf{v}}},)$
corresponding to $(X_i, \xi_i, F^a, n^a, \rho^a, T^a, p^a, v^a_1, n, \rho, T, p, v_i)$
by the following relations:

\begin{align}
  x_i &= \frac{X_i}{(\sqrt{\pi/2}) l_\infty}, \quad \xi_i = \frac{\xi_i}{(2kT_\infty m^a)^{1/2}}, \\
  \mathbf{F}^a &= \frac{(2kT_\infty m^a)^{1/2}}{n_{\infty}} F^a, \\
  \mathbf{n}^a &= \frac{n^a}{n_{\infty}}, \quad \mathbf{\rho}^a = \frac{\rho^a}{m^a n_{\infty}}, \quad \mathbf{\hat{n}}^a = \frac{T^a}{T_{\infty}}, \\
  \mathbf{\hat{\rho}}^a &= \frac{\rho^a}{p_{\infty}}, \quad \mathbf{\hat{v}}^a = \frac{v^a_1}{(2kT_{\infty} m^a)^{1/2}}, \\
  \mathbf{\hat{n}} &= \frac{n}{n_{\infty}}, \quad \mathbf{\hat{\rho}} = \frac{\rho}{m^a n_{\infty}}, \quad \mathbf{\hat{T}} = \frac{T}{T_{\infty}}, \\
  \mathbf{\hat{\mathbf{v}}} &= \frac{\mathbf{v}}{(2kT_\infty m^a)^{1/2}},
\end{align}

Here, $l_\infty$ is the mean free path of the vapor molecules in the
equilibrium state at rest with temperature $T_\infty$ and pressure $p_{\infty}$
(see Appendix A); for example, $l_\infty= [2(\pi d^A)^{5/2} n_{\infty}]^{-1}$ for
hard-sphere molecules, where $d^A$ is the diameter of a vapor
molecule [Eq. (A9b)], and $l_\infty= (2\sqrt{\pi})(2kT_\infty/m^A)^{1/2}K^{AB} n_{\infty}$
for the GSB model, where $K^{AB}$ is a constant (see Appen-
dix B).

Then, the Boltzmann equation for a binary mixture in the
present steady and spatially one-dimensional problem is
written as

\begin{equation}
  \xi_i \frac{\partial \mathbf{F}^a}{\partial x_1} = \sum_{\alpha=A,B} \mathcal{J}_{\alpha} (\mathbf{F}^B, \mathbf{F}^\alpha) \quad (\alpha=A,B),
\end{equation}

where $\mathcal{J}_{\alpha} (\mathbf{F}^B, \mathbf{F}^\alpha)$ is the dimensionless form of the collision term that expresses the effect of molecular collisions be-
tween molecules of the $\alpha$ and $\beta$ components on the change of $F^\alpha$. Its explicit form is given in Appendix A [Eq. (A1)]

The boundary condition on the condensed phase is given by

\begin{equation}
  \mathbf{F}^A = \pi^{-3/2} n_{\infty} T_\infty^{3/2} \exp \left( -\frac{T_\infty \xi_i^2}{T_{\infty}} \right),
\end{equation}

\begin{equation}
  \mathbf{F}^B = \pi^{-3/2} n_{\infty} T_\infty^{3/2} \exp \left( -\frac{m^B T_\infty \xi_i^2}{m^A T_{\infty}} \right),
\end{equation}

for $\xi_i > 0$, at $x_1 = 0$,

where

\begin{equation}
  \sigma^B_{\infty} = -2 \sqrt{\pi} \left( \frac{m^B}{m^A} \right)^{1/2} \frac{T_\infty^{1/2}}{T_{\infty}^{1/2}} \int_{\xi_i > 0} \xi_i \mathbf{F}^B d^3 \xi,
\end{equation}

with $d^3 \xi = dx_1 dx_2 dx_3$. The condition at infinity is

\begin{equation}
  \mathbf{F}^A \to \pi^{-3/2} \exp \left( -\frac{\xi_i}{\sqrt{2kT_\infty m^A}} \right),
\end{equation}

\begin{equation}
  \mathbf{F}^B \to 0,
\end{equation}

as $x_1 \to -\infty$.

where $v_{\infty1} = (v_{\infty1}, v_{\infty2}, 0)$. For convenience of the later use, we introduce the normal and tangential Mach numbers $M_{n_{\infty}} (>0)$ and $M_{t_{\infty}} (\geq 0)$ at infinity,

\begin{equation}
  M_{n_{\infty}} = \frac{-v_{\infty1}}{\sqrt{5kT_\infty/3m^A}} \quad M_{t_{\infty}} = \frac{v_{\infty2}}{\sqrt{5kT_\infty/3m^A}}.
\end{equation}

Then, the dimensionless velocity $v_{\infty1}/\sqrt{2kT_\infty m^A}$ in Eq. (5a)
is written as

\begin{equation}
  \frac{v_{\infty1}}{\sqrt{2kT_\infty m^A}} = \sqrt{\frac{5}{6}} (-M_{n_{\infty}}, M_{t_{\infty}}, 0).
\end{equation}

Here, we have given the basic equations and boundary con-
ditions in a dimensionless form. The corresponding dimen-
sional form is readily obtained by the use of Eqs. (1a)–(1f) and
the relations relevant to the collision integrals suma-
rized in Appendix A.
As is seen from Eqs. (2)–(7), the parameters imposed externally, that is, $T_w$, $n_w$, $T_\infty$, $n_\infty$, $v_{1\infty}$, and $v_{2\infty}$, appear as the following set:

$$
M_{n,\infty} = \frac{T_w}{T_\infty}, \quad \frac{n_\infty}{n_w} \left( \text{or} \frac{p_\infty}{p_w} \right),
$$

in the nondimensionalized boundary-value problem [note that Eq. (2) does not contain these parameters]. On the other hand, it is physically obvious that we need to specify a parameter related to the amount of the noncondensable gas to single out a solution. In Ref. 24, the following $\Gamma$ is used as this parameter:

$$
\Gamma = \frac{2}{\sqrt{\pi}} \frac{1}{n_{\infty} \lambda_{\infty}} \int_0^\infty n_B dx_1 = \int_0^\infty n_B dx_1.
$$

It corresponds to the total number of molecules of the noncondensable gas per unit area of the condensed phase.

