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Representation of AC Hysteretic Characteristics of Silicon Steel Sheet Using Simple Excess Eddy-Current Loss Approximation

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A stop model is combined with a one-dimensional finite-difference method or a homogenization method to represent ac hysteretic characteristics of a silicon steel sheet. Eddy-current analysis without excess eddy-current loss fails to give an accurate eddy-current field. Representation of ac $B$–$H$ loops is improved by increased conductivity that is obtained from an excess loss evaluation by the Pry and Bean model.

Index Terms—AC hysteresis, anomaly factor, excess eddy-current loss, homogenization, Pry and Bean model, stop model.

I. INTRODUCTION

GRAIN-ORIENTED silicon steel sheets are widely used as a core material, not only for transformers, but also for segmented core motors. The grain-oriented steel sheets have a large excess (or anomaly) eddy-current loss [1] because of their relatively large magnetic domain size compared with nonoriented steel sheets [2]. An anomaly factor [2] is often used to evaluate the excess eddy-current loss, which does not require an accurate $B$–$H$ field distribution inside the steel sheet.

Recent development of computer technology enables us to simulate detailed $B$–$H$ fields in electrical machines. However, accurate computation of the $B$–$H$ field is not yet easy because of complex magnetic properties of the steel sheet including the excess eddy-current loss, hysteresis [1], and a vector property [3].

On the other hand, several precise dc hysteresis models have been developed, such as play and stop models [4], [5]. Those models are sufficiently efficient to be applied to magnetic field analyses. For example, a stop model having an input-dependent shape function [6], [7] has been proposed to accurately represent dc hysteretic characteristics of silicon steel sheets.

This study combines the stop model with a one-dimensional (1-D) finite-difference method or a homogenization method to represent ac hysteretic characteristics of a grain-oriented silicon steel sheet. A homogenized model is derived from the Pry and Bean model [8], where the effect of excess eddy-current loss is evaluated simply by an anomaly factor. This simple loss evaluation is also applied to the 1-D finite-difference analysis.

II. EDDY-CURRENT ANALYSIS WITHOUT EXCESS LOSS

This study analyzes an eddy-current field in a grain-oriented silicon steel sheet (JIS: 30P105). This steel sheet has electric conductivity $\sigma$ of $2 \times 10^6$ S/m.

One-dimensional magnetic properties of the silicon steel sheet are measured using a single sheet tester [9] that applies the magnetic flux sinusoidally.

One-dimensional eddy-current fields are described by

$$\frac{\partial^2 H}{\partial y^2} = \sigma \frac{\partial B}{\partial t}$$  \hspace{1cm} (1)

where the $y$ direction is perpendicular to the rolling and transverse directions of the silicon steel sheet.

The finite-difference method is used to simulate the eddy-current field, where a stop model having the input-dependent shape function [6], [7] represents the hysteretic relation $H = H(B)$. The backward Euler time-difference scheme leads to

$$\frac{H(B^n_{i+1}) - 2H(B^n_{i}) + H(B^n_{i-1})}{(\Delta y)^2} = \sigma \frac{B^n_i - B^{n-1}_i}{\Delta t}$$ \hspace{1cm} (2)

where $B^n_i = B^n_{y=i\Delta y,t=n\Delta t}$. The surface magnetic field is given by the measured data as the boundary condition. The computed $B$ is averaged along the $y$ direction for comparison with measured data.

Simulated ac $B$–$H$ loops along the rolling and transverse directions are plotted in Figs. 1 and 2, respectively. The exciting frequencies are set at 20 and 50 Hz. Figs. 1 and 2 show that the simulated loops disagree with measured ones because this analysis neglects the excess eddy-current loss [1] that is attributable to the concentration of eddy currents around the magnetic domain walls.

III. PRY AND BEAN MODEL

Pry and Bean [8] have evaluated the excess eddy-current loss assuming a 1-D periodic magnetic domain structure. Fig. 3 illustrates this structure: $2L$ is the average domain width, $x_W$ is the wall displacement from the demagnetized position, and $d$ is the sheet thickness.

The electric current density $j = (j_x, j_y)$ satisfies the following equations in the analyzed region: $-L \leq x \leq L$ and $-d/2 \leq y \leq d/2$

$$\text{div} \, j = 0, \quad \text{curl} \, j = 0 \ (x \neq x_W).$$  \hspace{1cm} (3)
is the saturation magnetization and \( v_W = \frac{dx_W}{dt} \) is the wall velocity.

Thereby, the instantaneous power loss per unit volume \( P(= \int \int (\mathbf{F}_x^2 + \mathbf{F}_y^2)/\sigma \, dx \, dy / \int \int dx \, dy) \) becomes

\[
P = \frac{16\sigma L d}{\pi^3} \left( \frac{dB}{dt} \right)^2 \sum_{n \text{ odd}} \frac{\cosh \frac{n\pi(L-x_w)}{d} \cosh \frac{n\pi(L-x_w)}{d}}{n^3 \sinh \frac{2n\pi d}{L}}
\]

(6)

where \( B = M_{sEW} / L \) is the average magnetic flux density. When \( L \ll d \), \( P \) is reduced to the classical eddy-current loss \( P_C \) given by

\[
P_C = \frac{\sigma d^2}{12} \left( \frac{dB}{dt} \right)^2.
\]

(7)

On the other hand, when \( B \) is small (\( x_w \ll L \)), \( P \) is approximated as

\[
P = \frac{8\sigma L d}{\pi^3} \left( \frac{dB}{dt} \right)^2 \sum_{n \text{ odd}} \frac{1}{n^3 \coth \frac{n\pi L}{d}} = \frac{k_E d^2}{12} \left( \frac{dB}{dt} \right)^2
\]

(8)

where \( k_E \) is the anomaly factor given by

\[
k_E = \frac{P}{P_C} = \frac{96L}{\pi^3 d} \sum_{n \text{ odd}} \frac{1}{n^3 \coth \frac{n\pi L}{d}}.
\]

(9)

For simplicity, this paper uses this factor to approximate \( P \) as \( P = k_E P_C \), independently of \( B \).

