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IRREGULARITIES OF MICROHYPERBOLIC OPERATORS
(マイクロ双曲形作用素の非正則度)

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ABSTRACT. We consider well-posedness of microhyperbolic Cauchy problems in the category of microfunctions which are the singularity spectrums of ultradistributions. To obtain a precise result, we define the irregularities of microhyperbolic operators, and prove the relation between irregularities and ultradistribution orders.

1. Introduction.

It is well-known that a microhyperbolic Cauchy problem is always well-posed in the category of microfunctions (See [2]). Let us consider its well-posedness in the category of microfunctions which are the singularity spectrums of ultradistributions. There is a fundamental result of Kajitani and Wakabayashi for this problem. However, there are some special but important cases for which their theory does not give a satisfactory result. Therefore we want to ameliorate it.

Let (x, ξ) be the variables of $\sqrt{-1}T^*\mathbf{R}^n$, and let $x = (x_1, x') = (x_1, \cdots, x_n)$. Let $x^* \in \sqrt{-1}T^*\mathbf{R}^n$ (resp. $x^{*\prime} \in \sqrt{-1}T^*\mathbf{R}^{n-1}$) be the point defined by $x = 0, \xi = (0, \cdots, 0, \sqrt{-1})$ (resp. $x' = 0, \xi' = (0, \cdots, 0, \sqrt{-1})$). We denote by $\mathcal{B}$, $\mathcal{C}$, $\mathcal{E}$, $\mathcal{O}$ the sheaves of hyperfunctions, microfunctions, microdifferential operators, and holomorphic functions, respectively. For $1 < s < \infty$ we denote Gevrey functions with compact supports by $\mathcal{G}^{\{s\}}_{\mathrm{cpt}}$ and $\mathcal{G}^{(s)}_{\mathrm{cpt}}$:

$$\mathcal{G}^{\{s\}}_{\mathrm{cpt}}(\omega) = \{f(x);\ f \text{ is an infinitely differentiable function with compact support } \subset \omega, \text{ and there exists some } C \text{ such that } |D^\alpha f(x)| \leq C^{|\alpha|+1}\alpha!^s\},$$

$$\mathcal{G}^{(s)}_{\mathrm{cpt}}(\omega) = \{f(x);\ f \text{ is an infinitely differentiable function with compact support } \subset \omega, \text{ and for any } \varepsilon > 0 \text{ there exists some } C_\varepsilon \text{ such that } |D^\alpha f(x)| \leq C_\varepsilon \varepsilon^{|\alpha|}\alpha!^s\}$$

for an open subset ω of $\mathbf{R}^n$. Let

$$\mathcal{G}^{\{s\}\prime}_{\mathrm{cpt}\,0} = \varinjlim_{0\in\omega} \mathcal{G}^{\{s\}\prime}_{\mathrm{cpt}}(\omega),\ \mathcal{G}^{(s)\prime}_{\mathrm{cpt}0} = \varinjlim_{0\in\omega} \mathcal{G}^{(s)\prime}_{\mathrm{cpt}}(\omega)$$

be the set of germs of ultradistributions at the origin.

Let $\mathrm{sp} : \mathcal{B}_{\mathbf{R}^n,0} \longrightarrow \mathcal{C}_{\mathbf{R}^n,x^*}$ and $\mathrm{sp}' : \mathcal{B}_{\mathbf{R}^{n-1},0} \longrightarrow \mathcal{C}_{\mathbf{R}^{n-1},x^{*\prime}}$ be the canonical maps, and let

$$\mathcal{C}^{\{s\}}_{\mathbf{R}^n,x^*} = \mathrm{sp}(\mathcal{G}^{\{s\}\prime}_{\mathrm{cpt}\,\mathbf{R}^n,0}),\ \mathcal{C}^{\{s\}}_{\mathbf{R}^{n-1},x^{*\prime}} = \mathrm{sp}'(\mathcal{G}^{\{s\}\prime}_{\mathrm{cpt}\,\mathbf{R}^{n-1},0})\ (1 \leq s \leq \infty)$$
$$\mathcal{C}^{(s)}_{\mathbf{R}^n,x^*} = \mathrm{sp}(\mathcal{G}^{(s)\prime}_{\mathrm{cpt}\mathbf{R}^n,0}),\ \mathcal{C}^{(s)}_{\mathbf{R}^{n-1},x^{*\prime}} = \mathrm{sp}'(\mathcal{G}^{(s)\prime}_{\mathrm{cpt}\mathbf{R}^{n-1},0})\ (1 < s \leq \infty),$$

