Harmonic Langmuir waves. II. Turbulence spectrum

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The Langmuir wave turbulence generated by a beam–plasma interaction has been studied since the early days of plasma physics research. In particular, mechanisms which lead to the quasi-power-law spectrum for Langmuir waves have been investigated, since such a spectrum defines the turbulence characteristics. Meanwhile, the generation of harmonic Langmuir modes during the beam–plasma interaction has been known for quite some time, and yet has not been satisfactorily accounted for thus far. In paper I of this series, nonlinear dispersion relations for these harmonics have been derived. In this paper (paper II), generalized weak turbulence theory which includes multiharmonic Langmuir modes is formulated and the self-consistent particle and wave kinetic equations are solved. The result shows that harmonic Langmuir mode spectra can indeed exhibit a quasi-power-law feature, implying multiscale structure in both frequency and wave number space spanning several orders of magnitude.
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I. INTRODUCTION

The Langmuir turbulence spectrum arising from the nonlinear interaction of an electron beam with the background plasma has been an active research area during the last few decades.[1] Frequently, in events where the ratio of the electric field over plasma energy densities is small, i.e.,

$$I_k/(8\pi\hat{n}k_BT_e)\ll 1,$$

where I_k is the wave spectral intensity, $\hat{n}$ is the electron bulk density, $k_B=1.38\times 10^{-16}$ erg/K is the Boltzmann constant and T_e is the electron temperature, emission in harmonics of the fundamental frequency has been observed in laboratory,[2–6] in space plasma observations[7,8] or in numerical simulations.[9–14]

The concept of turbulence originates from hydrodynamics. In neutral fluids, turbulence is generated by the shear-flow instability. The characteristic scale of the largest eddy is determined by the system size, the flow speed, and the fluid viscosity, but smaller scale eddies are created by the shear in the eddy flow itself. Thus, the cascading of the disturbance from largest to smaller scales ensues, until the process is arrested by the dissipation when the eddy scale size becomes sufficiently small. Such a fluid turbulence is famously characterized by the Kolmogorov-type power-law spectrum for the steady-state turbulence,

$$\mathcal{E}_k\propto k^{-5/3},$$

where $\mathcal{E}_k$ and k are the kinetic energy and the wave number associated with the fluid eddies, respectively.[15] The above relation was derived on the basis of a dimensional argument.

The power-law spectrum associated with the turbulence implies that the fluctuation appears to be scale-free within a certain range. Such a quasiscale free, fractal-like structure associated with the disturbance in any continuous media is often said to define what a turbulence is. When such a concept is generalized to plasmas, however, the situation becomes rather complicated. In magnetohydrodynamic (MHD) turbulence, a similar dimensional argument involving Alfvén waves leads to the so-called Iroshnikov–Kraichnan spectra,[16,17]

$$\mathcal{E}_k\propto k^{-3/2}.$$

However, while two-dimensional (2D) MHD turbulence appears to follow such a scaling law, it is found on the basis of numerical simulations that fully three-dimensional (3D) situation behaves more like a neutral fluid,[18] a finding which is not completely understood.

For high-frequency plasma (i.e., Langmuir) turbulence, the situation is even worse. Macroscopic dimensional analysis such as the shear-flow instability (hydrodynamics) or Alfvén dynamics (MHD) is simply not available for Langmuir turbulence, since it involves microscopic wave-particle interaction. However, in the strong Langmuir turbulence (SLT) regime, defined by

$$I_k/(8\pi\hat{n}k_BT_e)\gg k^2\lambda_{\rm De}^2,$$

where

$$\lambda_{\rm De}^2=k_BT_e/4\pi\hat{n}e^2$$

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is the square of the Debye length, Zakharov[19] formulated a macroscopic theory of plasma turbulence by ignoring the microscopic wave-particle interaction. Ever since, a substantial amount of theoretical efforts have been focused on the solutions of the Zakharov equation,[19] which describes the mechanism of *wave collapse*.

According to the generally accepted picture, SLT consists of randomly distributed coherent field structures characterized by high field amplitude and low plasma density called *cavitons*, embedded in the low-level, incoherent background of the weakly turbulent spectrum. Cavitons form due to the increase of the ponderomotive force in certain localized regions, leading to the reduction of the plasma density. The cavitons act as a potential well, trapping Langmuir oscillations and generating islands of high-intensity coherent modes inside the plasma. Eventually, the cavitons start to collapse, effectively spreading the SLT spectrum to higher values of wave number along the Langmuir dispersion relation curve, until dissipation effects such as Landau damping become sufficiently important to absorb the coherent modes.

Relying on this model, Zakharov predicted that the stationary SLT spectrum would be a power law given by,

$$I_k \propto k^{-7/3}.$$

Although several subsequent theoretical works have found a power-law spectrum (see Ref. 1 for a review), the exponent $-7/3$ has not been confirmed. Moreover, experimental[20] and simulation[21,22] results suggest that the saturated spectrum should not be of a Kolmogorov type, but rather it possesses a combination of power-law and exponential dependence on the wave number. Recent experimental results[23] seem also to indicate that, besides the wave collapse, another competing mechanism for energy transfer to large wave numbers is the conversion of Langmuir waves induced by the ion-sound turbulence.

In the weakly turbulent regime,

$$I_k/(8\pi\hat{n}k_BT_e) \lesssim k^2\lambda_{\mathrm{De}}^2,$$

on the other hand, the effect of Landau damping becomes important and the rate of caviton formation is low. Thus, there is no such mechanism to account for the cascading toward the large wave-number region. However, experimental observation of the beam–plasma system has shown that the spectrum comprised of the harmonics of the plasma frequency forms a power law both in frequency and in wave number,[2,3] and that this spectrum readily emerges in the linear growth phase of the beam–plasma interaction process.

The traditional quasilinear and weak turbulence theories cannot reproduce this spectrum. Quasilinear theories only account for a finite, relatively narrow emission band around the characteristic wave number associated with the fundamental Langmuir mode,

$$k_{L_1} \sim \omega_{pe}/V_0$$

[where $\omega_{pe} = (4\pi\hat{n}e^2/m_e)^{1/2}$ is the plasma frequency, and V_0 represents the average speed associated with the electron beam, which excites the Langmuir mode, L_1], encompassed in wave-number space, by modes that are strongly damped.

