Lepton flavor violation with supersymmetric Higgs triplets in the TeV region for neutrino masses and leptogenesis

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Lepton flavor violating processes such as $\mu \to 3e$ and $\mu \to e\gamma$ are investigated with a supersymmetric Higgs triplet pair Δ and $\bar{\Delta}$ in light of neutrino masses and experimentally verifiable leptogenesis. The Higgs triplet mass M_Δ is expected to be in the range of 1–100 TeV. The branching ratios of these charged lepton decays are evaluated in terms of M_Δ and the coupling $fL\Delta L$ of the Higgs triplet Δ with lepton doublet pairs LL, which is proportional to the neutrino mass matrix. They may be reached in future collider experiments. In particular, the $\mu \to 3e$ decay would be observed indicating the existence of Higgs triplets with $M_\Delta \sim 1$–100 TeV for $|f| \sim 0.1$–1, while the $\mu \to e\gamma$ decay can be significant irrespective of M_Δ in the supersymmetric model due to the flavor violation in the slepton mass matrices induced by the renormalization effects.

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I. INTRODUCTION

The neutrino masses may be generated naturally by introducing the electroweak Higgs triplet Δ [1]. The effective higher-dimensional operators $L_i L_j H_u H_u$ of the lepton and Higgs doublets (with indices i,j to denote the generations) are indeed provided by the exchange of a Higgs triplet, as well as the right-handed neutrinos in the usual seesaw mechanism. The lepton number violation with a Higgs triplet or right-handed neutrinos may further realize the generation of lepton number asymmetry, leptogenesis, in the early Universe. Then, the sufficient baryon-to-entropy ratio can be provided from the lepton number asymmetry through the electroweak anomalous effect [2].

Leptogenesis has been investigated extensively in the literature in connection with neutrino mass generation. In most scenarios of leptogenesis via lepton number nonconserving decays the relevant particles such as right-handed neutrinos and Higgs triplets are supposed to be much heavier than the electroweak scale. On the phenomenological point of view, however, these particles are expected to be alive in the TeV region. In this respect, it is interesting that the leptogenesis can be realized with supersymmetric Higgs triplets via multiscalar coherent evolution after the inflation [3,4]. While the Higgs triplet mass M_Δ was originally supposed to be in the range 10^9–10^{14} GeV [3], it has been found by reanalyzing this leptogenesis scenario that the successful leptogenesis is possible even with the Higgs triplet mass in the TeV region $M_\Delta \sim 1$–100 TeV [5]. That is, just after the inflation the lepton number asymmetry appears via multiscalar coherent motion on the flat manifold of a pair of Higgs triplets Δ, $\bar{\Delta}$ and the antislepton $\tilde{e}^c$ in the manner of Affleck-Dine mechanism [6,7]. Then, the lepton number asymmetry is fluctuating during some period, and it is fixed to some significant value due to the effect of the Higgs triplet mass terms. It is here essential for fixing the lepton number asymmetry that the Higgs triplet mass terms should prevail over the negative thermal log term, requiring a condition on M_Δ. This condition can really be satisfied for $M_\Delta \gtrsim 1$ TeV depending on the reheating temperature of the universe $T_R < 10^9$ GeV and the mass scale $M/\lambda \sim 10^{20}$–10^{23} GeV of the nonrenormalizable superpotential terms for leptogenesis.

If the Higgs triplet mass is $M_\Delta \sim 1$ TeV, quite interesting phenomenology is provided in the electroweak to TeV region [8–12]. Then, the leptogenesis scenario as well as the neutrino mass generation with supersymmetric Higgs triplets can be verified by the future collider experiments. The Higgs triplets may be discovered by direct production, and their effects on lepton flavor violation may also be found in the decays of charged leptons, $\mu \to 3e$, $\mu \to e\gamma$, $\tau \to 3\mu$, $\tau \to \mu\gamma$, and so on. It is particularly interesting in the Higgs triplet model that these lepton flavor violating processes are related to each other through the neutrino mass matrix, which is proportional to the Yukawa coupling $f_{ij} L_i L_j \Delta$. The experimental observations on the atmospheric and solar neutrinos now provide important information about the neutrino masses and mixings [13–16]. Then, these relations among the lepton flavor violating processes in the Higgs triplet model will be tested in the feasible experiments, as investigated in the literature for the nonsupersymmetric model with $M_\Delta \sim 100$ GeV–1 TeV [10,11] and the supersymmetric model with $M_\Delta \sim 10^{11}$–10^{14} GeV through the renormalization effects on the slepton masses [17].

We here investigate these lepton flavor violating effects of the supersymmetric Higgs triplets in the light of neutrino masses and experimentally verifiable leptogenesis. The Higgs triplet mass is expected to be $M_\Delta \sim 1$–100 TeV. This paper is organized as follows. In Sec. II, we present the supersymmetric Higgs triplet model, and describe the neutrino mass generation, discussing how the Higgs triplets in the TeV region can develop naturally the desired tiny vacuum expectation values. In Sec. III, we examine the lepton flavor violating terms provided with the Higgs triplets, including the renormalization effects. In Sec. IV, we investigate the lepton flavor violating processes and the muon anomalous magnetic moment, which are related to each other

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through the neutrino mass matrix. Section V is devoted to the summary. The one-loop contributions of supersymmetric Higgs triplets to the charged lepton radiative decay amplitudes are calculated in the Appendix.