which we call microlocal ultradistributions. For the sake of convenience we denote by $\mathcal{G}^{\{1\}\prime}_{\mathrm{cpt}\,\mathbf{R}^n,0}$ the set of hyperfunctions, by $\mathcal{G}^{\{\infty\}\prime}_{\mathrm{cpt}\,\mathbf{R}^n,0}$ and $\mathcal{G}^{(\infty)\prime}_{\mathrm{cpt}\,\mathbf{R}^n,0}$ the set of distributions. Therefore $\mathcal{C}^{\{1\}}_{\mathbf{R}^n,x^*}$ is the usual set of microfunctions.

Let $P(x,D) \in \mathcal{E}_{x^*}$ be written in the form

$$\text{(1)} \qquad \begin{cases} P(x,D) = D_1^m + \sum\limits_{0\leq j\leq m-1} P_j(x,D')D_1^j, \\ \operatorname{ord} P_j \leq m-j \quad (0 \leq j \leq m-1). \end{cases}$$

Here we define $D = \partial/\partial$. We assume that

$$\text{(2)} \qquad \begin{cases} \text{for } 1 \leq j \leq m \text{ there exist } \Lambda_j(x,\xi) = \xi_1 - \lambda_j(x,\xi') \\ \in \mathcal{O}_{\mathbf{C}^{2n},x^*} \text{which are homogeneous in } \xi \text{ of degree } 1 \\ \text{vanishing at } x^*, \text{ and we have } \sigma_m(P) = \prod\limits_{1\leq j\leq m} \lambda_j(x,\xi), \end{cases}$$

where $\sigma_m(P)$ denotes the principal symbol of P. We finally assume that P is microhyperbolic, i.e.,

$$(3) \qquad (x,\xi') \in \mathbf{R}^n \times \sqrt{-1}\mathbf{R}^{n-1} \quad \Longrightarrow \quad \lambda_j(x,\xi') \in \sqrt{-1}\mathbf{R}$$

for $1 \le j \le m$. We do not assume any further conditions explicitly among these characteristic roots.

Let us consider the following Cauchy problem:

$$(4) \qquad P(x,D)u(x) = f(x), \quad D_1^{j-1}u(0,x') = v_j(x') \ (1 \le j \le m).$$

Precisely speaking, in order to ascertain that $D_1^{j-1}u(0,x')$ is well-defined, we must assume that $(0, \pm\sqrt{-1}dx_1) \notin \operatorname{sp} u$. For this purpose it suffices to assume $(0, \pm\sqrt{-1}dx_1) \notin \operatorname{sp} f$. However, we are considering in a neighborhood of x^*, and we may assume that $f \in \mathcal{C}_{\mathbf{R}^n,x^*}$ is extended as a global section of $\mathcal{C}_{\mathbf{R}^n}$, whose support does not contain $(0, \pm\sqrt{-1}dx_1)$. Since the solution $u \in \mathcal{C}_{\mathbf{R}^n,x^*}$ does not depend on such an extention, this is well-defined, and we consider (4) in this sense.

We say that P is $\{s\}$ well-posed if for any $f \in \mathcal{C}^{\{s\}}_{\mathbf{R}^n,x^*}$ and $v_1, \cdots, v_m \in \mathcal{C}^{\{s\}}_{\mathbf{R}^{n-1},x^{*\prime}}$, there exists $u \in \mathcal{C}^{\{s\}}_{\mathbf{R}^n,x^*}$ which satisfies (4) (The solution is always unique). Similarly we define (s) well-posedness. Kajitani and Wakabayashi [1] proved the following

Theorem 1. *If $1 \le s < m/(m-1)$, then P is $\{s\}$ well-posed. If $1 < s \le m/(m-1)$, then P is (s) well-posed.*

To see that we cannot generally improve this result anymore, let us consider the following

