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AUTHOR(S):

Matsuo, T; Tadamatsu, A; Shimasaki, M

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3-D Magnetohydrodynamic Field Computation of Supersonic Duct Flow of Weakly Ionized Plasma

Tetsuji Matsuo, *Member, IEEE*, Akihiro Tadamatsu, and Masaaki Shimasaki, *Member, IEEE*

Abstract—Supersonic magnetohydrodynamic field analysis is carried out for energy conversion using a weakly ionized plasma. The magnetic field induced by a generated electric current is taken into account. A three-dimensional simulation shows that the Lorentz force induces a shock wave and boundary-layer separation in the plasma duct flow.

Index Terms—Boundary-layer separation, induced magnetic field, magnetohydrodynamics (MHD), shock wave.

I. INTRODUCTION

HIGH-SPEED flows of weakly ionized plasma have a wide range of application. These include magnetohydrodynamic (MHD) generator/accelerators and MHD hypersonic flow control. The analysis of such high-speed plasma flows results in the coupled problem of electromagnetics with high-power density and gasdynamics with shock waves and boundary-layer separation.

Analyses of MHD generators usually ignore the magnetic field induced by the generated electric current because the magnetic Reynolds number of typical MHD generators is much less than one. Some MHD generators, however, have relatively large induced magnetic fields because of their high-electric power density, where the magnetic Reynolds number becomes of the order of 0.1. The application to MHD hypersonic flow control also requires analysis taking into account the induced magnetic field.

This paper carries out a three-dimensional (3-D) MHD field analysis taking into account the induced magnetic field for a supersonic plasma flow with a power density of more than 300 MW/m³.

II. ANALYZED MHD FLOW

A plasma flow of one of the generator ducts of the Pamir-3U MHD power system [1], [2] is analyzed in this paper. Pamir-3U is a portable, self-excited pulsed-MHD power system with a maximum power output of 15 MW.

Fig. 1 shows the generator duct, which has a pair of continuous electrodes (a cathode and an anode). The x , y , and z directions in the present analysis represent the directions of the core flow, the Faraday current induced and the applied magnetic field, respectively. The half domain of the duct ($z \geq 0$)

Fig. 1. Generator duct (unit: millimeters).

is analyzed on account of its symmetry. The generator duct is nonuniformly divided into $244 \times 40 \times 16$ cells.

The working fluid in the generator duct is a weakly ionized plasma of solid-propellant combustion gas with cesium seed. The applied magnetic flux density is 3–5 T. The electrical conductivity of the gas is 70–120 S/m with a Hall parameter of 0.4–1. The gas velocity is more than 1 km/s.

The weakly ionized plasma in the generator duct is analyzed under the following assumptions: 1) the induced magnetic field is smaller than the applied magnetic field since the magnetic Reynolds number is less than one; 2) the displacement current is much smaller than the conduction current and is negligible; and 3) the time variation in the magnetic flux density is negligible.

III. METHOD OF MHD FIELD ANALYSIS

A. Electromagnetic Field Analysis

The electromagnetic field in the plasma flow is governed by

$$\text{rot} \left(\frac{\mathbf{B}}{\mu} \right) = \mathbf{J}, \quad \text{div} \mathbf{B} = 0 \quad (1)$$

$$\text{rot} \mathbf{E} = 0, \quad \text{div} \mathbf{J} = 0 \quad (2)$$

$$\mathbf{J} = \Sigma(\mathbf{B})(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \quad (3)$$

where $\mathbf{B} = (B_x, B_y, B_z)$ is the magnetic flux density, $\mathbf{J} = (J_x, J_y, J_z)$ is the electric current density, $\mathbf{E} = (E_x, E_y, E_z)$ is the electric field, $\mathbf{u} = (u_x, u_y, u_z)$ is the gas velocity, and $\Sigma(\mathbf{B})$ is the conductivity tensor given by

$$\Sigma = \sigma \begin{bmatrix} 1 & mB_z & -mB_y \\ -mB_z & 1 & mB_x \\ mB_y & -mB_x & 1 \end{bmatrix}^{-1} \quad (4)$$

where σ is the electrical conductivity, $m = \beta / \|\mathbf{B}\|$, and β is the Hall parameter. The dependences of σ and β on the gas temperature and pressure are given in [5]. The permeability μ is set at $4\pi \times 10^{-7}$ (H/m).

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T. Matsuo and M. Shimasaki are with the Graduate School of Engineering, Kyoto University, Kyoto 606-8501, Japan (e-mail: tmatsuo@kuee.kyoto-u.ac.jp; shimasaki@kuee.kyoto-u.ac.jp).

A. Tadamatsu was with the Graduate School of Engineering, Kyoto University, Kyoto 606-8501, Japan. He is now with NTT DoCoMo, Inc., Tokyo 105-0001, Japan.