### C. Macroscopic quantities

The macroscopic quantities $\hat{n}^A$, $\hat{\rho}^A$, $\hat{\phi}^A$, $\hat{T}^A$, $\hat{n}$, $\hat{\rho}$, $\hat{\phi}$, and $\hat{T}$ are defined as follows:

$$
\hat{n}^A = \int \hat{F}^A d\xi, \quad \hat{\rho}^A = m^A \hat{n}^A,
$$

$$
\hat{\phi}^A = \frac{1}{\hat{\rho}^A} \int \hat{\rho}^A \hat{n}^A d\xi,
$$

$$
\hat{\rho}^A = \frac{2}{3} \hat{n}^A \int (\xi - \hat{\phi}^A)^2 \hat{F}^A d\xi,
$$

$$
\hat{n} = \int \sum_{\beta=A,B} \hat{F}^\beta d\xi, \quad \hat{\rho} = \int \sum_{\beta=A,B} \hat{\rho}^\beta \hat{F}^\beta d\xi,
$$

$$
\hat{\phi} = \frac{2}{3} \int (\xi - \hat{\phi}^A)^2 \sum_{\beta=A,B} \hat{\rho}^\beta \hat{F}^\beta d\xi,
$$

where $\hat{n}^A = 1$ and $\hat{\rho}^A = m^B/m^A$. The domain of integration of the integrals with respect to $\xi_1$ in Eqs. (10a)–(10f) is the whole space of $\xi_1$. The same rule applies to all the integrals with respect to $\xi_1$ in this paper unless the contrary is stated. The macroscopic quantities for the total mixture are expressed in terms of those for individual components as

$$
\hat{n} = \sum_{\beta=A,B} \hat{n}^\beta, \quad \hat{\rho} = \sum_{\beta=A,B} \hat{\rho}^\beta, \quad \hat{\phi} = \sum_{\beta=A,B} \hat{\phi}^\beta,
$$

$$
\hat{\phi} = \frac{2}{3} \hat{\rho} (\hat{\phi} - \hat{\phi}^A)^2.
$$

It should be noted that in the literature, the pressure $\hat{\rho}^A$ and temperature $\hat{T}^A$ of each component are often defined in a different way, i.e., by Eq. (10c) with $\hat{\phi}^A$ replaced by $\hat{\phi}$. Then, the pressure $\hat{\rho}$ of the total mixture becomes the simple sum of $\hat{\rho}^A$ and $\hat{\rho}^B$ rather than Eq. (11b).

The integration of Eq. (2) over the whole $\xi$ space leads to $\hat{n}_1^A A_1 = \text{const}$ because the right-hand side vanishes in the integration. For the noncondensable gas, $\hat{n}_1^B A_1 = 0$ holds on the condensed phase because of the diffuse reflection (3b) and (4) or at infinity because of Eq. (5b). Therefore, $\hat{n}_1^B A_1 = 0$ or $\hat{\phi}_1^A = 0$ holds identically for $x_1 = 0$.

### III. MECHANICALLY IDENTICAL MOLECULES

Following Ref. 24, we now introduce the assumption that the molecules of the vapor and those of the noncondensable gas are mechanically identical. Then, we have

$$
m^A = m^B \quad (\text{or} \quad \hat{m}^A = 1),
$$

$$
\hat{j}^A(\hat{F}^A, \hat{F}^A) = \hat{j}(\hat{F}^A, \hat{F}^A),
$$

where $\alpha, \beta = A, B$, and $\hat{j}$ is given in Eq. (A10). The discussion in this section is essentially the same as that given in Ref. 24. The only difference is that the case $v_{2\infty} = 0$ (or $M_{1\infty} = 0$) is considered there. But, we will repeat it for the reader’s convenience. The words such as the unique solution will be used in the physical sense, not in the rigorous mathematical sense.

### A. Preliminary transformation

Let $\hat{F}$ be the (dimensionless) velocity distribution function of the total mixture defined by

$$
\hat{F} = \hat{F}^A + \hat{F}^B.
$$

Then, we can transform the boundary-value problem (2)–(5) for $(\hat{F}^A, \hat{F}^B)$ to the problem for $(\hat{F}, \hat{F}^B)$, which is summarized as follows: the equations are

$$
\xi_1 \frac{\partial \hat{F}}{\partial x_1} = \hat{j}(\hat{F}, \hat{F}^B),
$$

$$
\xi_1 \frac{\partial \hat{F}^B}{\partial x_1} = \hat{J}(\hat{F}, \hat{F}^B),
$$

the boundary conditions on the condensed phase are

$$
\hat{F} = \frac{n_0}{n_\infty} \frac{T_\infty}{T_w} \left( \text{or} \frac{T_\infty}{T_w} \right)^{3/2} \left( \frac{T_\infty}{T_w} \right)^{3/2},
$$

$$
\hat{F}^B = \frac{n_0}{n_\infty} \frac{T_\infty}{T_w} \left( \text{or} \frac{T_\infty}{T_w} \right)^{3/2} \left( \frac{T_\infty}{T_w} \right)^{3/2},
$$

for $\xi_1 > 0$, at $x_1 = 0$,

with

$$n_0 = n_w + \sigma_w^B,
$$

$$\sigma_w^B = -\frac{n_\infty}{n_\infty} \frac{T_\infty}{T_w} \int_{\xi_1 < 0} \xi_1 \hat{F}^B d\xi,
$$

and the conditions at infinity are

$$\hat{F} \to \pi^{-3/2} \exp \left( -\left( \xi_1 - \frac{v_{1\infty}}{\sqrt{2kT_w/m^A}} \right)^2 \right),
$$

(17a)
\[
\hat{F}^B = 0, \\
\text{as } x_1 \to \infty.
\]  

Here, Eq. (14a), which is the Boltzmann equation for a single-component gas, is obtained by adding Eq. (2) with \( \alpha = A \) and \( B \) and by taking into account the bilinearity of \( \hat{J} \). Equation (14b) is Eq. (2) (with \( \alpha = B \)) with \( \hat{F}^B \) being eliminated by the use of Eq. (13). Equations (15a) and (17a) are, respectively, the sum of Eqs. (3a) and (3b) and that of Eqs. (5a) and (5b).

**B. Half-space problem for a pure vapor**

Now, let us suppose that \( n_0 \) (or \( n_0/n_w \)) in Eq. (15a) is a given parameter. Then, Eqs. (14a), (15a), and (17a) form a boundary-value problem equivalent to the half-space condensation problem for a pure vapor, namely, Eqs. (2)–(5b) with \( \hat{F}^B = 0 \), which has been studied comprehensively in Refs. 1, 5–7, 11, and 12; \( \hat{F} \) and \( n_0 \) correspond, respectively, to the (dimensionless) velocity distribution function of the vapor and to the saturation number density of the vapor at temperature \( T_w \). Then, this problem is characterized by the following set of parameters:

\[
M_{n_w}, \quad M_{p_w}, \quad \frac{T_w}{T_w}, \quad \frac{n_w}{n_0} \left( \text{or } \frac{p_w}{p_0} \right),
\]

where \( p_0 = k n_0 T_w \), which corresponds to the saturation vapor pressure at temperature \( T_w \).