The traditional weak turbulent theory goes beyond the plateau formation process described by the quasilinear theory to include wave decay and induced scattering process. However, the weak turbulence theory only involves the linear eigenmodes of the plasma, i.e., the fundamental Langmuir and ion-sound waves (in the case of unmagnetized, longitudinal plasma, which is of our concern). Therefore, the harmonic generation cannot be discussed on the basis of such a theory. Moreover, the processes depicted by weak turbulence theory are only effective after the saturation of the quasilinear wave growth, and thus are not able to explain the harmonic power spectrum observed before the saturation stage.[2,3] The usual weak turbulence theory only describes processes such as the formation of the backscattered mode and the so-called condensation of wave modes in small wave-number regime, $k \sim 0$. The combined effects of Landau damping and wave condensation result in a relatively narrow spectrum along the Langmuir dispersion relation curve. In short, the issue of the formation of power-law spectrum for Langmuir turbulence, especially in the weak turbulence regime, still remains open.

In this paper, we will argue and present evidence that the generation of quasi-power-law spectra associated with weak Langmuir turbulence involves a physical mechanism fundamentally different from fluid or MHD turbulence. The basic mechanism in hydrodynamic turbulence is the shear-flow instability, while in MHD, Alfvén waves play such a role. In weak Langmuir turbulence, however, as we will show, the generation of turbulence spectrum involves not only Langmuir waves, but an entire series of higher harmonics.

As already explained in the paper, paper I [P. H. Yoon *et al.*, Phys. Plasmas **10**, 364 (2003)], the excitation of electrostatic Langmuir harmonics can be viewed as either forced nonlinear perturbations (in which case, the harmonics are not the eigenmodes of a plasma),[10,24–27] or in terms of nonlinear eigenmodes of a turbulent plasma.[28,29] Both theories might be applicable under certain conditions, but we believe that the eigenmode theory is better capable of explaining the available simulation results. In particular, numerical simulations in Refs. 13 and 14 compared favorably with our theory.[28,29] However, these analyses are incomplete in that they pertain only to the first harmonic mode with frequency, $\omega \sim 2\omega_{pe}$.

In an effort to generalize the theory, put forth in Refs. 28 and 29, to include all higher harmonics, we first derived the nonlinear dispersion relations for higher-harmonic Langmuir modes in paper I of the present series. In the present paper, which forms paper II, we complete the analysis by also deriving and solving the entire particle and wave kinetic equations, which involves fundamental and higher-harmonic Langmuir waves as well as the low-frequency ion-sound waves. Since both experiments and simulations present evidence that the harmonic spectra develop during the quasilinear growth phase, the numerical analysis carried out in this paper will also be restricted to this time interval. The evolution of the wave spectra during the later time, fully nonlinear, stage was not considered in this work because the evolution of the first harmonic, studied in Ref. 29, seems to indicate that the harmonic spectra does not play a relevant role on the decay and/or nonlinear scattering processes. However, this assumption needs to be further investigated in future works. We find that under certain initial conditions, the saturated wave spectra involving higher harmonics form a power-law dependence among the peaks of individual harmonic spectra. We have also carried out the Vlasov simulations, the details of which are the subject of the third and final installment of the present series [paper III; T. Umeda *et al.*, Phys. Plasmas **10**, 382 (2003)]. In paper III, we will discuss that the formation of quasi-power-law saturated spectrum seems to be a natural consequence of the harmonic Langmuir turbulence process.

II. WAVE KINETIC EQUATION

The basic equation in the present analysis is the nonlinear spectral balance equation for electrostatic oscillations in an unmagnetized plasma,[28,30–33] i.e., Eq. (2) of paper I. The real part of the nonlinear spectral balance equation gives us the dispersion relations for all modes, as was already analyzed in paper I. The focus of the present paper is the imaginary part, which will provide the wave kinetic equations. However, in formulating and solving the wave kinetic equation, the eigenmodes must be chosen in accordance with the solution of the real part. Thus, the results of paper I are directly utilized in the present discussion.

We assume that the total wave intensity, $I(\mathbf{k},\omega)$, can be expanded as the sum of all possible eigenmodes in the plasma,

$$I(\mathbf{k},\omega)=\sum_{\sigma=\pm1}\sum_{\alpha} I_{\alpha}^{\sigma}(\mathbf{k})\,\delta(\omega-\sigma\omega_{\mathbf{k}}^{\alpha}) \tag{1}$$

(here, we ignore, for the sake of simplicity, the possibility of broadening of frequency spectrum by the turbulence—this is the standard approximate procedure adopted in the literature), where $\alpha=S,L_1,L_2,L_3,\ldots$ denotes the eigenmodes, with S and L_1 being the usual ion-sound and (fundamental) Langmuir modes, respectively. Mode designations, $\alpha=L_2,L_3,\ldots$, correspond to the first harmonic mode (L_2), the second harmonic (L_3), and so on. In Eq. (1), the quantity $\omega_{\mathbf{k}}^{\alpha}$ corresponds to the dispersion relation for a given mode, designated by α. The quantity $I_{\alpha}^{\sigma}(\mathbf{k})$ corresponds to the wave intensity carried by the mode α. Finally, the quantity $\sigma=\pm1$ takes into account the existence of both forward- and backward-propagating waves, for a given eigenmode.

The linear plasma theory provides us with the dispersion relations for the ion-sound and (fundamental) Langmuir modes,

$$\begin{aligned}\omega_{\mathbf{k}}^{S}&=\frac{kc_s(1+3T_i/T_e)^{1/2}}{(1+k^2\lambda_{\mathrm{De}}^2)^{1/2}},\\ \omega_{\mathbf{k}}^{L_1}&=\omega_{pe}\left(1+\frac{3}{4}\frac{k^2v_e^2}{\omega_{pe}^2}\right),\end{aligned} \tag{2}$$

respectively, where $v_e^2=2k_BT_e/m_e$ is the square of the thermal speed associated with the background electrons.