II. NEUTRINOS WITH HIGGS TRIPLETS

We investigate an extension of the minimal supersymmetric standard model by introducing a pair of Higgs triplets Δ and $\overline{\Delta}$, which are specified in terms of $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ as

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix} \sim (\mathbf{1},\mathbf{3},1), \tag{1}$$

$$\overline{\Delta} = \begin{pmatrix} \overline{\Delta}^-/\sqrt{2} & \overline{\Delta}^0 \\ \overline{\Delta}^{--} & -\overline{\Delta}^-/\sqrt{2} \end{pmatrix} \sim (\mathbf{1},\mathbf{3},-1). \tag{2}$$

The lepton doublets $L_i = (\nu_i, l_i)$, antilepton singlets l_i^c ($i = e, \mu, \tau$), and the Higgs doublets H_u, H_d are given as usual. The generic lepton number conserving superpotential for the leptons and Higgs fields is given by

$$W_0 = h_{ij} L_i H_d l_j^c + \mu H_u H_d + \frac{1}{\sqrt{2}} f_{ij} L_i \Delta L_j + M_\Delta \overline{\Delta}\Delta, \tag{3}$$

where the lepton basis is taken at the electroweak scale M_W with the diagonal Yukawa coupling h, and the dilepton coupling is given by a symmetric matrix $f = f^{\mathrm{T}}$. The lepton numbers are assigned to the Higgs triplets as

$$Q_L(\Delta) = -2, \quad Q_L(\overline{\Delta}) = 2. \tag{4}$$

Then, the lepton number violating terms may also be included in the superpotential as

$$W_{\mathrm{LV}} = \xi_1 H_u \overline{\Delta} H_u + \xi_2 H_d \Delta H_d. \tag{5}$$

The Higgs triplets are R-parity even, and we here do not consider the R-parity violation for definiteness.

The Higgs triplets develop nonzero vacuum expectation values (VEV's) due to the effects of W_{LV} as

$$\langle \Delta^0 \rangle = -c_1 \frac{\xi_1 \langle H_u \rangle^2}{M_\Delta}, \langle \overline{\Delta}^0 \rangle = -c_2 \frac{\xi_2 \langle H_d \rangle^2}{M_\Delta}. \tag{6}$$

The factors $c_1, c_2 \sim 1$ for $\xi_1 \sim \xi_2$ are determined precisely by minimizing the scalar potential including the soft supersymmetry breaking terms with the mass scale $m_0 \sim 10^3$ GeV ($c_1 = c_2 = 1$ in the limit of $\mu, m_0 \to 0$). It should be noted here that these VEV's are induced by the ξ_1 and ξ_2 couplings explicitly violating the lepton number conservation. Hence the so-called triplet Majoron does not appear from the Δ and $\overline{\Delta}$ fields, which rather acquire masses $\simeq M_\Delta$. The slepton fields $\tilde{L}_i$, $\tilde{l}_i^c$ do not develop VEV's since the R-parity is still preserved by the VEV's of Higgs triplets.

The neutrino mass matrix is provided by the VEV of the Higgs triplet as

$$M_\nu = f\sqrt{2}\langle \Delta^0 \rangle, \tag{7}$$

which is diagonalized with a unitary matrix U as

$$U^{\mathrm{T}} M_\nu U = \mathrm{diag}(m_1, m_2, m_3). \tag{8}$$

The charged lepton mass matrix is also given as

$$M_l = h\langle H_d \rangle = \mathrm{diag}(m_e, m_\mu, m_\tau). \tag{9}$$

Here, the flavor structure of leptons is described at M_W by the f coupling with the diagonal h coupling. This neutrino mass matrix (7) should reproduce the masses and mixing angles inferred from the data of neutrino experiments [13–16]. Then, by considering Eqs. (6) and (7) with $m_i \lesssim 10^{-1}$ eV, a constraint on the magnitude of f coupling is placed roughly as

$$|f| \lesssim 10^{-1} \left(\frac{\xi}{10^{-10}} \right)^{-1} \left(\frac{M_\Delta}{10^3 \ \mathrm{GeV}} \right). \tag{10}$$

Here, the magnitude of the lepton number violating couplings is supposed to be very small as $\xi_1, \xi_2 \sim \xi \sim 10^{-10}$ for $M_\Delta \sim 10^3$ GeV.

These tiny lepton number violating couplings ξ_1, ξ_2 inducing the VEV's of Higgs triplets may be explained as follows [3]. Suppose that the lepton number/R-parity violation originates in the Planck scale physics. Then, it may be provided with certain higher-order effective superpotential terms as

$$W'_{\mathrm{LV}} = \xi'_1 \frac{\overline{S} H_u \overline{\Delta} H_u}{M_{\mathrm{P}}} + \xi'_2 \frac{S H_d \Delta H_d}{M_{\mathrm{P}}} \tag{11}$$

with the reduced Planck mass $M_{\mathrm{P}} = m_{\mathrm{P}}/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV. Here, some singlet superfields S and $\overline{S}$ of R-parity odd with $Q_L = 1, -1$, respectively, are also considered. The lepton number/R-parity violating terms $S H_u H_d$ and $S \Delta \overline{\Delta}$ are hence excluded. These singlet fields may have the lepton number/R preserving superpotential terms,

$$W_S = M_S S\overline{S} + \lambda_S \frac{SS\overline{S}\overline{S}}{M_{\mathrm{P}}}, \tag{12}$$

where the Higgs singlet mass is assumed to be $M_S \sim 10^3$ GeV as well as the Higgs triplet mass $M_\Delta \sim 10^3$ GeV. Without cubic terms for the Higgs singlets, they are considered as flatons [18], and may develop large VEV's with vanishing F terms $|F_S|, |F_{\overline{S}}| \approx 0$ as

$$\langle S \rangle \sim \langle \overline{S} \rangle \sim \sqrt{M_S M_{\mathrm{P}}} \sim 10^{10} \ \mathrm{GeV}. \tag{13}$$

Then, the lepton number violating couplings ξ_1 and ξ_2 are derived effectively as

$$\xi_1 = \xi'_1 (\langle \overline{S} \rangle / M_{\mathrm{P}}), \xi_2 = \xi'_2 (\langle S \rangle / M_{\mathrm{P}}) \tag{14}$$

with the tiny factor desired for ξ in Eq. (10),

$$\langle S \rangle / M_{\mathrm{P}}, \langle \overline{S} \rangle / M_{\mathrm{P}} \sim \sqrt{M_S / M_{\mathrm{P}}} \sim 10^{-8}. \tag{15}$$

It is also notable that the smallness of the Higgs triplet VEV's may be explained elegantly in the context of large extra dimensions [9].