IV. HOMOGENIZED MODEL

By setting

\[
H_E = k_E \frac{\sigma d^2}{12} \frac{dB}{dt}
\]

(10)

the power loss \( P \) is factorized as

\[
P = H_E \frac{dB}{dt}.
\]

(11)

The field \( H_E \) can be regarded as an averaged magnetic field at the domain walls [1], which is required to move the domain walls against the counter force by the eddy current. Accordingly, the applied field \( H_a \) is decomposed into

\[
H_a = H_E + H_{DC}
\]

(12)

where \( H_{DC} \) is a rate-independent field [1] that causes a dc hysteresis.

Consequently, a homogenized model including the excess eddy-current loss is given as

\[
H_a(t) = H_{DC}(B(t)) + k_E \frac{\sigma d^2}{12} \frac{dB}{dt}
\]

(13)

where \( H_{DC}(B) \) represents the dc hysteresis property.

Several homogenized models having similar forms to (13) have already been presented in the literature. For example, homogenization without the excess loss (\( k_E = 1 \)) is given in [10]. A more mathematically general form than (13) is discussed in [11].

AC \( B-H \) loops of the grain-oriented silicon steel sheet are simulated by the homogenized model (13), where the applied
field $H_a(t)$ is given by the measured data for comparison with the finite-difference eddy-current analysis.

Figs. 4 and 5 show simulated $B$–$H$ loops along the rolling direction, where $k_E = 1$ and 2.5 are used for the homogenized model, respectively. The factor $k_E = 2.5$ corresponds to $2L/d ≈ 1.5$. Fig. 4 shows that homogenization without the excess eddy-current loss engenders a large discrepancy between the simulated and measured loops in the same way as in the 1-D finite-difference analysis failed. Fig. 5 shows that the factor $k_E = 2.5$ improves the representation of ac $B$–$H$ loops effectively.

Figs. 6 and 7 show simulated $B$–$H$ loops along the transverse direction, where $k_E = 1$ and 10 (corresponding to $2L/d ≈ 6$) are used, respectively. Fig. 6 shows that the simulation without the excess eddy-current loss fails to yield accurate $B$–$H$ loops. The factor $k_E = 10$ improves the representation of ac $B$–$H$ loops. The anomaly factor required for the transverse direction is much larger than that for the rolling direction because of the large domain size. However, a large discrepancy remains between the simulated and measured loops mainly because the Pry and Bean model cannot describe the $90^\circ$ domain wall motion that dominates the magnetic property along the transverse direction.

V. FINITE-DIFFERENCE ANALYSIS INCLUDING EXCESS LOSS

This paper simply multiplies electric conductivity by the factor $k_E$ for the 1-D finite-difference analysis to approximate the effect of excess eddy-current loss in the same way as in the homogenized model

$$\frac{\partial^2 H}{\partial t^2} = \sigma' \frac{\partial B}{\partial t}, \quad \sigma' = k_E \sigma. \quad (14)$$

Fig. 8 shows simulated $B$–$H$ loops along the rolling direction obtained by analysis using $k_E = 2.5$, where the increased conductivity improves the representation of the $B$–$H$ loops compared with Fig. 1. Table I lists the discrepancy (%) between the simulated and measured power loss $W = \frac{1}{2} \int \dot{H} d \mathcal{L}$ of five $B$–$H$ loops at 50 Hz, where $B_{\text{max}}$ denotes the maximum magnetic field of each measured $B$–$H$ loop. Table I shows that the factor $k_E = 2.5$ greatly improves the power loss evaluation. However, more than 10% of the discrepancy remains between the computed and measured losses because the excess loss evaluation by the constant anomaly factor is too simple to represent an accurate eddy-current field. Table I and the comparison between Figs. 5 and 8 show that the finite-difference model does...
not greatly improve the representation accuracy of the eddy-current field compared with the homogenization model. There is little improvement because the excitation frequency is too low for the skin effect to become sufficiently large.

Fig. 9 shows \( B-H \) loops along the transverse direction simulated with \( k_E = 10 \), where the representation of the \( B-H \) loops is improved compared with Fig. 2. However, a large discrepancy remains between the simulated and measured small \( B-H \) loops. This fact means that the excess loss approximation by the constant anomaly factor is not valid for the transverse direction because of the 90° domain wall motion that the Pry and Bean model cannot describe.

**VI. CONCLUSION**

This paper combines a 1-D finite-difference method or a homogenization method with a stop model to describe ac hysteretic properties of a grain-oriented silicon steel sheet. A homogenized model is derived from the Pry and Bean model, where the excess eddy-current loss is evaluated by an anomaly factor. An increased electric conductivity by the anomaly factor improves the representation of ac \( B-H \) loops by the 1-D finite-difference method as effectively as by the homogenization method.

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**REFERENCES**