According to the paper, paper I, the following dispersion relation for the harmonic Langmuir modes is applicable for a beam–plasma system when the characteristic wave number associated with the fundamental Langmuir mode is given by k_{L_1}:

$$\omega_{\mathbf{k}}^{L_n}=\omega_{pe}\left[n+\frac{3}{4}\frac{k^2v_e^2}{\omega_{pe}^2}-\frac{3(n-1)}{2}\left(\frac{k}{k_{L_1}}-\frac{n}{2}\right)\frac{k_{L_1}^2v_e^2}{\omega_{pe}^2}\right], \tag{3}$$

$n=1,2,3,\ldots$. Note that in paper I, we have specifically assumed that the characteristic wave number associated with the fundamental Langmuir mode is given by

$$k_{L_1}\approx\omega_{pe}/V_0,$$

where V_0 is the average speed of the beam electrons, but it is also noted there that a more accurate estimate is

$$k_{L_1}=\omega_{pe}/(V_0-\sqrt{k_BT_b/m_e}),$$

where T_b is the thermal spread associated with the beam component.

The general procedure for obtaining the wave kinetic equation for various plasma eigenmodes is already outlined in Refs. 28 and 29. Among the various terms in the wave kinetic equation, let us retain only the terms that are important. For the fundamental Langmuir wave (L_1), the most important terms are those which represent the induced emission (the first term on the right-hand side of the equation below), three-wave decay process involving L_1 and S modes (the second term), and the induced scattering off ions (the third term),

$$\begin{aligned}\frac{\partial I_{L_1}^{\sigma}(\mathbf{k})}{\partial t}&=\pi\sigma\omega_{\mathbf{k}}^{L_1}\frac{\omega_{pe}^2}{k^2}\int d\mathbf{v}\,\delta(\sigma\omega_{\mathbf{k}}^{L_1}-\mathbf{k}\cdot\mathbf{v})\mathbf{k}\cdot\frac{\partial f_e(\mathbf{v})}{\partial\mathbf{v}}I_{L_1}^{\sigma}(\mathbf{k})+\frac{\pi}{2}\frac{e^2}{k_B^2T_e^2}\sum_{\sigma',\sigma''=\pm1}\sigma\omega_{\mathbf{k}}^{L_1}\int d\mathbf{k}'\frac{\mu_{\mathbf{k}-\mathbf{k}'}(\mathbf{k}\cdot\mathbf{k}')^2}{k^2k'^2|\mathbf{k}-\mathbf{k}'|^2}\\ &\quad\times\delta(\sigma\omega_{\mathbf{k}}^{L_1}-\sigma'\omega_{\mathbf{k}'}^{L_1}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{S})\{\sigma\omega_{\mathbf{k}}^{L_1}I_{L_1}^{\sigma'}(\mathbf{k}')I_S^{\sigma''}(\mathbf{k}-\mathbf{k}')-[\sigma'\omega_{\mathbf{k}'}^{L_1}I_S^{\sigma''}(\mathbf{k}-\mathbf{k}')+\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L_1}I_{L_1}^{\sigma'}(\mathbf{k}')]I_{L_1}^{\sigma}(\mathbf{k})\}\\ &\quad+\frac{\pi}{\omega_{pe}^2}\frac{e^2}{m_em_i}\sigma\omega_{\mathbf{k}}^{L_1}\sum_{\sigma'=\pm1}\int d\mathbf{k}'\int d\mathbf{v}\frac{(\mathbf{k}\cdot\mathbf{k}')^2}{k^2k'^2}\delta[\sigma\omega_{\mathbf{k}}^{L_1}-\sigma'\omega_{\mathbf{k}'}^{L_1}-(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}](\mathbf{k}-\mathbf{k}')\cdot\frac{\partial f_i(\mathbf{v})}{\partial\mathbf{v}}I_{L_1}^{\sigma'}(\mathbf{k}')I_{L_1}^{\sigma}(\mathbf{k}).\end{aligned} \tag{4}$$

In the above, $\sigma=1$ designates forward-propagating L_1 mode, while $\sigma=-1$ represents the backward mode, and the quantity $\mu_{\mathbf{k}}$ is defined by

$$\mu_{\mathbf{k}}=|k|^3\lambda_{\mathrm{De}}^3\left(\frac{m_e}{m_i}\right)^{1/2}\left(1+\frac{3T_i}{T_e}\right)^{1/2}.$$

In Refs. 28 and 29, other terms including those that depict the induced scattering involving L_2 and L_1, as well as the three-wave decay/coalescence between L_2 mode and two L_1 modes, can be found. However, these terms are found to be generally insignificant. Moreover, the induced scattering process involving two L_1 modes but mediated by the electrons (i.e., nonlinear Landau damping by the electrons) is also known to be unimportant. As a result, these terms were ignored at the outset in the present discussion.

The wave kinetic equation for the ion-sound mode (S) can also be found in Refs. 28, 29, and 34,

$$\begin{aligned}\frac{\partial I_S^\sigma(\mathbf{k})}{\partial t}&=\pi\mu_{\mathbf{k}}\sigma\omega_{\mathbf{k}}^L\frac{\omega_{pe}^2}{k^2}\int d\mathbf{v}\,\delta(\sigma\omega_{\mathbf{k}}^S-\mathbf{k}\cdot\mathbf{v})\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}\right)\left(f_e(\mathbf{v})+\frac{m_e}{m_i}f_i(\mathbf{v})\right)I_S^\sigma(\mathbf{k})\\&+\frac{\pi}{4}\frac{e^2}{k_B^2T_e^2}\sum_{\sigma',\sigma''=\pm1}\sigma\omega_{\mathbf{k}}^{L_1}\int d\mathbf{k}'\frac{\mu_{\mathbf{k}}[\mathbf{k}'\cdot(\mathbf{k}-\mathbf{k}')]^2}{k^2k'^2|\mathbf{k}-\mathbf{k}'|^2}\delta(\sigma\omega_{\mathbf{k}}^S-\sigma'\omega_{\mathbf{k}'}^{L_1}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L_1})\{\sigma\omega_{\mathbf{k}}^{L_1}I_{L_1}^{\sigma'}(\mathbf{k}')I_{L_1}^{\sigma''}(\mathbf{k}-\mathbf{k}')\\&-[\sigma'\omega_{\mathbf{k}'}^{L_1}I_{L_1}^{\sigma''}(\mathbf{k}-\mathbf{k}')+\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L_1}I_{L_1}^{\sigma'}(\mathbf{k}')]I_S^\sigma(\mathbf{k})\}.\end{aligned}\tag{5}$$

In the above, the first term on the right-hand side represents the induced emission (or ion-sound damping) process, while the second term corresponds to the three-wave decay process. The induced scattering process for the ion-sound modes involves the ions and two S-mode waves, which is a very slow process compared with the time scale of interest to us in the present discussion. Therefore, such a term is ignored.