Example 1. Let $P = D_1^m - D_n^{m-1}$ and let us consider

$$P(x,D)u(x) = 0, \quad D_1^{j-1}u(0,x') = \delta_{j1}v(x') \ (1 \le j \le m).$$

It is easy to see that the microfunction solution is given by

$$u(x) = \frac{1}{m}\sum_{0 \le j \le m-1} \exp\left(\frac{2\pi\sqrt{-1}j}{m}x_1 D_n^{(m-1)/m}\right) v(x').$$

If we restrict ourselves to microlocal ultradistributions,

$$\exp\left(\frac{2\pi\sqrt{-1}j}{m}x_1 D_n^{(m-1)/m}\right) : \mathcal{C}^{\{s\}}_{\mathbf{R}^n,x^*} \longrightarrow \mathcal{C}^{\{s\}}_{\mathbf{R}^n,x^*}$$

is well-defined if, and only if, $1 \leq s < m/(m-1)$, and Theorem 1 is the best possible result in this sense.

However, this criterion is not satisfactory for the following cases:

Example 2 (regular involutive operators). Let $n \geq 3$ and let $P = D_1(D_1+D_2)+\alpha D_2$, $\alpha \in \mathbf{C}$. The above theorem means that if $1 \leq s < 2$ (resp. $1 < s \leq 2$), then it is $\{s\}$ well-posed (resp. (s) well-posed). However Okada [5] proved that it is $\{\infty\}$ well-posed.

Example 3 (non-involutive operators). Let $P = D_1(D_1 + x_1^q D_n) + \alpha x_1^{q-1} D_n$. It is well-known that P is $\{s\}$ well-posed (resp. (s) well-posed) for any s (Among many papers, we refer to [6]).

Example 4 (constant multiple operators). Assume that $\lambda_1 = \cdots = \lambda_m = 0$ in (1). Komatsu [3] defined the irregularity ι for this case by

$$\iota = \max\{1, \max_{0 \leq j \leq m-1}\{\frac{m-j}{m-j-\operatorname{ord} P_j}\}\}$$

In this case it follows that P is $\{s\}$ well-posed (resp. (s) well-posed) if $1 \leq s < \iota/(\iota-1)$ (resp. $1 < s \leq \iota/(\iota-1)$). We have $\iota \leq m$, and this is a stronger result than the above theorem. Since our theory is strongly influenced by [3], we briefly sketch the discussions there:

(i) A hyperbolic partial differential operator P with constant multiplicity can be written in a special form, which he called De Paris decomposition.

(ii) Rewriting P in such a form, we can define its irregularity ι similarly as above.

(iii) P is $\{s\}$ well-posed if $1 \leq s < \iota/(\iota-1)$.

As we shall see in the next section, we can extend this theory to the general case.

Our aim is to give a criterion which improves Theorem 1, and also contains all these examples. The main result is the following

Theorem 2. *If P satisfies* (1)-(3), *then we can define* $\operatorname{Irr} P$, *which is a rational number satisfying* $1 \leq \operatorname{Irr} P \leq m$. *Furthermore, if* $1 \leq s < \operatorname{Irr} P/(\operatorname{Irr} P - 1)$, *then P is $\{s\}$ well-posed, and if* $1 < s \leq \operatorname{Irr} P/(\operatorname{Irr} P - 1)$, *then P is (s) well-posed.*

Remark. If $\operatorname{Irr} P = 1$, then we define $\operatorname{Irr} P/(\operatorname{Irr} P - 1) = \infty$. Since $1 \le \operatorname{Irr} P \le m$, Theorem 2 is always stronger than (or equivalent to) Theorem 1.

In the above examples, it will turn out that

$$\begin{aligned} &\operatorname{Irr} P = m \quad \text{in Example 1,} \\ &\operatorname{Irr} P = 1 \quad \text{in Examples 2,3,} \\ &\operatorname{Irr} P = \iota (= \text{the above number}) \quad \text{in Example 4,} \end{aligned}$$

which coincides with the well-known results.