According to the references quoted above, there is a solution only when these parameters satisfy the following relations:

\[
\frac{p_w}{p_0} \geq F_A \left( M_{n_w}, M_{p_w}, \frac{T_w}{T_w} \right) \quad (M_{n_w} < 1),
\]

\[
\frac{p_w}{p_0} \geq F_A \left( 1, \frac{M_{n_w}}{M_{p_w}}, \frac{T_w}{T_w} \right) \quad (M_{n_w} = 1),
\]

\[
\frac{p_w}{p_0} \geq F_B \left( M_{n_w}, M_{p_w}, \frac{T_w}{T_w} \right) \quad (M_{n_w} > 1).
\]

The functions \( F_A \) and \( F_B \) have been constructed numerically in Refs. 7 (for \( M_{p_w} = 0 \)) and 5, using the BGK model. According to these results, \( F_A \) and \( F_B \) have the following properties: (i) Both functions are weakly dependent on \( M_{p_w} \) and \( T_w/T_w \). (ii) For any fixed \( M_{p_w} \) and \( T_w/T_w \), \( F_A \) is a monotonically increasing function in \( M_{n_w} \), whereas \( F_B \) is a monotonically decreasing function in \( M_{n_w} \). (iii) \( F_A(0, 0.1) = F_A(0, 0.1, T_w/T_w) = 1 \) and \( F_B(0, 0.1, T_w/T_w) = F_B(0, 0, 0.1, T_w/T_w) \) (see, e.g., Ref. 1).

Equations (19a)–(19c) indicate that in order to obtain a unique solution to Eqs. (14a), (15a), and (17a), one needs to specify three parameters, say \( M_{n_w}, M_{p_w}, \) and \( T_w/T_w \), out of the four parameters in Eq. (18) when \( M_{n_w} < 1 \) (subsonic condensation) and all the four parameters satisfying the inequality (19b) or (19c) when \( M_{n_w} \equiv 1 \) (supersonic condensation).

It should be mentioned that \( F_A \) (with \( M_{p_w} = 0 \)) has also been computed for hard-sphere molecules by means of the direct simulation Monte Carlo (DSMC) method\(^{35,36}\) by Sone and Sasaki.\(^{37}\) The result is quite close to that for the BGK model.

**C. Half-space problem in the presence of a noncondensable gas**

Let us suppose that we have obtained the solution \( \hat{F}_G \) corresponding to a given value of \( n_0 \) (or the parameter \( n_0/n_w \)). Then, Eqs. (14b), (15b), (16b), and (17b) reduce to a linear and homogeneous boundary-value problem for \( \hat{F}_B \). Therefore, a solution multiplied by an arbitrary constant is also a solution. The unique solution is determined by specifying \( \Gamma \) [Eq. (9)]. Let us denote by \( \hat{F}_G^B \) and \( \Gamma \) the solution and the value of \( \Gamma \) corresponding to the case \( \sigma_w^B = n_0 \) in Eq. (15b). Since \( \Gamma \) is linear in \( \hat{F}_G^B \) [see Eqs. (9) and (10a)], the solution for an arbitrary \( \Gamma \) is expressed in terms of \( \hat{F}_G^B \) and \( \Gamma \) as

\[
\hat{F}_G^B = (\Gamma/\Gamma \ast) \hat{F}_G^B.
\]

Then, Eq. (16b) yields

\[
\sigma_w^B = (\Gamma/\Gamma \ast) n_0,
\]

which together with Eq. (16a) gives the relationship between \( n_w \) and \( n_0 \) or that between \( p_w \) and \( p_0 \):

\[
n_w = (1 - \Gamma/\Gamma \ast) n_0,
\]

\[
p_w = (1 - \Gamma/\Gamma \ast) p_0.
\]

To summarize, we first obtain a solution \( \hat{F}_G \) to the problem (14a), (15a), and (17a) that corresponds to a given \( n_0 \) (or \( n_0/n_w \)). Then we solve the problem (14b), (15b), and (17b) for \( \sigma_w^B = n_0 \) (or \( \sigma_w^B/n_w = n_0/n_w \)) to obtain \( \hat{F}_G^B \), from which we compute \( \Gamma \ast \) using Eqs. (9) and (10a). Then, for a given \( \Gamma \), we obtain \( \hat{F}_G^B \) from Eq. (20) and \( \hat{F}_G^A \) from Eq. (13). The \( \hat{F}_G^A \) and \( \hat{F}_G^B \) thus obtained solve the original problem (2)–(5b) with Eqs. (12a) and (12b) for the \( \Gamma \) and the saturation number density \( n_w \) given by Eq. (22a). Since \( n_w \) is not negative physically, Eq. (22a) yields

\[
0 \leq \Gamma \leq \Gamma \ast
\]

That is, \( \Gamma \ast \) is the maximum \( \Gamma \) for a given \( \hat{F}_G \).

One might think that the above scheme for the solution is practically inconvenient because specifying the artificial parameter \( n_w/n_0 \) rather than the physically inherent parameter \( n_w/n_w \) of the system seems to be crucial. In the case of subsonic condensation, however, the inconvenience does not arise because the solution \( \hat{F}_G \) and \( \hat{F}_G^B \) can be determined by specifying \( M_{n_w}, M_{p_w}, T_w/T_w \), and \( \Gamma \) only, that is, we do not need to specify \( n_w/n_0 \) or \( n_w/n_w \) (see Sec. III D). On the other hand, the scheme gives a clear understanding of the relationship among the parameters which admits a solution even in the case of supersonic condensation (see Ref. 24).

**D. Existence range of a solution: Subsonic condensation**

Now we return to the original problem. In the present paper, we restrict ourselves to the case when \( M_{n_w} < 1 \).
IV. NUMERICAL ANALYSIS AND RESULTS

In this section, we carry out actual numerical analysis to obtain \( \hat{F}_B \) and \( \Gamma \), which gives the solution \( \hat{F}^A \) and \( \hat{F}^B \) of the original problem. Since the numerical technique is essentially the same as that used in the case of a single-component system in Refs. 7 and 5, where the detailed description of the method is given, we omit it here giving some remarks in Sec. IV A and concentrate on the results of analysis. Information about the accuracy of the computation is given in Appendix C.