A preliminary version of the kinetic equation for L_2 mode was also derived in Ref. 29. One of the findings in Ref. 29 is that the nonlinear terms of the wave kinetic equation only play a small role on the long-term dynamics of the mode (i.e., these terms remain insignificant until $\omega_{pe}t\sim10^4$ or so). Therefore, in the present study, which is restricted to relatively early time period, we shall ignore nonlinear wave coupling associated with L_2 and higher harmonics. This is justified in view of the evidence from experiments as well as numerical simulations. Since the late 1960s, it has been shown both in beam-plasma experiments[2–5,8,35–37] and in numerical simulations,[9–14,38] that harmonic waves start to grow while the system is still in the quasilinear growth phase of the fundamental mode. It is also known that harmonics reach saturation at about the same instant as the fundamental does. Therefore, it is apparent that the dynamics of the nonlinear harmonic modes is dominated by the quasilinear, or equivalently, an induced emission or inverse Landau damping process. Accordingly, in this work we will be concerned only with the quasilinear evolution of the harmonic modes.

In view of the above discussion, instead of considering the complete expression for the higher-harmonic wave kinetic equations, which can be represented as

$$\frac{\partial I_{L_n}^\sigma(\mathbf{k})}{\partial t}=\left(\left.\frac{\partial}{\partial t}\right|_{\text{ind. emiss.}}+\left.\frac{\partial}{\partial t}\right|_{\text{decay}}+\left.\frac{\partial}{\partial t}\right|_{\text{ind. scatt.}}\right)I_{L_n}^\sigma(\mathbf{k}),$$

where the first term describes induced emission, and the second and third terms represent decay and induced scattering processes, respectively, we will keep only the first term. Therefore, the wave kinetic equations for the harmonic eigenmodes is given by

$$\frac{\partial I_{L_n}^\sigma(\mathbf{k})}{\partial t}=-\frac{2\,\mathrm{Im}\,\varepsilon(\mathbf{k},\sigma\omega_{\mathbf{k}}^{L_n})}{\partial\,\mathrm{Re}\,\varepsilon(\mathbf{k},\sigma\omega_{\mathbf{k}}^{L_n})/\partial(\sigma\omega_{\mathbf{k}}^{L_n})}I_{L_n}^\sigma(\mathbf{k}),$$

where $n\geqslant2$. Noting that

$$\frac{\partial\,\mathrm{Re}\,\varepsilon(\mathbf{k},\sigma\omega_{\mathbf{k}}^{L_n})}{\partial(\sigma\omega_{\mathbf{k}}^{L_n})}\approx\frac{2\omega_{pe}^2}{(\sigma\omega_{\mathbf{k}}^{L_n})^3}\approx\frac{2}{n^2\sigma\omega_{\mathbf{k}}^{L_n}},$$

$$\mathrm{Im}\,\varepsilon(\mathbf{k},\sigma\omega_{\mathbf{k}}^{L_n})=-\pi\frac{\omega_{pe}^2}{k^2}\int d\mathbf{v}\,\mathbf{k}\cdot\frac{\partial f_e(\mathbf{v})}{\partial\mathbf{v}}\delta(\sigma\omega_{\mathbf{k}}^{L_n}-\mathbf{k}\cdot\mathbf{v}),$$

we obtain the desired kinetic equation for L_n mode,

$$\begin{aligned}\frac{\partial I_{L_n}^\sigma(\mathbf{k})}{\partial t}&=n^2\pi\sigma\omega_{\mathbf{k}}^{L_n}\frac{\omega_{pe}^2}{k^2}\int d\mathbf{v}\,\delta(\sigma\omega_{\mathbf{k}}^{L_n}-\mathbf{k}\cdot\mathbf{v})\\&\quad\times\mathbf{k}\cdot\frac{\partial f_e(\mathbf{v})}{\partial\mathbf{v}}I_{L_n}^\sigma(\mathbf{k}).\end{aligned}\tag{6}$$

Equation (6) has the same structure as the conventional quasilinear wave kinetic equation, except for the overall coefficient proportional to n^2. Therefore, even without solving this equation, one can readily see that the harmonic modes will start to grow in the linear regime, provided that a minimum level of spectral intensity exists for these modes.

This brings us to an important question of the physical origin of the seed perturbation for the harmonics. The present theory relies upon the presence of the harmonic modes at noise level at time $t=0$ (however small they may be). We expect two mechanisms which can bring about the seed perturbation. One is the spontaneous thermal emission from the plasma (which is a single-particle effect, and thus cannot be discussed on the basis of Vlasov equation), and the other possibility is the coherent nonlinear dynamics. In the present formalism we simply impose a certain level of fluctuations for each harmonic mode at the outset. At this moment, a complete theory which includes the spontaneously generated noise, or self-consistently generated harmonics by the coherent nonlinear dynamics, in a general scheme of self-consistent theory, is not forthcoming.