2. Lascar decomposition.

We first want to express P in a special form similarly to [3]. If $0 \le q \le m$ we define S_{mq} to be the set of all q-tuples $\mu = (\mu_1, \mu_2, \cdots, \mu_q)$ such that $\mu_1, \mu_2, \cdots, \mu_q \in \{1, 2, \cdots, m\}$ are mutually distinctive. Here we distinguish different arrangements of the same numbers. Although S_{m0} does not make sense, we assume that it consists of only one element, which we denote by $\emptyset$. We define $S = \bigcup_{0 \le q \le m} S_{mq}$, and $S' = \bigcup_{0 \le q \le m-1} S_{mq}$. If $\mu \in S_{mq}$, then we define $|\mu| = q$, and

$$\Lambda^{\mu}(x, D) = \Lambda_{\mu_q}(x, D) \cdots \Lambda_{\mu_1}(x, D).$$

Here $\Lambda_j(x, D)$ denotes the microdifferential operator whose complete symbol is $\Lambda_j(x, \xi)$. We also define $\Lambda^{\emptyset} = 1$. We define $\bar{\mathcal{E}}_{x^*}(j) = \{P \in \mathcal{E}_{x^*};\ [P, x_1] = 0,\ \operatorname{ord} P \le j\}$. By a Lascar decomposition we mean an expression of the following form:

$$(5) \qquad \begin{cases} P(x, D) = \Lambda_m(x, D) \cdots \Lambda_1(x, D) \\ \qquad\qquad + \sum_{\mu \in S'} (x_1^{-m+|\mu|} a_\mu(x, D') + b_\mu(x, D')) \Lambda^{\mu}(x, D), \\ a_\mu(x, D') \in \bar{\mathcal{E}}_{x^*}(0), \quad b_\mu(x, D') \in \bar{\mathcal{E}}_{x^*}(m - |\mu| - 1). \end{cases}$$

Here we consider a negative power of x_1 formally. It is easy to see that an arbitrary operator has an infinitely many Lascar decompositions.

Example 2 $^{\text{bis}}$. Let $n \ge 3$ and let

$$(6) \qquad P = D_1(D_1 + D_2) + \alpha D_2.$$

Here $\Lambda_1 = D_1 + D_2$, $\Lambda_2 = D_1$, and by a Lascar decomposition we mean an expression of the following form:

$$\begin{cases} P = \Lambda_2\Lambda_1 + (x^{-1}a_1 + b_1)\Lambda_1 + (x^{-1}a_2 + b_2)\Lambda_2 + (x^{-2}a_\emptyset + b_\emptyset), \\ \operatorname{ord} a_\mu \le 0, \ \operatorname{ord} b_j \le 0 \ (j = 1, 2), \ \operatorname{ord} b_\emptyset \le 1. \end{cases}$$

Note that (6) is a Lascar decomposition as it stands. In fact we may take $b_\emptyset = \alpha D_2$, and all the other coefficient operators to be 0. We also have another expression:

$$P = \Lambda_2\Lambda_1 + \alpha\Lambda_1 - \alpha\Lambda_2. \tag{7}$$

This means $b_1 = -b_2 = \alpha$, and all the other coefficient operators are 0. We have still other expressions, but they are not important. We shall see that some expressions are heavy, and some expressions are light.

Example 3 $^{\text{bis}}$. Let

$$P = D_1(D_1 + x_1^q D_n) + \alpha x_1^{q-1} D_n. \tag{8}$$

Here $\Lambda_1 = D_1 + x_1^q D_n$, $\Lambda_2 = D_1$. Again this is a Lascar decomposition as it stands. We also have another expression:

$$P = \Lambda_2\Lambda_1 + x_1^{-1}\alpha\Lambda_1 - x_1^{-1}\alpha\Lambda_2. \tag{9}$$

In (5), P is decomposed into three parts. Firstly, $\Lambda_m \cdots \Lambda_1$ denotes the principal part. The lower order terms are formally written in a form like an element of some $\mathcal{E}_{x^*}$-module generated by Λ^μ, $\mu \in S'$. For the sake of convenience, let us call Λ^μ the generator part, and $x_1^{-m+|\mu|}a_\mu + b_\mu$ the coefficient part. Roughly speaking we have

$$\begin{aligned} P(x, D) &= \text{principal part} + \text{lower order part} \\ &= \text{principal part} + (\text{coefficient part} \times \text{generator part}). \end{aligned}$$

If we calculate the amount of the lower order part ($=$ coefficient part $\times$ generator part), we can prove Theorem 1. To the contrary, if we calculate the amount of the coefficient part alone, we can prove Theorem 2. Of course less amount gives a better result, so the latter calculation is preferable. However, this amount depends on Lascar decompositions, and we determine the best one as follows.