III. LEPTON FLAVOR VIOLATION

We here examine the lepton flavor violating couplings provided with Higgs triplets, including the renormalization-group effects.

A. Yukawa couplings

The lepton basis is taken with the diagonal Yukawa coupling h at M_W in Eqs. (3) and (9):

$$h_{ij}=h_i\delta_{ij}. \tag{16}$$

(Henceforth M_W is omitted for the quantities at the electroweak scale.) Then, the lepton flavor violation, which is provided by the f coupling at M_W, is linked directly to the neutrino mass matrix, as seen in Eq. (7) [10,11,17]. This is a very interesting feature of Higgs triplet model. Specifically, the f coupling is given in terms of the neutrino masses (m_i), mixing angles (θ_{ij}), and CP violating phases (δ,α_1,α_2) as

$$f_{ij}=|f|\sum_k U^*_{ik}U^*_{jk}(m_k/m_{\mathrm{atm}}), \tag{17}$$

where $m_{\mathrm{atm}}\equiv\sqrt{\Delta m^2_{\mathrm{atm}}}$ with $\Delta m^2_{\mathrm{atm}}\sim 3\times 10^{-3}$ eV^2. The explicit form of the Maki-Nakagawa-Sakata (MNS) matrix U (lepton mixing matrix) [19] is given in a review by the Particle Data Group [20]. The mean magnitude of the f coupling is given suitably by

$$|f|\equiv m_{\mathrm{atm}}/(\sqrt{2}\langle\Delta^0\rangle), \tag{18}$$

which is constrained, as seen in Eq. (10), with $\langle\Delta^0\rangle$ in terms of ξ and M_Δ.

The flavor violation appears in the h coupling at a certain unification scale M_G such as the grand unification or gravitational scale through the renormalization effects. In the bottom-up view point $M_W\to M_G$, the relevant couplings at M_G are evaluated with those at M_W as

$$h_{ij}(M_G)=c_{hi}h_i\delta_{ij}+(\Delta_f h)_{ij}, \tag{19}$$

$$f_{ij}(M_G)=c_{fij}f_{ij}+(\Delta_f f)_{ij}, \tag{20}$$

where the sum is not taken over i,j. The factors c_{hi},c_{fij} ~ 1 are provided by the gauge and h couplings. The remaining terms provided by the f coupling are calculated in the leading-log approximation as

$$(\Delta_f h)_{ij}\simeq(3/2)h_i(f^\dagger f)_{ij}t_G, \tag{21}$$

$$(\Delta_f f)_{ij}\simeq 3(ff^\dagger f)_{ij}t_G, \tag{22}$$

where

$$t_G\equiv(1/8\pi^2)\ln(M_G/M_W)\sim 0.4. \tag{23}$$

B. Slepton mass terms

The flavor violation also appears in the soft supersymmetry breaking terms. We may assume the universality of the soft supersymmetry breaking terms at the unification scale M_G, i.e., the soft masses of scalar fields are given by the common mass m_0, and the A terms are given by a_0m_0 with $a_0\sim 1$. Then, in the top-down view point $M_G\to M_W$ the soft mass terms at M_W are calculated particularly for the left-handed slepton doublets $\tilde{L}$ and the right-handed charged slepton singlets $\tilde{l}^c$ [17] as

$$(M^2_{\tilde{L}})_{ij}=c_{\tilde{L}}m_0^2\delta_{ij}+(\Delta_{h+f}M^2_{\tilde{L}})_{ij}, \tag{24}$$

$$(M^2_{\tilde{l}^c})_{ij}=c_{\tilde{l}^c}m_0^2\delta_{ij}+(\Delta_h M^2_{\tilde{l}^c})_{ij}. \tag{25}$$

Here, the contributions of the gauge couplings are included in the factors $c_{\tilde{L}},c_{\tilde{l}^c}\sim 1$, and those of the h and f couplings are given by

$$\begin{aligned}\Delta_{h+f}M^2_{\tilde{L}}\simeq&-m_0^2[(3+a_0^2)h^\dagger(M_G)h(M_G)\\&-(9+3a_0^2)f^\dagger(M_G)f(M_G)]t_G,\end{aligned} \tag{26}$$

$$\Delta_h M^2_{\tilde{l}^c}\simeq -m_0^2(6+2a_0^2)h^\dagger(M_G)h(M_G)t_G. \tag{27}$$

The A_h term of the h coupling is also given at M_W by

$$(A_h)_{ij}=a_hm_0h_{ij}(M_G)+(\Delta A_h)_{ij} \tag{28}$$

with $a_h\sim a_0$ including the effects of gauge couplings and

$$\begin{aligned}\Delta A_h\simeq&-(9/2)a_0m_0h(M_G)[h^\dagger(M_G)h(M_G)\\&+f^\dagger(M_G)f(M_G)]t_G.\end{aligned} \tag{29}$$

