Vanishing Theorems in Hyperasymptotic Analysis

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In the former paper, we proved the following: Theorem[Commutative case]
Let \( \sigma \) be a rational number \( \geq 1 \) and

\[
\{ S(R, a_h, b_h) (h = 1, \ldots, N) \}
\]

be a good open sectorial covering for \( \sigma \) of \( D(R, \infty) = \{ z | +\infty > |z| > R \} \). For \( h = 1, \ldots, N \), let \( U_{h-1,h}(z) \) be an \( m \times n \) matricial function defined in \( S_{h-1,h}(R) \) and, for some non-zero constant \( \kappa_{h-1,h} \) with \( \arg \kappa_{h-1,h} = -\frac{a_h + b_{h-1}}{2\sigma} \), \( \exp(-\kappa_{h-1,h}z^{\frac{1}{\sigma}}) \) is asymptotically developable to the formal power-series \( 0 \) and, for a complex number \( \mu_{h-1,h} \),

\[
z^{\mu_{h-1,h}} \exp(\kappa_{h-1,h}z^{\frac{1}{\sigma}}) U_{h-1,h}(z)
\]

is asymptotically developable to a formal power-series matrix \( \sum_{s=0}^{\infty} U_s^{h-1,h} z^{-s} \) in the sector \( S_{h-1,h}(R) \).

Then, there exist a positive number \( R'' (\geq R) \), a formal power-series matrix

\( \tilde{V}(z) = \sum_{r=0}^{\infty} T_r z^{-r} \)

and \( m \times n \) matricial functions \( V_h \) defined in \( S_h(R'') \) \( (h = 1, \ldots, N) \) such that

(i) the relation

\[
U_{h-1,h}(z) = -V_{h-1}(z) + V_h(z)
\]

holds for \( z \in S_{h-1,h}(R'') = S(R'', a_{h-1}, b_{h-1}) \cap S(R'', a_h, b_h) \).

(ii) \( V_h \) is asymptotically developable to the formal power-series matrix \( \tilde{V}(z) \) in \( S_h(R'') \), and for any sufficiently large number \( r \),

\[
T_r = \sum_{(h-1,h)} \sum_{s=0}^{M-1} \sigma U_{s}^{h-1,h}(\kappa_{h-1,h})^{(r-s)\sigma + \mu_{h-1,h}} \Gamma((r-s)\sigma - \mu_{h-1,h})
+ O\{\Gamma((r-M)\sigma - \Re\mu_{h-1,h})\}
\]

provided \( 1 \leq M < r \).
This theorem can be used to study the structure of divergent power-series solutions to the non-homogeneous differential equations associated to linear ordinary differential equations, for example, Bessel equations, Whittaker equations, Weber equations and so on. In this talk, we will give a refined version of the above theorem. The result will be published as a joint-work with C. J. Howls, and A. B. Olde Daalhuis.

References