For each Lascar decomposition (5) we define

$$\kappa = \max\{1, \max_{\mu \in S'}\{\frac{m - |\mu|}{m - |\mu| - \operatorname{ord} b_\mu}\}\}.$$

We have $1 \leq \kappa \leq m$. This number depends on the expression and we define $\operatorname{irr} P$ as the minimum value of κ among all the Lascar decompositions. Although there are infinitely many decompositions, the minimum value is well-defined.

Example 2 $^{\text{tris}}$. In (6) we have $m = 2$, and $\operatorname{ord} b_\emptyset = 1$, $|\emptyset| = 0$. Therefore we have

$$\kappa = \max\{1, (2-0)/(2-0-1)\} = 2$$

for this decomposition. On the other hand, in (7) we have $\operatorname{ord} b_1 = \operatorname{ord} b_2 = 0$, $|1| = |2| = 1$. Therefore we have

$$\kappa = \max\{1, (2-1)/(2-1-0)\} = 1$$

for this decomposition. This means that (7) is a better expression than (6). We obtain $\operatorname{irr} P = 1$.

We can similarly prove $\operatorname{irr} P = m, 1, \iota$. for Examples 1,3,4, respectively.

Remark. Although we have infinitely many Lascar decompositions, to construct the fundamental solution we can choose the best decomposition, and forget all the other expressions. This means that we only use the minimum value of κ, and we may neglect all the other values. Therefore we define $\operatorname{irr} P = \min\{\kappa;$ Lascar decompositions$\}$.

We next consider permutations in the principal part. Let $\sigma \in S_{mm}$, and let us consider the following expression:

$$(10) \quad \begin{cases} P(x, D) = \Lambda^\sigma(x, D) \\ \qquad\qquad + \sum_{\mu \in S'} (x_1^{-m+|\mu|} a'_\mu(x, D') + b'_\mu(x, D'))\Lambda^\mu(x, D), \\ a'_\mu(x, D') \in \bar{\mathcal{E}}_{x^*}(0), \quad b'_\mu(x, D') \in \bar{\mathcal{E}}_{x^*}(m - |\mu| - 1). \end{cases}$$

We call (10) a Lascar decomposition subordinate to σ. We have infinitely many expressions again, and for each expression we define

$$\kappa' = \max\{1, \max_{\mu \in S'}\{\frac{m-|\mu|}{m-|\mu|-\operatorname{ord} b'_\mu}\}\}.$$

We define

$$\operatorname{irr}_\sigma P = \min\{\kappa'; \text{ Lascar decompositions subordinate to } \sigma\}.$$

Finally we define the irregularity $\operatorname{Irr} P$ of P by

$$\operatorname{Irr} P = \max\{\operatorname{irr}_\sigma P; \ \sigma \in S_{mm}\}.$$

In all the above examples we have $\operatorname{irr} P = \operatorname{irr}_\sigma P = \operatorname{Irr} P$.

Remark. R. Lascar considered an expression of the form (5) in [4]. In his paper he assumed that the characteristic variety of P is regularly involutive, and he assumed that $a_\mu = 0$, $\operatorname{ord} b_\mu \leq 0$. Under these assumptions he proved that the wave front set of the distribution solution of $Pu = 0$ propagates along the integral manifold defined by the characteristic variety.

3. $\operatorname{irr} P$ **and** $\operatorname{Irr} P$**.**

In the previous section we defined the irregularity in three steps. We first calculate κ, next $\operatorname{irr} P$, and finally $\operatorname{Irr} P$. One may think this uncomfortable, and it may be preferable if we can omit the last step. This is possible in two special cases. The first case is the following

Lemma 1. *Assume that*

$$(11) \qquad \{\Lambda_i(x,\xi),\ \Lambda_j(x,\xi)\} \in x_1^{-1}\Lambda_i(x,\xi)\mathcal{O}_{x^*} + x_1^{-1}\Lambda_j(x,\xi)\mathcal{O}_{x^*}$$

for each i and j. Then we have

$$\operatorname{irr}_\sigma P = \operatorname{irr}_\tau P = \operatorname{Irr} P$$

for each $\sigma, \tau \in S_{mm}$.