The charged slepton mass matrix is given in the basis of $(\tilde{l},\tilde{l}^{c*})$ by

$$\mathcal{M}^2_{\tilde{l}}=\begin{pmatrix}M^2_{\tilde{l}LL} & M^2_{\tilde{l}LR}\\ M^2_{\tilde{l}RL} & M^2_{\tilde{l}RR}\end{pmatrix}, \tag{30}$$

where the submatrices are given by

$$M^2_{\tilde{l}LL}=M^2_{\tilde{L}}+M^2_l, \tag{31}$$

$$M^2_{\tilde{l}RR}=M^2_{\tilde{l}^c}+M^2_l, \tag{32}$$

$$M^2_{\tilde{l}LR}=M^{2\dagger}_{\tilde{l}RL}=\langle H_d\rangle A_h+\tan\beta\mu M_l, \tag{33}$$

with $\tan\beta\equiv\langle H_u\rangle/\langle H_d\rangle$. The sneutrino mass matrix is also given by

$$\mathcal{M}^2_{\tilde{\nu}}=M^2_{\tilde{L}}, \tag{34}$$

where the tiny lepton number violating term related to the Majorana neutrino mass matrix is neglected in a good approximation. The flavor changing components are particularly calculated in the leading order of $|f|^2$ as

$$\frac{(M^2_{\tilde{l}LL})_{ij}|_{i\neq j}}{m_0^2} \simeq -3(3+a_0^2)(1+c_{hi}h_i^2 t_G)t_G(f^\dagger f)_{ij}, \tag{35}$$

$$\frac{(M^2_{\tilde{l}RR})_{ij}|_{i\neq j}}{m_0^2} \simeq -6(3+a_0^2)c_{hi}h_i^2 t_G^2(f^\dagger f)_{ij}, \tag{36}$$

$$\frac{(M^2_{\tilde{l}LR})_{ij}|_{i\neq j}}{m_0^2} \simeq -\frac{9}{2}a_0\frac{m_{l_i}}{m_0}(1+3c_{hi}h_i^2 t_G)t_G(f^\dagger f)_{ij}, \tag{37}$$

$$\frac{M^2_{\tilde{\nu}}|_{i\neq j}}{m_0^2} \simeq -3(3+a_0^2)(1+c_{hi}h_i^2 t_G)t_G(f^\dagger f)_{ij}, \tag{38}$$

where the values of f and h couplings are taken at M_W. It is noticed that these leading contributions of flavor violation are determined essentially by $t_G(f^\dagger f)_{ij}$ $(i\neq j)$ [17] with the significant log factor $t_G \sim 0.4$ in the present scheme of $M_\Delta \sim 10^3$ GeV.

IV. CHARGED LEPTON PROCESSES

We investigate the charged lepton processes in order, to which the supersymmetric Higgs triplets in the TeV region may provide significant contributions. Such effects are expected to show the evidence of Higgs triplets particularly related to the neutrino masses and mixings.

A. $\mu\rightarrow 3e$ and $\tau\rightarrow 3\mu$

The leading contribution to the $\mu\rightarrow 3e$ decay is provided at the tree level mediated by the Higgs triplet. The supersymmetric contributions, on the other hand, appear at the one-loop level through the flavor violation in the slepton sectors [21]. They are, however, negligible compared to the tree-level contribution for $M_\Delta \sim 10^3$ GeV. The branching ratio is calculated [22] as

$$\mathrm{Br}(\mu\rightarrow 3e) = \frac{|f^*_{ee}f_{\mu e}|^2}{8g^4}\left(\frac{M_W}{m_\Delta}\right)^4 = 3\times 10^{-13}\left(\frac{1\ \mathrm{TeV}}{m_\Delta}\right)^4\left(\frac{|I_{\mu\rightarrow 3e}|}{0.01}\right)^2\left(\frac{|f|}{0.1}\right)^4, \tag{39}$$

where

$$f^*_{ee}f_{\mu e} \equiv I_{\mu\rightarrow 3e}|f|^2, \tag{40}$$

and the mass of scalar Higgs triplet is given including the contribution of soft supersymmetry breaking $(c_\Delta \sim 1)$ by

$$m_\Delta = \sqrt{M_\Delta^2 + c_\Delta m_0^2}. \tag{41}$$

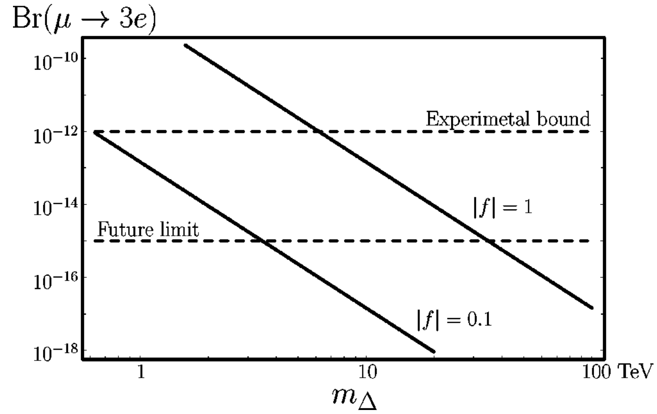

FIG. 1. A typical estimate of the branching ratio of $\mu\rightarrow 3e$ is shown depending on the Higgs triplet mass m_Δ for $|f|=1$ and $|f|=0.1$.

The experimental bound is, on the other hand, placed as $\mathrm{Br}(\mu\rightarrow 3e) < 1.0\times 10^{-12}$ [23]. The flavor changing factor $|I_{\mu\rightarrow 3e}| = 0.01$ is taken in Eq. (39) as a reference value. Its value is evaluated precisely from Eq. (17) with the neutrino masses and MNS matrix U, which are inferred from the data of neutrino experiments [13–16]. Numerically, we have $|I_{\mu\rightarrow 3e}| \lesssim 0.03$ (HI), $\lesssim 0.06$ (DG), and $\lesssim 0.2$ (IH), respectively, for the hierarchical (HI) case $m_1 \ll m_2 \ll m_3$, the degenerate (DG) case $m_1 \sim m_2 \sim m_3$, and the inverted-hierarchical (IH) case $m_1 \sim m_2 \gg m_3$.