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The magnetic flux density $\mathbf{B}$ consists of the applied magnetic flux density ($\mathbf{B}_a$) and the induced magnetic flux density ($\mathbf{B}_i$). It is assumed that $\mathbf{B}_a$ is known and that $\|\mathbf{B}_a\| \gg \|\mathbf{B}_i\|$. The fields $\mathbf{B}_a$ and $\mathbf{B}_i$ satisfy (5) and (6) in the generator duct, respectively.

$$\text{rot } \mathbf{B}_a = 0 \quad (5)$$

$$\text{rot } \mathbf{B}_i = \mu \mathbf{J} \quad (6)$$

It is assumed that the unknown field $\mathbf{B}_i$ is approximated by $\mathbf{B}_{i0}$. First, by setting $\mathbf{B} = \mathbf{B}_0 = \mathbf{B}_a + \mathbf{B}_{i0}$, (2) and (3) lead to

$$\text{div } \mathbf{J}_0 = \text{div} \{ \Sigma(\mathbf{B}_0) (-\text{grad } \phi_0 + \mathbf{u} \times \mathbf{B}_0) \} = 0 \quad (7)$$

where ϕ_0 is the electric potential satisfying

$$-\text{grad } \phi_0 = \mathbf{E}_0 \quad (8)$$

and $\mathbf{J}_0$ and $\mathbf{E}_0$ are $\mathbf{J}$ and $\mathbf{E}$ when $\mathbf{B} = \mathbf{B}_0$, respectively. It is assumed that $\mathbf{u}$ is known from the gasdynamical analysis. Equation (7) is solved for ϕ_0 by the Galerkin finite-element method (FEM) to obtain $\mathbf{J}_0$ and $\mathbf{E}_0$ from (3) and (8).

Next, $\mathbf{J}_0$ is used to determine $\mathbf{B}_i$ from (6) as

$$\text{rot}(\text{rot } \mathbf{A}_i) = \mu \mathbf{J}_0 \quad (9)$$

where $\mathbf{A}_i$ is the magnetic vector potential that satisfies

$$\text{rot } \mathbf{A}_i = \mathbf{B}_i. \quad (10)$$

The solution to (9) for $\mathbf{A}_i$ by the Galerkin FEM gives a more accurate $\mathbf{B}_i$.

After correcting $\mathbf{B}(= \mathbf{B}_a + \mathbf{B}_i)$, $\mathbf{E}$ and $\mathbf{J}$ are recalculated from (11)

$$\begin{aligned} \text{div } \mathbf{J} = \text{div} \left[\Sigma(\mathbf{B}_a + \mathbf{B}_i) \right. \\ \left. \cdot \{ -\text{grad } \phi + \mathbf{u} \times (\mathbf{B}_a + \mathbf{B}_i) \} \right] = 0 \end{aligned} \quad (11)$$

where ϕ satisfies

$$-\text{grad } \phi = \mathbf{E}. \quad (12)$$

The solution to (11) for ϕ by the Galerkin FEM gives $\mathbf{J}$ and $\mathbf{E}$ from (3) and (12).

B. Gasdynamical Analysis

The basic equations used for gasdynamics are the time-dependent 3-D compressible Navier–Stokes equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (13)$$

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \tau + \mathbf{J} \times \mathbf{B} \quad (14)$$

$$\frac{\partial \varepsilon}{\partial t} + \nabla \cdot \{ (\varepsilon + p) \mathbf{u} \} = \nabla \cdot (\tau \mathbf{u} - \mathbf{q}) + \mathbf{J} \cdot \mathbf{E} \quad (15)$$

where ρ is the mass density, $\mathbf{u}$ is the velocity, p is the pressure, $\varepsilon = \rho(e_0 + \|\mathbf{u}\|^2/2)$, where e_0 is the gas internal energy; τ is the shear stress tensor and $\mathbf{q}$ is the heat flux. The mixing-length turbulence model [3], [4] is used to describe the turbulent viscosity. The thermodynamical properties of the working fluid are given by [5].

The Navier–Stokes equations (13)–(15) are solved by the Harten–Yee upwind explicit total variation diminishing (TVD) scheme [6].

The multitime-step method [7] is applied to the gasdynamical analysis in order to give efficient time steps. The generator duct

Fig. 2. Distribution of B_z ($z = 0$); (a) along the channel center ($y = 0$) and (b) along the anode electrode.

is divided into three subdomains according to grid size, which correspond to the 1) boundary layer, 2) core-flow region, and 3) intermediate region between 1) and 2). These three subdomains have the time steps of 31.25, 125, and 500 ns, respectively.

The terms $\mathbf{J} \times \mathbf{B}$ and $\mathbf{J} \cdot \mathbf{E}$ in (14) and (15) are determined from the electromagnetic field analysis as described in the preceding subsection. The electromagnetic field is updated every 1 μs , where $\mathbf{B}_{i0}$ is set at the value of $\mathbf{B}_i$ at the previous time step.