Here $\{\Lambda_i(x,\xi),\ \Lambda_j(x,\xi)\}$ denotes the Poisson bracket. Regularly involutive operators and non-involutive operators satisfy (11). In such cases we only need to calculate $\operatorname{irr} P$ instead of $\operatorname{Irr} P$. We want to emphasize that the former number is more easy to calculate than the latter one. The second case is the following

Lemma 2. *If $\sigma, \tau \in S_{mm}$, then we have*

$$\operatorname{irr}_\tau P \leq \max(2, \operatorname{irr}_\sigma P), \ \operatorname{Irr} P \leq \max(2, \operatorname{irr}_\sigma P).$$

This result is very interesting. Sometimes we are interested in microlocal ultradistributions of some special order s_0. Theorem 2 means that P is $\{s_0\}$ well-posed if

$$\operatorname{Irr} P(\leq \max(2, \operatorname{irr} P)) < s_0/(s_0 - 1). \tag{12}$$

Assume that $1 \leq s_0 < 2$. (12) is equivalent to $\operatorname{irr} P < s_0/(s_0 - 1)$, which means that we can use $\operatorname{irr} P$ instead of $\operatorname{Irr} P$, and otherwise we must calculate $\operatorname{Irr} P$. The author thinks that it coincides with historical experience: The well-posedness is an easy problem in hyperfunction theory (where $s = 1$), and is a difficult problem in distribution theory (where $s = \infty$). Even in the case $2 \leq s_0 \leq \infty$, the situation is not so bad if either we can use Lemma 1 or m is not large. In distribution theory it is usual to assume such an assumption. Otherwise we need to calculate $\operatorname{irr}_\sigma$ for $\sigma \in S_{mm}$, which contains $m!$ elements. Then the criterion may be very complicated.

At the end we consider the case of $m = 2$ as an example. In this case $\operatorname{Irr} P \in \{1, 2\}$, and we have

$$\begin{aligned}
&\operatorname{Irr} P = 1 \iff \operatorname{irr}_{(1,2)} P = \operatorname{irr}_{(2,1)} P = 1 \\
&\iff \begin{cases} P \in \Lambda_2\Lambda_1 + x_1^{-1}\bar{\mathcal{E}}_{x^*}(0)\Lambda_1 + x_1^{-1}\bar{\mathcal{E}}_{x^*}(0)\Lambda_2 + x_1^{-2}\bar{\mathcal{E}}_{x^*}(0), \\ P \in \Lambda_1\Lambda_2 + x_1^{-1}\bar{\mathcal{E}}_{x^*}(0)\Lambda_1 + x_1^{-1}\bar{\mathcal{E}}_{x^*}(0)\Lambda_2 + x_1^{-2}\bar{\mathcal{E}}_{x^*}(0) \end{cases} \\
&\iff \begin{cases} P \in \Lambda_2\Lambda_1 + x_1^{-1}\bar{\mathcal{E}}_{x^*}(0)\Lambda_1 + x_1^{-1}\bar{\mathcal{E}}_{x^*}(0)\Lambda_2 + x_1^{-2}\bar{\mathcal{E}}_{x^*}(0), \\ [\Lambda_1, \ \Lambda_2] \in x_1^{-1}\bar{\mathcal{E}}_{x^*}(0)\Lambda_1 + x_1^{-1}\bar{\mathcal{E}}_{x^*}(0)\Lambda_2 + x_1^{-2}\bar{\mathcal{E}}_{x^*}(0). \end{cases}
\end{aligned}$$

This is equivalent to

$$P \in \Lambda_2\Lambda_1 + x_1^{-1}\bar{\mathcal{E}}_{x^*}(0)\Lambda_1 + x_1^{-1}\bar{\mathcal{E}}_{x^*}(0)\Lambda_2 + x_1^{-2}\bar{\mathcal{E}}_{x^*}(0), \tag{13}$$

and

(14) Λ_1 and Λ_2 satisfy (11).

If (13) and (14) are true, then $\operatorname{Irr} P = 1$ and P is $\{s\}$ well-posed for any s. Otherwise $\operatorname{Irr} P = 2$ and P is $\{s\}$ well-posed for $1 \leq s < 2$. In other words, according to our result we must assume (13) and (14) for the case $2 \leq s \leq \infty$. (13) means that the lower order terms must vanish according to some rule, and is not surprising. However as far as our theory applies, we must also assume condition (14) for the principal symbol.

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