Finally, the kinetic equation for the particle distribution, $f_a(\mathbf{v},t)$, is given by the customary quasilinear diffusion equation,

$$\frac{\partial f_a(\mathbf{v})}{\partial t} = \frac{\pi^2 e^2}{m_a^2}\frac{\partial}{\partial v_i}\sum_{\sigma=\pm 1}\sum_{\alpha}\int d\mathbf{k}\,\frac{k_i k_j}{k^2} I_\alpha^\sigma(\mathbf{k}) \times \delta(\sigma\omega_{\mathbf{k}}^\alpha - \mathbf{k}\cdot\mathbf{v})\frac{\partial f_a(\mathbf{v})}{\partial v_j}. \tag{7}$$

In the subsequent numerical computation, we will hold the ion distribution as quasiconstant in time. Therefore, the above diffusion equation will be solved only for the electrons.

In what follows, we have restricted ourselves to a one-dimensional situation. Let us adopt the following normalization convention:

$$\begin{aligned} &T=\omega_{pe}t, \quad u=v/v_e, \quad x=\omega/\omega_{pe}, \quad \kappa=kv_e/\omega_{pe},\\ &I(\kappa)=I(k)/(8\pi\hat{n}k_B T_e), \quad \tau=T_i/T_e, \quad \mu=m_e/m_i,\\ &q=(8\mu)^{1/2}(1+3\tau)^{1/2}/3, \quad \kappa_{L_1}=v_e/V_0. \end{aligned} \tag{8}$$

Then, the one-dimensional normalized versions of wave dispersion relations for harmonic Langmuir modes (including the fundamental) and the ion-sound mode are given by

$$\begin{aligned} &x_\kappa^{L_n} = n + (3/4)[\kappa^2 + (n-1)\kappa_{L_1}(n\kappa_{L_1} - 2\kappa)],\\ &x_\kappa^S = (3q/4)[\kappa(1+\kappa^2/2)^{-1/2}], \end{aligned} \tag{9}$$

while the one-dimensional normalized form of self-consistent kinetic equations for the waves and particles are given, respectively, by

$$\partial f_e/\partial T = (\partial/\partial u)(D\partial f_e/\partial u), \quad D(u) = \frac{2\pi}{|u|}\sum_{n=1,2,3,\ldots}\sum_{\sigma=\pm 1}\Theta(\sigma u)[I_{L_n}^\sigma(\kappa)]_{\kappa=1/|u|},$$

$$\begin{aligned}
\frac{\partial I_{L_1}^\pm(\kappa)}{\partial T} = {} & \frac{\pm\pi x_\kappa^{L_1}}{\kappa^2}\left[\frac{\partial f_e}{\partial u}\right]_{u=\pm x_\kappa^{L_1}/\kappa} I_{L_1}^\pm(\kappa)\\
& + \pi q x_\kappa^{L_1}[x_\kappa^{L_1} I_{L_1}^\mp(\kappa+q) I_S^\mp(2\kappa+q) - x_{\kappa+q}^{L_1} I_S^\mp(2\kappa+q) I_{L_1}^\pm(\kappa) + x_{2\kappa+q}^{L_1} I_{L_1}^\mp(\kappa+q) I_{L_1}^\pm(\kappa)]\\
& + \pi q\Theta(\kappa-q) x_\kappa^{L_1}[x_\kappa^{L_1} I_{L_1}^\mp(\kappa-q) I_S^\pm(2\kappa-q) - x_{\kappa-q}^{L_1} I_S^\pm(2\kappa-q) I_{L_1}^\pm(\kappa) - x_{2\kappa-q}^{L_1} I_{L_1}^\mp(\kappa-q) I_{L_1}^\pm(\kappa)]\\
& + \pi q\Theta(q-\kappa)\Theta(2\kappa-q) x_\kappa^{L_1}[x_\kappa^{L_1} I_{L_1}^\pm(q-\kappa) I_S^\pm(2\kappa-q) - x_{q-\kappa}^{L_1} I_S^\pm(2\kappa-q) I_{L_1}^\pm(\kappa) - x_{2\kappa-q}^{L_1} I_{L_1}^\pm(q-\kappa) I_{L_1}^\pm(\kappa)]\\
& + \pi q\Theta(q-\kappa)\Theta(q-2\kappa) x_\kappa^{L_1}[x_\kappa^{L_1} I_{L_1}^\pm(q-\kappa) I_S^\pm(q-2\kappa) - x_{q-\kappa}^{L_1} I_S^\pm(q-2\kappa) I_{L_1}^\pm(\kappa) + x_{q-2\kappa}^{L_1} I_{L_1}^\pm(q-\kappa) I_{L_1}^\pm(\kappa)]\\
& \mp\frac{2\pi x_\kappa^{L_1}}{\tau}\int_0^\infty d\kappa'\,\frac{|\kappa-\kappa'|}{\kappa-\kappa'}[u f_i(u)]_{u=\pm 3(\kappa+\kappa')/4} I_{L_1}^\pm(\kappa') I_{L_1}^\pm(\kappa)\\
& \mp\frac{2\pi x_\kappa^{L_1}}{\tau}\int_0^\infty d\kappa'\,\frac{|\kappa+\kappa'|}{\kappa+\kappa'}[u f_i(u)]_{u=\pm 3(\kappa-\kappa')/4} I_{L_1}^\mp(\kappa') I_{L_1}^\pm(\kappa),
\end{aligned} \tag{10}$$

$$\begin{aligned}
\frac{\partial I_S^\pm(\kappa)}{\partial T} = {} & \frac{\pm 3\pi q}{8} x_\kappa^{L_1}\kappa\left(\frac{\partial f_e(u)}{\partial u} - \frac{2u}{\tau} f_i(u)\right)_{u=\pm x_\kappa^S/\kappa} I_S^\pm(\kappa) + \frac{\pi q}{2}\Theta(q-\kappa) x_\kappa^{L_1}\left[x_\kappa^{L_1} I_{L_1}^\pm\left(\frac{\kappa+q}{2}\right) I_{L_1}^\pm\left(\frac{q-\kappa}{2}\right)\right.\\
& \left. - x_{(\kappa+q)/2}^{L_1} I_{L_1}^\pm\left(\frac{q-\kappa}{2}\right) I_S^\pm(\kappa) + x_{(q-\kappa)/2}^{L_1} I_{L_1}^\pm\left(\frac{\kappa+q}{2}\right) I_S^\pm(\kappa)\right] + \frac{\pi q}{2}\Theta(\kappa-q) x_\kappa^{L_1}\left[x_\kappa^{L_1} I_{L_1}^\pm\left(\frac{\kappa+q}{2},t\right) I_{L_1}^\mp\left(\frac{\kappa-q}{2}\right)\right.\\
& \left. - x_{(\kappa+q)/2}^{L_1} I_{L_1}^\mp\left(\frac{\kappa-q}{2}\right) I_S^\pm(\kappa) + x_{(\kappa-q)/2}^{L_1} I_{L_1}^\pm\left(\frac{\kappa+q}{2}\right) I_S^\pm(\kappa)\right],
\end{aligned}$$