A detailed estimate of $\mathrm{Br}(\mu\rightarrow 3e)$ is presented in Fig. 1 depending on the Higgs triplet mass m_Δ. Typical values of the neutrino masses and mixings are taken in the HI case as

$$(m_1, m_2, m_3) = (10^{-3}\ \mathrm{eV}, 8\times 10^{-3}\ \mathrm{eV}, 5\times 10^{-2}\ \mathrm{eV}),$$

$$(\sin\theta_{12}, \sin\theta_{23}, \sin\theta_{13}) = (1/2, 1/\sqrt{2}, 0.1),$$

and the zero CP violating phases, which provides

$$I_{\mu\rightarrow 3e} = 0.7\times 10^{-2}.$$

The upper and lower solid lines represent the results for $|f| = 1$ and $|f| = 0.1$, respectively. The present experimental bound 1.0×10^{-12} and a future sensitivity $\sim 10^{-15}$ achieved by proposed experiments [24] are also shown with the upper and lower dashed lines, respectively. It is interesting here that through the $\mu\rightarrow 3e$ decay the evidence of Higgs triplets may be seen up to the mass $M_\Delta \simeq m_\Delta = 100$ TeV for $|f| \sim 1$. This will be promising especially for obtaining the experimental evidence of leptogenesis in TeV region with the supersymmetric Higgs triplets. On the other hand, as discussed later, the Higgs triplet contributions to the $\mu\rightarrow e\gamma$ decay are significant even for $M_\Delta \gg 100$ TeV through renormalization effects.

The branching ratio of $\tau\rightarrow 3\mu$ is also estimated as

$$\mathrm{Br}(\tau\rightarrow 3\mu) = 2\times 10^{-11}\left(\frac{1\ \mathrm{TeV}}{m_\Delta}\right)^4\left(\frac{|I_{\tau\rightarrow 3\mu}|}{0.2}\right)^2\left(\frac{|f|}{0.1}\right)^4, \tag{42}$$

where

$$f_{\mu\mu}^{*}f_{\tau\mu}\equiv I_{\tau\rightarrow 3\mu}|f|^{2}. \tag{43}$$

We have numerically $|I_{\tau\rightarrow 3\mu}|\simeq 0.1$–$0.3$ (HI), $\simeq 0.1$–0.2 (DG), and $\simeq 0.1$–0.3 (IH), respectively. This Higgs triplet contribution to the $\tau\rightarrow 3\mu$ decay is far below the experimental bound $\mathrm{Br}(\tau\rightarrow 3\mu)<3.8\times 10^{-7}$ [25] for $m_{\Delta}\sim 1$ TeV and $|f|\lesssim 0.1$. Similar estimates are made for the leptonic three-body decays, $\tau\rightarrow\overline{e}\mu\mu$, and so on [11].

B. $\mu\rightarrow e\gamma$ and $\tau\rightarrow\mu\gamma$

The flavor changing radiative decays such as $\mu\rightarrow e\gamma$ and $\tau\rightarrow\mu\gamma$ are induced by the one-loop diagrams. In the Higgs triplet model, the nonsupersymmetric contribution is given by the L-Δ loop, which is almost independent of the mass of the internal lepton for $m_{l}\ll m_{\Delta}$ [10,11]. The supersymmetric partner of this contribution is given by the $\tilde{L}$-$\tilde{\Delta}$ loop. The flavor violation appears even in the internal slepton line through the renormalization effects, though its contribution is sufficiently small for $|f|\lesssim 10^{-1}$. The flavor violation in the slepton mass matrices also provides the supersymmetric contributions of the $\tilde{l}$-$\tilde{\chi}^{0}$ (neutralino) loop and the $\tilde{\nu}$-$\tilde{\chi}^{-}$ (chargino) loop, as in the minimal supersymmetric standard model [21]. For the case of very large Higgs triplet mass such as $M_{\Delta}\sim 10^{11}$–10^{14} GeV, the $\tilde{l}$-$\tilde{\chi}^{0}$ and $\tilde{\nu}$-$\tilde{\chi}^{-}$ contributions are dominant [17], while the L-Δ and $\tilde{L}$-$\tilde{\Delta}$ contributions are negligible due to the suppression factor $(m_{0}/M_{\Delta})^{2}$. On the other hand, for the case of $M_{\Delta}\sim m_{0}\sim 1$ TeV, as motivated for the direct detection of Higgs triplet, these contributions may be comparable.

In this interesting case of $M_{\Delta}\sim m_{0}\sim 1$ TeV, we investigate the charged lepton radiative decays and their intimate relation to the leptonic three-body decays of charged leptons through the neutrino mass matrix proportional to the f coupling. In particular, the supersymmetric contributions of the $\tilde{l}$-$\tilde{\chi}^{0}$ and $\tilde{\nu}$-$\tilde{\chi}^{-}$ loops may become most significant for a certain range of the model parameters, while those of the $\tilde{L}$-$\tilde{\Delta}$ loop are comparable to or even larger than their nonsupersymmetric partners of the L-Δ loop for $M_{\tilde{\Delta}}=M_{\Delta}<m_{\Delta}$. Then, the relations between the decays $\mu\rightarrow 3e$, etc., and the decays $\mu\rightarrow e\gamma$, etc., as found in the nonsupersymmetric case [10,11], may be modified to some extent, since the radiative decays are enhanced due to the supersymmetric contributions [17] with the log factor $t_{G}\sim 0.4$.