A. Some remarks on numerical analysis

As in Refs. 24 and 25, we employ the GSB model for Eq. (2). The model collision term is given in Appendix B. The original Boltzmann equation has the property that, when the molecules of the A component are mechanically identical with those of the B component, the equation for \( \hat{F} = \hat{F}^A + \hat{F}^B \) coincides with the Boltzmann equation (14a) for a single-component gas, i.e., the equation in the case where \( \hat{F}^A = \hat{F} = \hat{F}^B = 0 \). Although several model Boltzmann equations have been proposed for gas mixtures \(^{34,38-40} \) only the GSB model (and the model in Ref. 40 under some restrictions) satisfies this property. Since it plays an essential role in the present approach, we adopt the GSB model. In addition, Eq. (14a) for this model reduces to the BGK model (Appendix B). Therefore, for the solution \( \hat{F} \) to Eqs. (14a), (15a), and (17a) and the function \( F_s \) in Eq. (25), we can use the detailed data given in Refs. 7 and 5. We just need to solve the linear system (14b) [with Eq. (B6)], (15b) [with \( \sigma_n^2 = n_0 \)], and (17b) numerically to obtain \( \hat{F}_B \) and the corresponding \( \Gamma \).

We solve this problem by means of a finite-difference method. In both systems, Eqs. (14a), (15a), and (17a) and Eqs. (14b), (15b), and (17b), the model collision terms allow us to eliminate the molecular-velocity variables \( \xi_2 \) and \( \xi_3 \) from the systems, as in the case of the BGK model for a single-component gas.\(^{41} \) That is, by multiplying the equations and boundary conditions by 1, \( \xi_2 \), and \( \xi_2^2 + \xi_3^2 \) and by integrating the respective results over the whole range of \( \xi_2 \) and \( \xi_3 \), we obtain the equations and boundary conditions of the respective sets (\( H_a, H_b, H_c \)) and (\( H^B_a, H^B_b, H^B_c \)) of marginal velocity distribution functions defined by

\[
\begin{align}
(H_a, H_b, H_c) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (1, \xi_2, \xi_2^2 + \xi_3^2) \hat{F} \, d\xi_2 \, d\xi_3, \\
(H^B_a, H^B_b, H^B_c) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (1, \xi_2, \xi_2^2 + \xi_3^2) \hat{F}_B \, d\xi_2 \, d\xi_3.
\end{align}
\]

The solution method for the system for (\( H_a, H_b, H_c \)) is described in detail in Ref. 5, and that for the system (\( H^B_a, H^B_b, H^B_c \)) is essentially the same as the former. Therefore, we avoid the description of the methods for conciseness.

B. Existence range of a solution

In this section, we show some of the numerical results for the existence range of a solution already discussed in Sec. III D, namely the numerical data for the function \( F_s(M_{\infty}, M_{\infty}, T_{\infty}/T_w, \Gamma) \) in Eq. (24). For this purpose, we need to present the data for \( F_s \) and \( \Gamma \).

The data for \( F_s \) have already been given in Ref. 5. That is, Fig. 3 and Tables I–IV in Ref. 5 show the numerical data of \( F_s(M_{\infty}, M_{\infty}, T_{\infty}/T_w) \) for \( T_{\infty}/T_w = 0.5, 1, 1.5, \) and 2. In the present study, we recomputed all the cases in Ref. 5 and confirmed the accuracy of the data given there. To be more precise, in Tables II–IV of Ref. 5, the last figure should be changed by one in several data, and in Table I there, the last figure should be changed by one in about 20 data for \( M_{\infty} = 0.9 \) and by two to seven in the data for \( M_{\infty} = 0.99 \). We also made additional computations to supplement these data. Some of the results are shown in Table I, the more comprehensive data being given in Tables I–IV in Ref. 42. The numerical results for \( \Gamma(M_{\infty}, M_{\infty}, T_{\infty}/T_w) \) obtained in
the present study are shown in Table II, the more detailed tables being given as Tables V–VIII in Ref. 42. With these data, we can construct the function \( F_s \). The result for \( T_w/T_w = 1 \) is shown in Fig. 2, where \( F_s \) versus \( M_{n*} \) is shown for various \( \Gamma \) at four values of \( M_{n*} \), i.e., \( M_{n*} = 0, 1, 2, \) and 3. The similar figures for \( T_w/T_w = 0.5, 1.5, \) and 2 are given as Figs. 3, 1, and 4 in Ref. 42 (Fig. 2 in Ref. 42 is the same as Fig. 2 here). The \( F_s \) is an increasing function of \( M_{n*} \), and its curve moves upward with the increase of \( \Gamma \). The \( \Gamma \) in the figures is a critical value of \( \Gamma \), depending on \( M_{n*} \) and \( T_w/T_w \), introduced by Eq. (27) below. When \( \Gamma < \Gamma_c \), \( F_s \) takes a finite value at \( M_{n*} = 1 \). But, when \( \Gamma = \Gamma_c \), \( F_s \) becomes infinitely large as \( M_{n*} \) approaches a critical value \( M_c (\approx 1) \) that depends on \( M_{n*} \), \( T_w/T_w \), and \( \Gamma (\Gamma_c = 1 \text{ when } \Gamma = \Gamma_c) \). That is, \( M_{n*} = M_c \) is the asymptote of the curve. Therefore, when \( \Gamma > \Gamma_c \), there is no solution in the interval \( M_{n*} < M_c < \infty \). More detailed information about \( \Gamma_c \) and \( M_c \) will be given below. Because the dependence of \( F_s \) and \( \Gamma_c \) on \( M_{n*} \) is not strong, the function \( F_c \) does not depend much on \( M_{n*} \). Therefore, the features of \( F_s \) are essentially the same as those described in Refs. 24 and 25 for the case of \( M_{n*} = 0 \). In particular, for \( M_{n*} \) smaller than around 1 and \( M_{n*} \) smaller than around 0.5, \( F_s \) is almost independent of \( M_{n*} \). The dependence of \( F_s \) on \( T_w/T_w \) is also weak in general (see Figs. 1–4 in Ref. 42). The \( F_s \) is an increasing function of \( M_{n*} \), whereas \( \Gamma_c \) is its decreasing function. Therefore, as is seen from Eq. (25), \( F_c \) is an increasing function of \( M_{n*} \) (see Fig. 2). Numerical results show that \( F_s \to 1 \) as \( M_{n*} \to 0 \), and \( F_c \) and \( \Gamma_c \) approach finite values as \( M_{n*} \to 1 \). The values of \( F_s \) and \( \Gamma_c \) at \( M_{n*} = 1 \) in Tables I and II (and in Tables I–IV and V–VIII in Ref. 42) are the values obtained by extrapolation, in which many additional data that are not included in the tables have also been used. Let us set

\[
\Gamma_c (M_{n*}, T_w, T_w) = \lim_{M_{n*} \to 1} (M_{n*}, T_{n*}, T_w)
\]