$$\frac{\partial I_{L_n}^\pm(\kappa)}{\partial T} = \frac{\pm n^2\pi x_\kappa^{L_n}}{\kappa^2}\left[\frac{\partial f_e}{\partial u}\right]_{u=\pm x_\kappa^{L_n}/\kappa} I_{L_n}^\pm(\kappa).$$

In the above, $\Theta(x)=1$ for $x>0$, and $\Theta(x)=0$ for $x<0$.

The initial electron distribution function is given by a Maxwellian (thermal) core plus an energetic beam component, while the ions are treated as quasistationary. In normalized forms, the initial electron and immobile ion distributions are given, respectively, by

$$f_e(u,0)=\frac{1-\delta}{\sqrt{\pi}}e^{-u^2}+\frac{\delta}{\sqrt{\pi\rho}}\exp\left(-\frac{(u-U_0)^2}{\rho}\right),$$
$$f_i(u)=(\pi\mu\tau)^{-1/2}\exp[-u^2/(\mu\tau)], \tag{11}$$

where the various dimensionless parameters are

$$\delta=\frac{n_b}{n_0},\quad \rho=\frac{T_b}{T_e},\quad U_0=\frac{V_0}{v_b},\quad v_{Tb}=\sqrt{\frac{2k_BT_b}{m_e}}. \tag{12}$$

III. NUMERICAL ANALYSIS

We have numerically solved the complete set of wave and particle kinetic equations, Eq. (10), with the initial choice of parameters,

$$\delta=10^{-3},\quad U_0=5,\quad \rho=1,\quad \frac{T_e}{T_i}=\frac{1}{\tau}=7,\quad \mu=\frac{1}{1836}.$$

To solve the equations numerically, we have defined a set of $N_u=120$–240 points along the normalized velocity (u), with a spacing of $\Delta_u=(u_{\max}-u_{\min})/(N_u-1)$, and a set of N_q points along the normalized wave number (q). The value of N_q increases with the total number of harmonics ($N_{\rm har}$) considered and the spacing (Δ_q) and the final value ($q_{\max}$) is chosen so that the effect of all harmonics is adequately considered in the linear resonance region. We have then a total of $N_{\rm eq}=N_u+2(N_{\rm har}+1)N_q$ equations. For a total number of harmonics $N_{\rm har}=11$, we have typically $N_q\approx 680$ and $N_{\rm eq}\approx 16\,560$. This set of equations is numerically solved using a fourth-order Runge–Kutta method with adaptive step size[39] and the solutions are stored at predefined time instants. This method insures that the total energy of the system composed by particles plus waves is conserved within 0.1%.

As mentioned already, the present formalism does not have a self-consistent generation mechanism which can account for the initial (seed) perturbation for harmonic modes with $n>2$. Such a conceptual difficulty does not arise for the fundamental Langmuir (L_1) and ion-sound (S) modes, however, since their initial noise levels can be attributed to thermal fluctuations (i.e., spontaneous emission). Therefore, although the present formalism ignores spontaneous fluctuations, the physical origin of the initial levels of L_1 and S modes (which we arbitrarily impose) can be easily justified on the basis of physical grounds.

As far as the harmonics are concerned, there is no theory yet available in the literature which accounts for the spontaneous emission levels, and the development of such theory is beyond the scope of the present work. In this case, we simply sidestep the precise issue of the physical origin of the seed perturbations, and treat the initial choice of mode intensities as somewhat of a free parameter. In the present scheme, we employ the following *ad hoc* procedure to define the initial state for all eigenmodes: For a given nth harmonic, the small level of initial spectrum is modeled by a Gaussian form,

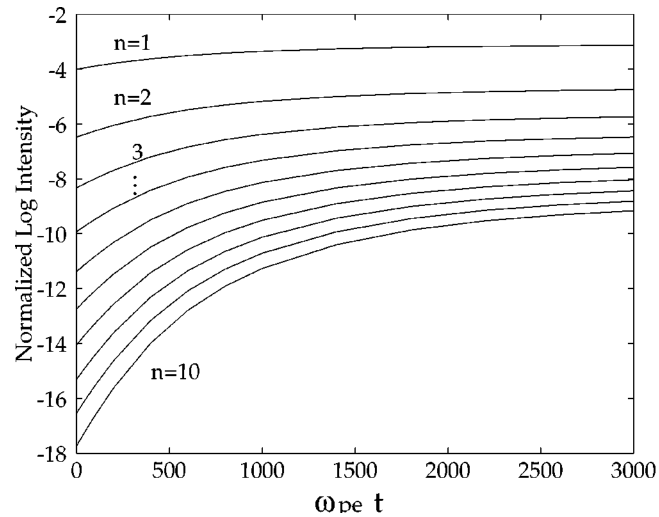

FIG. 1. Time evolution of the normalized wave intensity, max $I^+_{L_n}(t)/(8\pi\hat{n}k_BT_e)$, versus $\omega_{pe}t$, in logarithmic vertical scale, for the first ten harmonics.

$$I_{L_n}(\kappa)=\frac{I_n}{\sqrt{\pi}\Delta}\exp\left(-\frac{(\kappa-n\kappa_{L_1})^2}{\Delta^2}\right), \tag{13}$$

where κ_{L_1} is the normalized wave number associated with the primary L_1 mode. This choice is guided by the linear growth property which dictates that the nth harmonic mode should grow in the vicinity of $\kappa\sim n\kappa_{L_1}$, with some spread in wave number, characterized by Δ. The precise functional form is not crucial in this regard. The quantity I_n is determined by the expression

$$I_n=I_1e^{-\beta(n-1)}n^{-\alpha}, \tag{14}$$

where I_1, α, and β are all constants that can be arbitrarily chosen. It turns out that this particular profile allows for a combination of exponential and power-law dependence among the peaks of saturated harmonic mode spectra. Since the dynamical evolution of the harmonic modes is dictated by quasilinear equation, the choice of input will be directly reflected in the saturation spectra. Our choice (14) gives us sufficient freedom to adjust our theory to match the simulation result to be shown later (paper III).