We now estimate the branching ratio of the $\mu\rightarrow e\gamma$ decay. The decay amplitude is generally given by

$$T(\mu\rightarrow e\gamma)=e\epsilon^{\alpha *}\overline{u_{e}}[i\sigma_{\alpha\beta}q^{\beta}(A_{L}P_{L}+A_{R}P_{R})]u_{\mu}. \tag{44}$$

Then, the decay rate is given by

$$\Gamma(\mu\rightarrow e\gamma)=\frac{e^{2}}{16\pi}m_{\mu}^{3}(|A_{L}|^{2}+|A_{R}|^{2}), \tag{45}$$

and the branching ratio is calculated by

$$\mathrm{Br}(\mu\rightarrow e\gamma)=\frac{\Gamma(\mu\rightarrow e\gamma)}{G_{F}^{2}m_{\mu}^{5}/192\pi^{3}}. \tag{46}$$

The left-handed and right-handed decay amplitudes are calculated in the leading order by combining the one-loop contributions:

$$A_{L,R}=A_{L,R}^{\tilde{\chi}^{0}}+A_{L,R}^{\tilde{\chi}^{-}}+A_{L,R}^{\Delta}+A_{L,R}^{\tilde{\Delta}}. \tag{47}$$

The formulas for calculating the contributions $A_{L,R}^{\tilde{\chi}^{0}}$ and $A_{L,R}^{\tilde{\chi}^{-}}$ of the neutralinos and charginos are presented in the literature [21]. The contributions $A_{L,R}^{\Delta}$ and $A_{L,R}^{\tilde{\Delta}}$ of the supersymmetric Higgs triplets are calculated in the Appendix. Then, the decay amplitudes are given specifically as

$$A_{L}=\frac{m_{\mu}}{32\pi^{2}}I_{\mu\rightarrow e\gamma}|f|^{2}\left[\frac{G_{L}^{\tilde{\chi}}}{m_{0}^{2}}+\frac{G_{L}^{\Delta}}{m_{\Delta}^{2}}+\frac{G_{L}^{\tilde{\Delta}}}{M_{\Delta}^{2}}\right], \tag{48}$$

$$A_{R}=\frac{m_{\mu}}{32\pi^{2}}I_{\mu\rightarrow e\gamma}|f|^{2}\left[\frac{G_{R}^{\tilde{\chi}}}{m_{0}^{2}}+\frac{G_{R}^{\Delta}}{m_{\Delta}^{2}}+\frac{G_{R}^{\tilde{\Delta}}}{M_{\Delta}^{2}}\right], \tag{49}$$

where

$$\sum_{k}f_{ek}^{*}f_{\mu k}\equiv I_{\mu\rightarrow e\gamma}|f|^{2}. \tag{50}$$

These leading contributions to the decay amplitudes are proportional to the flavor changing factor $(f^{\dagger}f)_{e\mu}=\Sigma_{k}f_{ek}^{*}f_{\mu k}$ $(f=f^{\mathrm{T}})$, as seen in Eq. (50). This is realized for the $\tilde{l}$-$\tilde{\chi}^{0}$ and $\tilde{\nu}$-$\tilde{\chi}^{-}$ loops by using the mass-insertion method with the flavor changing elements of slepton mass matrices in Eqs. (35)–(38). As for the L-Δ and $\tilde{L}$-$\tilde{\Delta}$ loops, the flavor dependence of the masses of intermediate states can be neglected in a good approximation. Then, the factor $(f^{\dagger}f)_{e\mu}$ is extracted from the two vertices in the loop diagram (see also the Appendix for details). It should, however, be remarked that the contribution of $k=\tau$ in Eq. (50) may be modified to some extent for $\tan\beta\gtrsim 30$. This is because the renormalization effects on the $\tilde{\tau}$ and $\tilde{\nu}_{\tau}$ masses by the Yukawa coupling h_{τ} become significant especially for the $\tilde{\tau}$-$\tilde{\Delta}^{++}$ and $\tilde{\nu}_{\tau}$-$\tilde{\Delta}^{+}$ loops. Furthermore, the renormalization effects may modify significantly the flavor structure of these amplitudes for the large f coupling as $|f|\sim 0.5$–1. At present, there is no strong motivation to pursue such special cases.

As a typical example, the factors $G_{L,R}^{\tilde{\chi}}$, $G_{L,R}^{\Delta}$ and $G_{L,R}^{\tilde{\Delta}}$ are evaluated numerically as

$$G_{L}^{\tilde{\chi}}=0.20,\ \ G_{L}^{\Delta}=0.8\times 10^{-3},\ \ G_{L}^{\tilde{\Delta}}=1.0\times 10^{-3},$$

$$G_{R}^{\tilde{\chi}}=1.35,\ \ G_{R}^{\Delta}=0.17,\ \ G_{R}^{\tilde{\Delta}}=0.21 \tag{51}$$

by taking the parameters as $M_{\tilde{\Delta}}=M_{\Delta}=700$ GeV, $m_{\Delta}=1000$ GeV, $m_{0}=700$ GeV, $a_{0}=1$, $\tan\beta=3$, $\mu=1000$ GeV, $M_{1}=300$ GeV, and $M_{2}=600$ GeV [M_{1} and M_{2} are the gaugino masses of $\mathrm{U}(1)_{Y}$ and $\mathrm{SU}(2)_{L}$, respectively]. Here, $G_R^{\tilde{\chi}}$ is somewhat enhanced by $\tan\beta$ coming from the $\tilde{\mu}_L$-$\tilde{\mu}_R$ flip with $(M_{\tilde{l}LR}^2)_{\mu\mu}$ and the μ_R-$\tilde{\mu}_L$-$\tilde{H}_d^0$ vertex [17,21]. The small ratio $G_L^{\Delta}/G_R^{\Delta}=G_L^{\tilde{\Delta}}/G_R^{\tilde{\Delta}}=m_e/m_\mu$ is attributed to the chirality flip of the external charged leptons. Then, the branching ratio is estimated as