IV. SIMULATION RESULTS

A. Simulation of Maximum Output Operation

First, an MHD field is analyzed to reconstruct the experimental results at hot-fire test no. 5 that achieved the maximum power output of Pamir-3U [2]. The inlet stagnation pressure and temperature are set at 46 atm and 3880 K, respectively. The channel voltage is 504 V. The applied magnetic flux density has the distribution of B_z shown by the dotted lines in Fig. 2. It is assumed that the electrode currents are taken through a pair of current leads along the y direction from the centers of the electrodes.

Figs. 2, 3, 4, and 5 depict distributions of steady-state MHD field in the power-generating region ($0 < x < 1$ m). Fig. 2 shows the distribution of B_z along the flow direction at $z = 0$ m. The solid lines in Fig. 2(a) and (b) show the distributions along the channel center ($y = 0$ m) and along the anode electrode, respectively. Figs. 3–5 depicts the distributions of electric current density, gas velocity, and static pressure, respectively. The channel current computed is 19.5 kA, which roughly agrees with the experimental data (20.8 kA).

Fig. 3. Distributions of electric current density—channel voltage 504 V. (a) Distribution of (J_x, J_y) at $z = 0$ m. (b) Distribution of (J_y, J_z) at $x = 0.32$ m.

Fig. 2(b) shows that the induced magnetic field becomes about 10% of the applied field near the electrode center. This is because the electrode current concentrates at the electrode center in order to be taken out through the current lead along the y direction.

Fig. 3(a) shows that not only a Faraday current (J_y) but also a Hall current (J_x) is generated, which is due to the Hall electric field (E_x) induced by J_y . A reverse current flow toward the anode electrode is seen along the insulator wall ($z = 0.079$ m) in Fig. 3(b) because the electromotive force ($\mathbf{u} \times \mathbf{B}$) becomes small in the boundary-layer region.

Fig. 4(a) depicts that the flow velocity distribution is almost symmetric along the Faraday direction. Fig. 4(b) shows that the distribution of u_y and u_z is also almost symmetric in the upstream region, whereas the distribution becomes asymmetric in the downstream region, as shown in Fig. 4(c). The flow toward both electrode walls in Fig. 4(b) is caused by the duct shape with divergent walls. Fig. 4(c) shows that the plasma flow toward the cathode electrode occurs in the core-flow region (near $z = 0$ m), whereas the flow toward the anode electrode is seen along the insulator wall. This asymmetric flow (secondary flow) is caused by the Hall current of which the Lorentz force pushes the core flow toward the cathode side.

The flow pushed toward the cathode side results in a higher pressure in the cathode side than in the anode side, as shown in Fig. 5. The pressure fluctuation in Fig. 5 is caused by an oblique shock wave due to the change of the duct angle at $x = 0$ m.

B. MHD Field With Excessive Electrode Current

Experimental and numerical studies [2], [5] have shown that the excessive channel current reduces the power output because of the boundary-layer separation. Next, the channel voltage is decreased to 200 V in order to examine the effect of the large Lorentz force due to the large Faraday current.

Figs. 6–8 depict an obtained steady-state MHD field. A comparison of Fig. 6(a) with Fig. 3(a) shows that the decrease in the

Fig. 4. Velocity distributions—channel voltage 504 V. (a) x - y distribution of u_x at $z = 0$ m. (b) Distribution of (u_y, u_z) at $x = 0.32$ m. (c) Distribution of (u_y, u_z) at $x = 0.96$ m.

Fig. 5. Distribution of static pressure at $z = 0$ m—channel voltage 504 V.

channel voltage results in an increase in the Faraday current. A reverse current flow is seen near the insulator wall in Fig. 6(b). The sharp deceleration of the core-flow and the negative flow velocity in Fig. 7(a) depict, respectively, the shock wave and the boundary-layer separation, which are induced by the Lorentz force due to the large Faraday current. Fig. 7(b) shows that the boundary-layer separation induces a complex secondary flow, which has not been revealed by two-dimensional analyses [5], [7]. The sharp increase in static pressure in Fig. 8 is caused by the shock wave. The MHD field in Figs. 6–8 is asymmetric in the Faraday direction. This asymmetry is caused by the Hall current of which the Lorentz force pushes the working fluid toward the cathode side.

Fig. 6. Distributions of electric current density—channel voltage 200 V. (a) Distribution of (J_x, J_y) at $z = 0$ m. (b) Distribution of (J_y, J_z) at $x = 0.32$ m.

Fig. 7. Velocity distributions—channel voltage 200 V. (a) x - y distribution of u_x at $z = 0$ m. (b) Distribution of (u_y, u_z) at $x = 0.32$ m.

Fig. 8. Distribution of static pressure at $z = 0$ m—channel voltage 200 V.

The dash-dotted lines in Fig. 2 show the distribution of B_z , which shows that the induced magnetic field increases with the channel current.

V. CONCLUSION

A 3-D MHD field analysis taking into account the induced magnetic field is carried out for a supersonic plasma duct flow. The analysis reveals that the Lorentz force induces an asymmetric shock wave and boundary-layer separation in the plasma flow.

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