Figure 1 shows the time evolution for the peak of each harmonic mode spectrum,

$$\max I^+_{L_n}(t)/(8\pi\hat{n}k_BT_e).$$

The parameters relevant to determine the initial spectra are $I_1=10^{-3}$, $\alpha=5$, and $\beta=2.236$. Note that the higher the harmonic mode number, the faster the mode grows initially, in agreement with the simulations,[10,11] and also with the linear growth rate computed in paper I (see Fig. 2 of that paper). Note also that all eigenmodes reach saturation at about the same time. In the displayed time scale, nonlinear effects are still not relevant enough to substantially affect the spectrum. Therefore, although we have fully retained the nonlinear

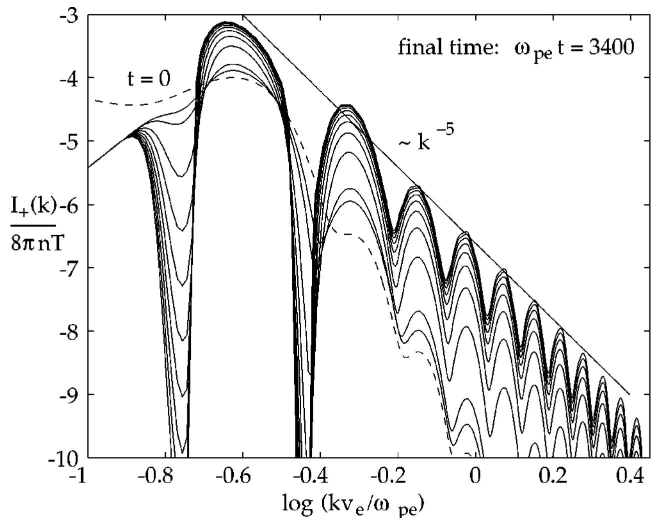

FIG. 2. Evolution of the total wave number spectrum, $I_+(k)/(8\pi\hat{n}k_BT_e)$ versus kv_e/ω_{pe} (in log–log scale), where $I_+(k)=\Sigma_{n=1}^{10}I_{L_n}^+(k)$, showing up to 10 harmonics. The parameters are the same as in Fig. 1. The dashed line is for $t=0$. The subsequent curves correspond to $\omega_{pe}t=100$, 200, 400, 600, 1000, 1400, 1800, 2200, 2600, 3000, and 3400.

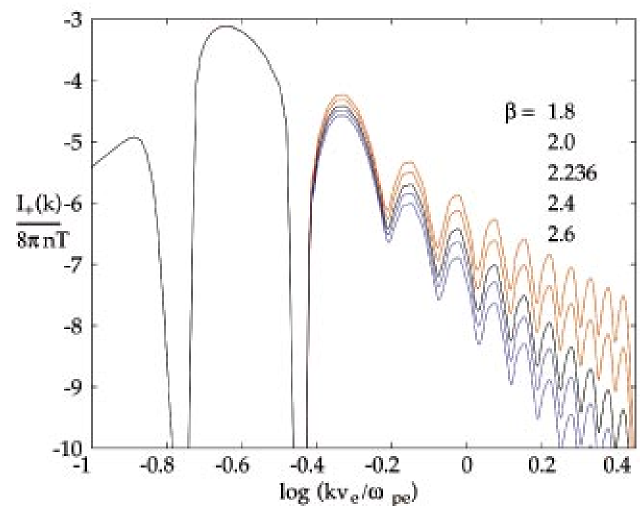

FIG. 3. (Color) Saturation-state spectra for five different choices of the parameter β. The black curve in the middle corresponds to the saturation state depicted in Fig. 2, namely $\beta=2.236$. The two red curves above the black line are for $\beta=1.8$ and $\beta=2.000$, the topmost curve being the case for the lower β value. These two curves show the power-law index slightly increasing for large k. The two blue curves underneath the solid line are for β values corresponding to $\beta=2.4$ and $\beta=2.6$, as indicated in the figure. The latter two curves exhibit combined power law and exponential decay for large k.

wave coupling terms in Eq. (10), time evolution shown in Fig. 1 is practically determined by the quasilinear process alone.

The evolution of the wave-number spectrum for weak harmonic Langmuir turbulence from the linear phase until quasisaturation stage can be seen in Fig. 2. In this figure, where we have plotted the superposition of all the harmonic wave intensities,

$$I^+(k)=\sum_{n=1,2,3,\ldots} I_{L_n}^+(k),$$

the total time interval ranges from $\omega_{pe}t=0$ to $\omega_{pe}t=3400$. Note that the initial form of superposed spectra (dashed line) is not in the power-law form, but it achieves a power-law spectral shape at the saturation stage. This is owing to the fact that the higher harmonics grow faster than the lower harmonics. Our choice of spectral shape parameters, α and β, and the specific form of initial spectra were partly designed to produce a power-law form at quasisaturation stage. If we connect the peaks of the individual harmonics, then one obtains an overall power-law,

$$I^+(k)\propto k^{-5}.$$

The spectrum shown in Fig. 2 bears a qualitative resemblance with some measurements made on weak beam–plasma systems (see, for instance, Fig. 4 of Ref. 3). The spectral property in terms of frequency, instead of wave number, follows the same power-law pattern and is not shown here.