The properties of \( F_s \) described in the preceding paragraph follow immediately from Eqs. (25) and (27) and the fact that \( \Gamma_c \) is a decreasing function of \( M_{n*} \). That is, when \( \Gamma < \Gamma_c \), the \( \Gamma / \Gamma_c \) in Eq. (25) is less than unity and thus \( F_s \) remains finite in the whole range of \( 0 < M_{n*} < 1 \). When \( \Gamma > \Gamma_c \) (or \( \Gamma = \Gamma_c \)), the \( \Gamma / \Gamma_c \) in Eq. (25) becomes unity at an \( M_{n*} < 1 \) (or \( M_{n*} = 1 \)). We denote this value of \( M_{n*} \) by \( M_c (M_c = 1 \text{ for } \Gamma = \Gamma_c) \). Then, \( F_s \) increases indefinitely as \( M_{n*} \) approaches \( M_c \). The \( \Gamma_c (M_{n*}, T_w, T_w) \) versus \( M_{n*} \) is shown in Fig. 3, where \( \Gamma_c \) versus \( M_{n*} \) is plotted for \( T_w/T_w = 0.5, 1, 1.5, \) and 2. The \( M_c (M_{n*}, T_w, T_w), \) which is the solution of

\[
\Gamma_c (M_{n*}, M_{n*}, T_w, T_w) - \Gamma_c = 0,
\]

is shown in Fig. 4, where \( M_c \) versus \( M_{n*} \) at \( T_w/T_w = 0.5, 1, \)
where the molecules of the two components are not identical mechanically where the molecules of the two components are not identical mechanically. In the present analysis, we have taken full advantage of the GSB model to obtain the numerical results such as shown in Fig. 2. A preliminary numerical analysis of the present problem using the DSMC method has also been carried out for hard-sphere molecules in a more general case where the molecules of the two components are not identical mechanically [Aoki, Takata, and Fujimoto (unpublished)]. The result suggests that the relation of the form (24) with $\mathcal{F}_s$ depending on $m^B/m^A$ and $d^B/d^A$ holds in this case ($d^A$ is the diameter of the molecules of the $A$ component). As an example, $\mathcal{F}_s$ for $m^B/m^A=2$ and $d^B/d^A=1$ is shown in Fig. 5 in the case $M_{nB}=0$ and $T_s/T_w=1$. The qualitative feature of the figure is the same as that of Fig. 2.

### C. Macroscopic quantities

In this section, we give some results for the macroscopic quantities. Figures 6–8 show the profiles of the macroscopic quantities for $T_s/T_w=1$ and for three different values of $M_{nB}$, i.e., Fig. 6 for $M_{nB}=0.1$, Fig. 7 for $M_{nB}=0.5$, and Fig. 8 for $M_{nB}=0.9$. In each figure, the result for $M_{nB}=1$ is shown in (a) and that for $M_{nB}=3$ in (b). In these figures, the dimensional quantities listed in the beginning of Sec. II B are used rather than their dimensionless counterparts, and the notation $a_n = (5kT_s/3m^A)^{1/2}$ has been introduced. It is seen from Eqs. (10a)–(10c) with $\alpha = B$ and with $\beta^B = 0$ (see the last part of Sec. II C) that $\hat{n}^B$, $\hat{v}^B$, and $\hat{p}^B = \hat{p}^B T^B$ are linear in $\hat{F}^B$. Therefore, because of the form (20), $(\Gamma_{nB}/\Gamma)\hat{n}^B$, $\hat{v}^B$, $\hat{T}^B$, and $(\Gamma_{nB}/\Gamma)\hat{p}^B$, i.e., $(\Gamma_{nB}/\Gamma)(n^B/n_{nB})$, $\nu^B/(2kT_s/m^A)^{1/2}$, $T^B/T_w$, and $(\Gamma_{nB}/\Gamma)(p^B/p_{nB})$ are independent of $\Gamma$. It should be noted that $n$, $v_1$, $v_2$, $T$, and $p$ of the total mixture are the same as $n^A$, $v_1^A$, $v_2^A$, $T^A$, and $p^A$ for $\Gamma = 0$, respectively.

The noncondensable gas extends far away when $M_{nB}$ is small (over 50$\ell_{\infty}$ when $M_{nB}=0.1$; see Fig. 6) but is confined in a narrower region when $M_{nB}$ is large. For large $M_{nB}$, the temperature near the condensed phase increases because of the strong friction. But the temperature rise is smaller for larger $M_{nB}$ because the heated gas near the condensed phase is removed by the strong condensation. These features are the same as those discussed in Refs. 5 (for $\Gamma = 0$) and 24 (for $M_{nB} = 0$). The acceleration of the vapor toward the condensed phase is larger for larger $\Gamma$ because $p_{nB}/p_{n\infty}$ is smaller, i.e., the suction effect on the condensed phase is stronger. As discussed in Ref. 24, because of $v_1^B = 0$, $F^B$ does not accommodate to $F^A$, which is close to the equilibrium distribution.

### Table II. $\Gamma_{nB}(M_{nB}, M_{nC}, T_s/T_w)$ as a function of $M_{nB}$, $M_{nC}$, and $T_s/T_w$.

<table>
<thead>
<tr>
<th>$M_{nB}$</th>
<th>$M_{nC}$</th>
<th>$T_s/T_w = 0.5$</th>
<th>$T_s/T_w = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.05</td>
<td>0.10</td>
<td>0.15</td>
</tr>
<tr>
<td>0.05</td>
<td>0.10</td>
<td>0.15</td>
<td>0.20</td>
</tr>
<tr>
<td>0.10</td>
<td>0.15</td>
<td>0.20</td>
<td>0.25</td>
</tr>
<tr>
<td>0.20</td>
<td>0.25</td>
<td>0.30</td>
<td>0.35</td>
</tr>
<tr>
<td>0.30</td>
<td>0.35</td>
<td>0.40</td>
<td>0.45</td>
</tr>
</tbody>
</table>

The abscissa for easy comparison with Fig. 2~a is $M_{nB}$ for $M_{nC}$ in Ref. 42.
As is seen from Figs. 6–8, there is a macroscopic motion of the noncondensable gas along the condensed phase (i.e., in the $X_2$ direction) when the vapor flow at infinity has a transversal component (i.e., $v_{\infty 2} > 0$). Let us denote by $N_f^*$ the dimensional total particle flux (per unit width in $X_2$ and per unit time) of the noncondensable gas in the $X_2$ direction and by $N_f$ its dimensionless counterpart defined by

$$N_f^* = (2/\sqrt{\pi})[n_{\infty} l_{\infty} (2kT_{\infty}/m^2)^{1/2}]^{-1} N_f.$$  

Then, $N_f^*$ is expressed as

$$N_f^* = \int_0^\infty \hat{f} \rho B_\perp^2 \, dx_1 = \int_0^\infty (1/2) \hat{f}_B \, d^2 \xi \, dx_1.$$  