$$\mathrm{Br}(\mu\to e\gamma)=7\times 10^{-12}\left(\frac{G}{3}\right)^2\left(\frac{1\ \ \mathrm{TeV}}{m_\Delta}\right)^4\left(\frac{|I_{\mu\to e\gamma}|}{0.1}\right)^2\left(\frac{|f|}{0.1}\right)^4, \tag{52}$$

which should be compared to the experimental bound $\mathrm{Br}(\mu\to e\gamma)<1.2\times 10^{-11}$ [20]. Here, we take $G=3$ as a reference value for

$$G\equiv\left(\sum_{K=L,R}|r_{\tilde{\chi}}G_K^{\tilde{\chi}}+G_K^{\Delta}+r_{\tilde{\Delta}}G_K^{\tilde{\Delta}}|^2\right)^{1/2} \tag{53}$$

with $r_{\tilde{\chi}}\equiv(m_\Delta/m_0)^2$ and $r_{\tilde{\Delta}}\equiv(m_\Delta/M_\Delta)^2$. This net G factor is actually calculated depending on the various parameters, as seen from Eq. (51). It is usually of O(1) for the reasonable parameter range. The weights of supersymmetric contributions are relatively enhanced in G due to $r_{\tilde{\chi}},r_{\tilde{\Delta}}>1$ for $m_\Delta>m_0$, M_Δ from Eq. (41), compared to the nonsupersymmetric ones. We have also numerically $|I_{\mu\to e\gamma}|\lesssim 0.2$ (HI), $\lesssim 0.1$ (DG), and $\lesssim 0.2$ (IH), respectively. This expected branching ratio $\mathrm{Br}(\mu\to e\gamma)$ really becomes larger by one order or so due to the supersymmetric contributions than that of the nonsupersymmetric case [10,11]. It should also be remarked that the $\mu\to e\gamma$ decay can be a good test to distinguish the supersymmetric Higgs triplets from the nonsupersymmetric ones. This is because in the nonsupersymmetric model the left-handed decay amplitude $A_L=A_L^{\Delta}$ is much smaller than the right-handed one $A_R=A_R^{\Delta}$ due to the suppression with m_e/m_μ.

We can make a similar estimate on the branching ratio of $\tau\to\mu\gamma$ as

$$\mathrm{Br}(\tau\to\mu\gamma)=3\times 10^{-11}\left(\frac{G}{3}\right)^2\left(\frac{1\ \ \mathrm{TeV}}{m_\Delta}\right)^4\left(\frac{|I_{\tau\to\mu\gamma}|}{0.5}\right)^2\left(\frac{|f|}{0.1}\right)^4, \tag{54}$$

where

$$\sum_k f_{\mu k}^* f_{\tau k}\equiv I_{\tau\to\mu\gamma}|f|^2. \tag{55}$$

We have numerically $|I_{\tau\to\mu\gamma}|\simeq 0.4$–$0.5$ (HI), $\simeq 0.1$–0.4 (DG), and $\simeq 0.4$–0.5 (IH), respectively. This Higgs triplet contribution to the $\tau\to\mu\gamma$ decay is much smaller than the experimental bound $\mathrm{Br}(\tau\to\mu\gamma)<3.1\times 10^{-7}$ [26] for $m_\Delta\sim 1$ TeV and $|f|\lesssim 0.1$.

C. Muon anomalous magnetic moment

The contributions of the f coupling to the muon anomalous magnetic moment mainly appear through the Δ-L and $\tilde{\Delta}$-$\tilde{L}$ loops. The magnitude of these contributions are estimated roughly for $M_\Delta\sim m_0$ as

$$|\Delta_f a_\mu|\sim\frac{1}{8\pi^2}\left(\frac{m_\mu}{m_\Delta}\right)^2\sum_k|f_{\mu k}|^2$$
$$\sim 10^{-12}\left(\frac{1\ \ \mathrm{TeV}}{m_\Delta}\right)^2\left(\frac{|f|}{0.1}\right)^2. \tag{56}$$

Hence the contributions of the f coupling to the muon anomalous magnetic moment are found to be harmlessly small.

V. SUMMARY

We have investigated the lepton flavor violating processes such as $\mu\to 3e$ and $\mu\to e\gamma$ with the supersymmetric Higgs triplets in the light of neutrino masses and experimentally verifiable leptogenesis. The Higgs triplet mass M_Δ is expected to be in the range of 1–100 TeV. The branching ratios of these charged lepton decays are evaluated in terms of M_Δ and the coupling $fL\Delta L$ of Higgs triplet Δ with lepton doublet pairs LL, which is proportional to the neutrino mass matrix. They may be reached in the future collider experiments. In particular, the $\mu\to 3e$ decay would be observed indicating the existence of Higgs triplets with $M_\Delta\sim 1$–100 TeV for $|f|\sim 0.1$–1, while $\mathrm{Br}(\mu\to e\gamma)$ can be significant irrespective of M_Δ in the supersymmetric model due to the flavor violation in the slepton mass matrices induced by the renormalization effects.

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APPENDIX: ONE-LOOP CONTRIBUTIONS OF SUPERSYMMETRIC Δ TO $l_j\to l_i+\gamma$

We here present the formulas for calculating the one-loop contributions of supersymmetric Higgs triplets to the decay amplitudes of $l_j\to l_i+\gamma$.