Since the results presented here practically correspond to the quasilinear stage of the kinetic evolution, the electron distribution function is mostly affected by the linear wave-particle interaction with the combined fundamental and nonlinear harmonic Langmuir modes. However, the energy content of the harmonic modes is very low compared to the fundamental mode. As a consequence, the temporal evolution of the distribution function is very similar to the customary quasilinear theory which does not include the harmonics. The change in the electron distribution is largely restricted to the inverse population region, $\partial f_e/\partial v>0$, resulting in the usual plateau formation at saturation. Thus, this process does not scatter the electrons to the tail, resulting in a power-law velocity distribution function, as is frequently observed simultaneously with the power-law spectrum for the waves. The velocity-space power-law distribution must result from the longer time-scale nonlinear interactions in the plasma, which may involve processes other than quasilinear diffusion, and is beyond the scope of the present discussion.

Admittedly, our choice of initial spectra affects the outcome of the dynamic evolution. The power-law form for the total spectrum is not the only possible solution at the saturation stage. The specific power-law index, -5 in the case of Fig. 2, is the result of our choice of form factor, $\alpha=5$ in Eq. (14), although the choice was guided by experimental observation.[3] However, other possible power-law indices, or even an alternative form for the spectra may also result depending on the choice of the initial spectral function, Eq. (14). For different choices of the constant β, the saturated spectrum may also deviate substantially from the pure power law.

As an example, Fig. 3 shows five different possibilities of β parameter. The black line corresponds to the case shown in Fig. 2, namely, $\beta=2.236$. This case displays an exact power-law profile with the index of -5. However, the neighboring curves were obtained with slightly different values for the parameter β. The specific values are as follows: For the red curves, we chose $\beta=1.8$ and 2.0; for the blue lines, we have made the choices of $\beta=2.4$ and 2.6. These curves display a combination of power-law and exponential profile, similar to that reported in Ref. 20 for a strong beam–plasma turbulence, although the present case is in a different context. The source of this ambiguity is again, the lack of information on the initial level of perturbation, as mentioned already, which is not yet included in the theory.

Given the fact that the final spectrum depends critically on the choice of initial conditions, one may naturally ask whether the quasi-power-law distribution of waves at the saturation stage is an artifact or not. To address this issue, we have also carried out Vlasov simulations. Since the Vlasov simulation also suffers from the lack of single-particle effects (i.e., the lack of spontaneous fluctuations), to a certain extent, the final outcome of the Vlasov simulation should also depend on the initial choice of seed perturbation. However, as we will discuss more thoroughly in paper III, the results of Vlasov simulations with a variety of initial spectral functions, including quasi-monochromatic initial perturbation and white noise, invariably produced quasi-power-law spectra at the end stage, albeit the detailed structure associated with the spectrum varied depending on the initial condition. From this, we conclude that the harmonic Langmuir wave spectrum exhibiting quasi-power-law feature is an essential characteristic of the Langmuir turbulence.

IV. CONCLUSIONS

In this paper, we have numerically solved the complete set of one-dimensional weak turbulent equations, including a new set of equations for the harmonic eigenmodes, along with the corresponding dispersion relations. The new equations were derived from the generalized weak turbulence theory,[28] which predicts the existence of harmonic modes in a weak beam–plasma system, as solutions to the nonlinear spectral balance equation when one considers nonlinear effects over both the real and imaginary parts of the said equation.

The usual linear plasma theory of the weak beam–plasma system only considers the existence of the ion-sound and Langmuir modes as the eigenmodes of an unmagnetized plasma interacting through electrostatic field in a uniform medium. In the conventional turbulence theories available in the literature, plasma turbulence is described in terms of mode couplings among these linear eigenmodes. In the generalized weak turbulence theory, on the other hand, the generation of harmonic eigenmodes is considered as part of the basic turbulent beam–plasma interaction process.

By including nonlinear terms not only on the imaginary part of the spectral balance equation, but also on the real part, and by carefully balancing the effects of the nonlinear terms against the linear response, the generalized weak turbulence theory provides the dispersion relations and the wave kinetic equations for the harmonic Langmuir eigenmodes. The spectra for the harmonic modes, obtained from the numerical solution of the wave kinetic equations, show that a possible solution for the total spectrum of Langmuir turbulence has a power-law dependence on the peaks of the individual harmonic modes.

The possibility of a power-law spectrum in the relatively early quasilinear phase may provide an alternative scenario to the usual plasma turbulence models. This process is very different from the hydrodynamic turbulence where the quasistationary power-law spectrum is formed due to nonlinear cascading process of the shear-flow instability, or from strong Langmuir turbulence, where the same behavior is displayed by the plasma caviton collapse process. Both examples of the turbulence spectrum formation depend on the fact that a given energy threshold is reached before they can be triggered.

In the usual weak-turbulence theory, there is no similar basic structure such as a fluid eddy or a caviton. Since the linear mechanism of Landau damping is dominant over the nonlinear interaction effects, i.e., the wave decay and induced scattering, any wave packet with phase velocity outside the inverse-population region will be strongly damped. However, the generation of harmonics of the Langmuir mode has no energy threshold, and it occurs along different eigenmodes, but they all employ the same free energy source. In this way, the free energy contained in the distribution of particles is effectively partitioned among several wave modes before any nonlinear effect can take place. Moreover, the presence of a power-law spectrum composed by the harmonics of the fundamental frequency imply the well-known fractal or multiscale structures inside the plasma, both in time and space. This picture suggests, therefore, that the generation of harmonic eigenmodes of Langmuir waves is an alternative mechanism of high-frequency turbulence in a plasma within the weakly turbulent regime.

One of the shortcomings of the present approach is the lack of the effects due to single-particle fluctuations, which will describe the spontaneous emission of the plasma eigenmodes including the harmonic components. The customary theory of fluctuations describes the spontaneous emissions of linear eigenmodes, namely, the ion-sound and fundamental Langmuir modes. A generalization of the customary theory of fluctuations may provide the theory for the initial-state wave amplitudes for the harmonic modes as well, thus rendering the artificial initial condition implemented in this work unnecessary. Such extension is under development now.

Another shortcoming of the present approach is that coherent nonlinear dynamics is precluded at the outset. This situation is not easy to remedy, however. The evidence from Vlasov simulation (see paper III) suggests that the later development of harmonic generation is intimately related to the internal nonlinear dynamics which takes place beyond the initial quasilinear process.

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