If we use Eq. (20) in Eq. (30) and denote by $N_f^*$ the $\hat{N}_f$ corresponding to $\hat{F}_B^*$, then we have

$$\hat{N}_f^* = (\Gamma/\Gamma_*) \hat{N}_f^*.$$  

We recall that $\hat{F}_B^*$ depends on $M_{\infty \gamma}$, $M_{\infty \gamma}$, and $T_{\infty}/T_w$, so that $\hat{N}_f^*$, as well as $\Gamma_*$, is a function of these three parameters. Therefore, setting $G = \hat{N}_f^*/\Gamma_*$, we can write

$$\hat{N}_f^* = \Gamma G (M_{\infty \gamma}, M_{\infty \gamma}, T_{\infty}/T_w).$$  

The relation (32), or more generally, the $\hat{N}_f$ as the function of $M_{\infty \gamma}$, $M_{\infty \gamma}$, and $T_{\infty}/T_w$, and $\Gamma$, is required as a part of the boundary condition for the Euler set in the continuum limit in the situation described in Sec. I and in the end of Sec. III D (see Ref. 33 for the details). Some of the numerical results for $G$ are given in Fig. 9, where $G$ versus $M_{\infty \gamma}$ is plotted for typical $M_{\infty \gamma}$ and $T_{\infty}/T_w$, and in Table III (see Tables IX–XII in Ref. 42 for more detailed data). The values of $G$ at $M_{\infty \gamma} = 1$ in Table III (and in Tables IX–XII in Ref. 42) are those obtained by extrapolation using the data for $M_{\infty \gamma} < 1$ in the tables and many additional data not shown there. As is seen from Fig. 9 and the tables, $G$ is almost linear in $M_{\infty \gamma}$ and weakly dependent on $M_{\infty \gamma}$.
V. COMMENT ON THE CASE OF EVAPORATION

In this paper, we have exclusively considered the case of condensation, $v_{\infty} > 0$. Here, we give a short comment on the case of evaporation, i.e., the boundary-value problem $\sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \si
where $\kappa^{AB}$ is a positive constant depending on the constant in the intermolecular force law and on $\mu^{AB}$ defined in Eq. (A2b). Since $\phi_1^A > 0$ and $\bar{n}^B \phi_1^B = 0$ (see the last part of Sec. II C), the right-hand side of Eq. (33) is strictly positive. Therefore, $\int_1^X \xi F^B d\xi = 0$ is a monotonically increasing function of $x_1$. On the other hand, it should vanish at infinity because of the condition (5b). This is possible only when $\bar{F}^B$ is identically zero.

Most of the model equations for multicomponent mixtures, such as proposed in Refs. 34, 38–40, are designed in such a way that the model collision terms reproduce the momentum and energy transport between different species for Maxwellian molecules. Therefore, they satisfy the relation (34) with an appropriate constant corresponding to $\kappa^{AB}$. Thus, $\bar{F}^B = 0$ is also true for these model equations.

In connection with the above conclusion, it should be
mentioned that unsteady evaporation into a half-space initially filled with a uniform noncondensable gas is investigated numerically using the GSB model in Refs. 43 and 44. It is demonstrated that, if the initial number density of the noncondensable gas is smaller than the saturation number density of the vapor corresponding to the wall temperature, all the noncondensable gas is swept away to infinity by the vapor, and the final steady state is the pure-vapor evaporation, i.e., the solution to Eqs. (2)–(5b) with \( \dot{F}^B = 0 \). It is also demonstrated that, if the initial number density of the noncondensable gas is larger than the saturation number density of the vapor, the evaporation stops finally, and the mixture approaches an equilibrium state at rest in the entire half-space.

VI. CONCLUDING REMARKS

In this paper, we have considered a flow of a vapor condensing onto a plane condensed phase of the vapor at inci-
dence in the case where a noncondensable gas is present near the condensed phase. The present study is a continuation of Ref. 24, where the vapor is assumed to be condensing perpendicularly onto the condensed phase. Such an extension was required in connection with the general theory\textsuperscript{33} to describe the vapor flow around an arbitrarily shaped condensed phase in the continuum limit when a small amount of the noncondensable gas is contained in the system.

The approach to the problem in the present paper is essentially the same as that in Ref. 24. After formulating the problem in Sec. II, we introduced the assumption that the molecules of the vapor and those of the noncondensable gas are mechanically identical in Sec. III. This assumption enables us to decompose the original problem into two problems, one for the total mixture, which is equivalent to the half-space problem of strong condensation for a pure vapor, and the other for the noncondensable gas. Taking advantage of this property, we discussed the general features of the

FIG. 8. Profiles of the macroscopic quantities for $M_{\infty} = 0.9$ and $T_\infty/T_{\infty} = 1$. (a) $M_{\infty} = 1$, (b) $M_{\infty} = 3$. See the caption of Fig. 6.
solution, in particular, the relation among the parameters that admits a steady solution (Sec. III). Then, we carried out actual numerical computations using the GSB model to obtain the numerical solution of the problem, in particular, the numerical data for the relation to be satisfied by the parameters (Sec. IV). In the present paper, we have restricted ourselves to the case of subsonic condensation, leaving the case of supersonic condensation in the subsequent paper. The numerical data of the relation provide the essential part of the numerical boundary condition for the Euler set of equations on the condensing boundary in the continuum limit. Finally, we considered the half-space problem of strong evaporation in Sec. V and showed that the noncondensable gas cannot be present when evaporation is taking place in the case of Maxwellian molecules.

With the present numerical results incorporated as the boundary condition, the Euler system derived in Ref. 33 is now applicable to practical problems. Actually, such an example is already contained in Ref. 33. That is, the Euler system is applied to the analysis of the vapor flow evaporating from a plane condensed phase and condensing onto a wavy condensed phase of sinusoidal shape in the continuum limit, in the presence of a noncondensable gas of an infinitesimal average concentration. The result shows that such a trace of the noncondensable gas has a significant effect on the vapor flow. Further applications of the Euler system will be treated in a forthcoming paper.

**APPENDIX A: COLLISION TERMS**

The dimensionless collision term $\hat{J}^{\beta\alpha}$ is given as follows:

**TABLE III.** $G(M_{n^\infty}, M_{v^\infty}, T_v/T_w)$ as a function of $M_{n^\infty}$, $M_{v^\infty}$, and $T_v/T_w$.

<table>
<thead>
<tr>
<th>$M_{n^\infty}/M_{v^\infty}$</th>
<th>$T_v/T_w = 0.5$</th>
<th>$T_v/T_w = 1$</th>
<th>$T_v/T_w = 1.5$</th>
<th>$T_v/T_w = 2$</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>2</td>
<td>3</td>
</tr>
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</tr>
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</tr>
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\[ J^{\beta \alpha}(f, g) = \int [f(\xi_{\alpha i})g(\xi'_{\alpha i}) - f(\xi'_{\alpha i})g(\xi_{\alpha i})] \]
\[ \times \hat{B}^{\beta \alpha}(|e_j \hat{V}_j|, |\hat{V}_j|)d\Omega(e_j)d^3 \xi_{\alpha i}, \]  
where
\[ \xi'_{\alpha i} = \xi_{\alpha i} + \frac{\hat{\mu}^{\beta \alpha}}{\hat{m}^{\beta \alpha}}(e_j \hat{V}_j)e_i, \quad \xi_{\alpha i} = \xi_{\alpha i} - \frac{\hat{\mu}^{\beta \alpha}}{\hat{m}^{\beta \alpha}}(e_j \hat{V}_j)e_i, \]
(A2a)
\[ \hat{\mu}^{\beta \alpha} = \frac{2\hat{m}^{\alpha \beta} \hat{m}^{\beta \alpha}}{\hat{m}^{\alpha \alpha} + \hat{m}^{\beta \beta}}, \quad \hat{m}^{\beta \alpha} = m^{\beta \alpha}/m^{\alpha \alpha}, \]
(A2b)
\[ \nu_{\beta \alpha} = \frac{d^3 \xi_{\beta \alpha}}{d\xi_{\alpha i}} = \frac{d\xi_{\alpha i}}{d\xi_{\alpha i}}d\xi_{\alpha i}2d\xi_{\alpha \beta}, \]
(A3)
is such that the collision frequency \( \nu^{\beta \alpha} \) of an molecule for the collision with \( \beta \) molecules with the velocity distribution function \( F^{\beta \alpha} \) is expressed as
\[ \nu_{\beta \alpha} = \int F^{\beta \alpha}(|e_j V_j|, |V_j|)d\Omega(e_j)d^3 \xi_{\beta \alpha}, \]
(A4)
where \( d^3 \xi_{\beta \alpha} = d\xi_{\alpha i}d\xi_{\alpha j}d\xi_{\alpha \beta}, \) the domain of integration is the whole space of \( \xi_{\beta \alpha}, \) and all directions of \( e_i, \) and the arguments of \( F^{\beta \alpha} \) other than the molecular velocity are omitted. Let \( \nu_{\beta \alpha} \) be the mean collision frequency (i.e., the inverse of the mean free time) of the vapor molecules in the equilibrium state at rest with number density \( n_{\infty} \) and temperature \( T_{\infty} \), which is related to the mean free path \( l_{\infty} \) by
\[ \nu_{\beta \alpha} = \frac{1}{n_{\infty}} \int \nu^A F^{\beta \alpha}(\xi_{\beta \alpha})d\Omega(e_j)d^3 \xi_{\beta \alpha}, \]
(A6a)
\[ F^{\beta \alpha}(\xi_{\beta \alpha}) = \left( \frac{\pi}{2} \right)^{\frac{1}{2}} \exp \left( -\frac{\xi_{\beta \alpha}^2}{2\pi k T_\infty/m^{\alpha \alpha}} \right), \]
(A6b)
where \( d^3 \xi_{\beta \alpha} = d\xi_{\alpha i}d\xi_{\alpha j}d\xi_{\alpha \beta}, \) and the domain of integration is the whole space of \( \xi_{\beta \alpha}, \) that of \( \xi_{\alpha i}, \) and all directions of \( e_i. \)
The \( \hat{B}^{\beta \alpha} \) in Eq. (A1) has been normalized as
\[ \hat{B}^{\beta \alpha}(|e_j \hat{V}_j|, |\hat{V}_j|) = \left( \frac{\hat{m}^{\beta \alpha}}{\hat{m}^{\alpha \alpha}m^{\beta \beta}} \right)^{\frac{3}{2}} \exp \left( -\frac{\hat{m}^{\alpha \alpha}(\xi_{\beta \alpha} - \hat{\theta})^2}{\hat{T}} \right), \]
(B1a)
\[ \hat{T}^{\beta \alpha} = \frac{\hat{m}^{\beta \beta} \hat{m}^{\beta \alpha}}{\hat{m}^{\alpha \alpha} m^{\beta \beta}} \left( \hat{m}^{\beta \beta} \hat{m}^{\beta \beta} + \hat{m}^{\beta \alpha} \hat{m}^{\beta \alpha} \right) T^{\alpha \beta} + 2T^{\beta \alpha}, \]
(B2c)
\[ \hat{K}^{\beta \alpha} = \frac{\hat{K}^{\beta \alpha}}{K^{\alpha \alpha}}, \]
(B2d)
and \( K^{\beta \alpha} \) are constants. The collision frequency \( \nu^{\beta \alpha} \) of the \( \alpha \) molecules for their collisions with the \( \beta \) molecules is given by \( \nu^{\beta \alpha} = \hat{K}^{\beta \alpha} n^{\beta \alpha} \), and therefore, the mean free path \( l_{\infty} \) is
\[ l_{\infty} = \left( \frac{\pi k T_{\infty}}{m^{\alpha \alpha}} \right)^{\frac{1}{2}} n_{\infty}^{\beta \alpha}, \]
(A9b)
\[ \hat{B}^{\beta \alpha} = \frac{1}{4\sqrt{2} \pi} \left( \frac{d^\alpha + d^\beta}{2d^\lambda} \right) |e_j \hat{V}_j|, \]
(A9c)
given by \( l_c = (2/\sqrt{\pi})(2kT_\text{a}/m^A)^{1/2}/k^{AA}n_\infty \). The \( \tilde{n}^a \), \( \tilde{\eta}^a \), \( \tilde{\nu}^a \), \( \tilde{\tau} \), and \( \tilde{\varphi} \) in Eqs. (B1)–(B2c) are defined by Eqs. (10a)–(10f).

When the molecule of the vapor and that of the noncondensable gas are mechanically identical, we have

\[
\tilde{m}^a = 1, \quad \tilde{K}^{B\alpha} = 1.
\]  
(B3)

In this case, therefore, if we use Eq. (B1) in Eq. (2), then Eq. (14a) becomes the BGK model, that is, \( \tilde{J}(\tilde{F}, \tilde{F}) \) in Eq. (14a) reduces to

\[
\tilde{J}(\tilde{F}, \tilde{F}) = \hat{n}(\tilde{F}_e - \tilde{F}),
\]  
(B4)

where

\[
\tilde{F}_e = \pi^{-3/2} \frac{\hat{n}}{\tilde{T}^{3/2}} \exp\left(-\frac{(\zeta_i - \tilde{\varphi})^2}{\tilde{T}}\right),
\]  
(B5a)

\[
\hat{n} = \int \tilde{F} d^3 \zeta, \quad \tilde{\varphi}_i = \frac{1}{\hat{n}} \int \zeta_i \tilde{F} d^3 \zeta,
\]  
(B5b)

\[
\tilde{T} = \frac{2}{3\hat{n}} \int (\zeta_i - \tilde{\varphi}_i)^2 \tilde{F} d^3 \zeta.
\]  
(B5c)