The charged slepton mass eigenstates are determined by diagonalizing the mass matrix $\mathcal{M}_{\tilde{l}}^2$ in Eq. (30) with a unitary matrix $U^{\tilde{l}}$:

$$\tilde{l}_a=U_{ai}^{\tilde{l}}\tilde{l}_{Li}+U_{ai+3}^{\tilde{l}}\tilde{l}_{Ri}(a=1-6), \tag{A1}$$

where $\tilde{l}_L\equiv\tilde{l}$ and $\tilde{l}_R\equiv\tilde{l}^{c*}$. The sneutrino mass eigenstates are determined by diagonalizing the mass matrix $\mathcal{M}_{\tilde{\nu}}^2$ in Eq. (34) with a unitary matrix $U^{\tilde{\nu}}$:

$$\tilde{\nu}_b=U_{bi}^{\tilde{\nu}}\tilde{\nu}_{Li}(b=1-3), \tag{A2}$$

where $\tilde{\nu}_{L_i}\equiv\tilde{\nu}_i$. The interactions of bileptons with scalar Higgs triplet are given from Eq. (3) by

$$\mathcal{L}_{\Delta}=-\frac{1}{\sqrt{2}}f_{ij}\overline{l_i^c}P_L l_j\Delta^{++}-\frac{1}{2}f_{ij}\overline{l_i^c}P_L\nu_j\Delta^{+}-\frac{1}{2}f_{ij}\overline{\nu_i^c}P_L l_j\Delta^{+}+\frac{1}{\sqrt{2}}f_{ij}\overline{\nu_i^c}P_L\nu_j\Delta^{0}+\text{H.c.} \quad (A3)$$

The interactions of bisleptons with Higgsino triplet are given in terms of the mass eigenstates in Eqs. (A1) and (A2) by

$$\mathcal{L}_{\tilde{\Delta}}=-\sqrt{2}\mathcal{F}^{\tilde{l}}_{ia}\overline{l_i^c}P_L\tilde{\Delta}^{++}\tilde{l}_a-\mathcal{F}^{\tilde{\nu}}_{ib}\overline{l_i^c}P_L\tilde{\Delta}^{+}\tilde{\nu}_b-\mathcal{F}^{\tilde{l}}_{ia}\overline{\nu_i^c}P_L\tilde{\Delta}^{+}\tilde{l}_a+\sqrt{2}\mathcal{F}^{\tilde{\nu}}_{ib}\overline{\nu_i^c}P_L\tilde{\Delta}^{0}\tilde{\nu}_b+\text{H.c.}, \quad (A4)$$

where

$$\mathcal{F}^{\tilde{l}}_{ia}=(fU^{\tilde{l}\dagger})_{ia}\,,\mathcal{F}^{\tilde{\nu}}_{ib}=(fU^{\tilde{\nu}\dagger})_{ib}\,. \quad (A5)$$

The contributions of L-Δ loops are calculated by using the interactions in Eq. (A3) as

$$A_R^{\Delta}=\frac{1}{32\pi^2}\frac{m_{l_j}}{m_\Delta^2}\sum_k f_{ik}^{*}f_{jk}[F_1(0)+4F_1(x_k)-2F_2(x_k)], \quad (A6)$$

$$A_L^{\Delta}=(m_{l_i}/m_{l_j})A_R^{\Delta}, \quad (A7)$$

where $x_k\equiv(m_{l_k}/m_\Delta)^2$ with the scalar Higgs triplet mass m_Δ in Eq. (41). The functions F_1 and F_2 are given by

$$F_1(x)=\frac{1-6x+3x^2+2x^3-6x^2\ln x}{6(1-x)^4}, \quad (A8)$$

$$F_2(x)=\frac{2+3x-6x^2+x^3+6x\ln x}{6(1-x)^4}. \quad (A9)$$

The contributions of $\tilde{L}$-$\tilde{\Delta}$ loops are also calculated by using the interactions in Eq. (A4) as

$$A_R^{\tilde{\Delta}}=\frac{1}{32\pi^2}\frac{m_{l_j}}{M_\Delta^2}\left\{\sum_a \mathcal{F}^{\tilde{l}*}_{ia}\mathcal{F}^{\tilde{l}}_{ja}[-2F_1(x_a)+4F_2(x_a)]+\sum_b \mathcal{F}^{\tilde{\nu}*}_{ib}\mathcal{F}^{\tilde{\nu}}_{jb}F_2(x_b)\right\}, \quad (A10)$$

$$A_L^{\tilde{\Delta}}=(m_{l_i}/m_{l_j})A_R^{\tilde{\Delta}}, \quad (A11)$$

where $x_a\equiv(M_{\tilde{l}_a}/M_\Delta)^2$ and $x_b\equiv(M_{\tilde{\nu}_b}/M_\Delta)^2$, and the Higgsino triplet mass is given by $M_{\tilde{\Delta}}=M_\Delta$.

Here, two remarks should be made. (i) The suppression factor $m_{l_i}/m_{l_j}\ll 1$ appears in the left-handed contributions where the chirality is flipped in the final state l_i. This is due to the fact that only the left-handed lepton doublets participate in the f coupling of bileptons and Higgs triplet. (ii) These amplitudes are approximately proportional to $(f^\dagger f)_{ij}$. In the amplitudes $A^{\Delta}_{L,R}$ we have $F_{1,2}(x_k)\simeq F_{1,2}(0)$ for $x_k\ll 1$, so that the factor $(f^\dagger f)_{ij}=\Sigma_k f^*_{ik}f_{jk}$ $(f=f^{\mathrm{T}})$ is extracted. Similarly, in the amplitudes $A^{\tilde{\Delta}}_{L,R}$ we may neglect the mass differences among the sleptons for small enough $|f|\lesssim 0.1$, so that the factor $(f^\dagger f)_{ij}=\Sigma_a\mathcal{F}^{\tilde{l}*}_{ia}\mathcal{F}^{\tilde{l}}_{ja}=\Sigma_b\mathcal{F}^{\tilde{\nu}*}_{ib}\mathcal{F}^{\tilde{\nu}}_{jb}$ is extracted again with unitarity of $U^{\tilde{l}}$ and $U^{\tilde{\nu}}$. In other words, the flavor mixing of the intermediate sleptons is actually ineffective for $A^{\tilde{\Delta}}_{L,R}$ in the leading order of $|f|^2$.

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