Correspondingly, \( \tilde{J}(\tilde{F}, \tilde{F}^B) \) in Eq. (14b) reduces to

\[
\tilde{J}(\tilde{F}, \tilde{F}^B) = \hat{n}(\tilde{\Psi}_e - \tilde{F}^B),
\]  
(B6)

where

\[
\tilde{\Psi}_e = \frac{\hat{n}}{\tilde{n}} \tilde{F} \left(1 - \frac{(\tilde{\varphi}_i - \tilde{\varphi}^B_i)(\zeta_i - \tilde{\varphi}_i)}{\tilde{T}} - \frac{1}{2} \left(1 - \frac{2(\tilde{\varphi}_i - \tilde{\varphi}^B_i)^2}{3\tilde{T}} \right) \right) - \frac{\tilde{F}^B}{\tilde{T}} \left(\frac{(\zeta_i - \tilde{\varphi}_i)^2}{\tilde{T}} - \frac{3}{2}\right).
\]  
(B7)

If the definitions of \( \hat{n}^B \), \( \tilde{\varphi}^B \), and \( \tilde{T}^B \), i.e., Eqs. (10a)–(10c) with \( \alpha = B \), are used, it turns out that \( \tilde{\Psi}_e \) is linear in \( \tilde{F}^B \).

APPENDIX C: DATA ON NUMERICAL COMPUTATION

The lattice systems used here are essentially the same as those used in Ref. 5 (see Appendix A of Ref. 5). But, in the present computation, the higher accuracy is attained basically by using wider computational regions, more lattice points, and smaller lattice intervals. The details of the lattice systems are omitted here.

The accuracy of the computation was checked in various ways. For many cases included in Tables V–VIII in Ref. 42 (the cases in Tables I–IV in Ref. 42 are all included in Tables V–VIII there), we carried out computation with finer lattice systems with double lattice points either in \( x_1 \) or in \( \xi_1 \) and confirmed that the values of \( F_c \) and \( \Gamma_a \) in Tables I and II (and Tables I–VIII in Ref. 42) did not change. More specifically, concerning the \( x_1 \) lattices, this check was performed for all \( M_{n\infty} \) and for \( M_{\infty} = 0 \) and 3 in the cases included in Tables V–VIII in Ref. 42. The same check was also performed for many other cases in Tables V, VI, and VIII in Ref. 42. As for the \( \xi_1 \) lattices, the check was performed for about one third of the cases of Table VI in Ref. 42 and several cases for \( M_{\infty} = 3 \) of Tables V and VIII there. In general, accurate computation becomes more difficult as \( M_{\infty} \) increases. Although the lattice systems used in Ref. 5 are sufficient for obtaining \( (\mathcal{H}_1, \mathcal{H}_6, \mathcal{H}_c) \) [cf. Eq. (26a)] and \( F_\infty \), we need higher accuracy to obtain accurate results for \( (\mathcal{H}_{aB}, \mathcal{H}_{Kb}, \mathcal{H}_{CB}) \) [cf. Eq. (26b)] and \( \Gamma_a \).

As in Ref. 5, the conservation laws were also used for checking the accuracy. Let us set

\[
(I_1, I_2, I_3, I_4) = \int \xi(1, \xi_1, \xi_2, \xi_2^2) \tilde{F} d^3 \xi,
\]  
(C1a)

\[
I_1^B = \int \xi \tilde{F}^B d^3 \xi.
\]  
(C1b)

The \( n_\infty(2kT_\text{a}/m^A)^{1/2}I_1 \), \( 2p_\infty I_2 \), \( 2p_\infty \), and \( p_{\infty}(2kT_\text{a}/m^A)^{1/2}I_4^B \) are, respectively, the number of molecules, the \( X_1 \) component of the momentum, its \( X_2 \) component, and the energy of the total mixture transported in the positive \( X_1 \) direction across a unit area of the plane \( X_1 = \text{const} \) per unit time; \( n_\infty(2kT_\text{a}/m^A)^{1/2}I_1^B \) is the molecular flux of the noncondensable gas corresponding to \( n_\infty(2kT_\text{a}/m^A)^{1/2}I_1 \). It was shown in Sec. II C that \( I_1^B = 0 \). The integration of Eq. (14a) multiplied by \( 1, \xi_1, \xi_2, \xi_2^2 \) with respect to \( \xi \) over its whole space, under the condition (17a), yields

\[
I_m = \text{const} = I_{m\infty} \quad (m = 1, 2, 3, 4),
\]  
(C2)

where \( I_{m\infty} \) are the \( I_m \) at infinity and are given by

\[
I_{1\infty} = -(5/6)^{1/2}M_{\infty}, \quad I_{2\infty} = [(5/3)M_{\infty}^2 + 1]/2,
\]  
(C3)

\[
I_{3\infty} = -(5/6)M_{\infty}M_{\infty}, \quad I_{4\infty} = -(5/6)^{3/2}M_{\infty}(M_{\infty}^2 + M_{\infty}^2 + 3).
\]  
(C3)

Because of numerical error, \( I_m \) do not satisfy Eq. (C2) exactly and \( I_1^B \) does not vanish exactly. The deviations of the numerical values of \( I_m - I_{m\infty} \) and \( I_1^B \) from zero, where \( I_1^B \) is the \( I_1^B \) with \( \tilde{F}^B = \tilde{F}^B_\alpha \) (see the first paragraph in Sec. III C), are estimated as follows:

\[
|I_m - I_{m\infty}|/|I_{m\infty}| \quad \text{and} \quad |I_1^B|/|I_{1\infty}|
\]

\[
\begin{align*}
0.30 \times 10^{-4} & \quad (M_{\infty} = 0.01), \\
0.11 \times 10^{-4} & \quad (0.03 \leq M_{\infty} \leq 0.07), \\
0.22 \times 10^{-5} & \quad (0.1 \leq M_{\infty} \leq 0.3), \\
0.32 \times 10^{-6} & \quad (0.4 \leq M_{\infty}).
\end{align*}
\]  
(C4)

for all \( T_a = T_w \) and \( M_{\infty} \) \( (M_{\infty} = 0 \) is excluded for \( m = 3 \) because \( I_3 = I_{3\infty} = 0 \) in this case). The estimate naturally deteriorates for small \( M_{\infty} \) because \( I_{1\infty}, I_{3\infty}, \) and \( I_{4\infty} \) in the denominator are proportional to \( M_{\infty} \).


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40See EPAPS Document No. E-PHFLE6-15-504303 for tables and figures. A direct link to this document may be found in the online article’s HTML reference section. The document may also be reached via the EPAPS homepage (http://www.aip.org/pubservs/epaps.html) or from ftp.aip.org in the directory /epaps/. See the EPAPS homepage for more information.

