Vanishing Theorems in Hyperasymptotic Analysis (Asymptotic Analysis and Microlocal Analysis of PDE)

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Vanishing Theorems in Hyperasymptotic Analysis

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In the former paper, we proved the following: Theorem[Commutative case]
Let $\sigma$ be a rational number $\geq 1$ and

$$\{S(R, a_h, b_h) \mid h = 1, \ldots, N\}$$

be a good open sectorial covering for $\sigma$ of $D(R, \infty) = \{ z \mid + \infty > |z| > R \}$. For $h = 1, \ldots, N$, let $U_{h-1,h}(z)$ be an $m \times n$ matricial function defined in $S_{h-1,h}(R)$ and, for some non-zero constant $\kappa_{h-1,h}$ with $\arg \kappa_{h-1,h} = -\frac{a_h+b_{h-1}}{2\sigma}$, $\exp(-\kappa_{h-1,h}z^{\frac{1}{\sigma}})$ is asymptotically developable to the formal power-series $0$ and, for a complex number $\mu_{h-1,h}$,

$$z^{\mu_{h-1,h}} \exp(\kappa_{h-1,h}z^{\frac{1}{\sigma}})U_{h-1,h}(z)$$

is asymptotically developable to a formal power-series matrix $\sum_{s=0}^{\infty} U_{s}^{h-1,h}z^{-s}$ in the sector $S_{h-1,h}(R)$.

Then, there exist a positive number $R'' (\geq R)$, a formal power-series matrix

$$\tilde{V}(z) = \sum_{r=0}^{\infty} T_r z^{-r}$$

and $m \times n$ matricial functions $V_h$ defined in $S_h(R'') (h = 1, \ldots, N)$ such that

(i) the relation

$$U_{h-1,h}(z) = -V_{h-1}(z) + V_{h}(z)$$

holds for $z \in S_{h-1,h}(R'') = S(R'', a_{h-1}, b_{h-1}) \cap S(R'', a_h, b_h)$.

(ii) $V_h$ is asymptotically developable to the formal power-series matrix $\tilde{V}(z)$ in $S_h(R'')$, and for any sufficiently large number $r$,

$$T_r = \sum_{(h-1,h)} \sum_{s=0}^{M-1} \sigma U_{s}^{h-1,h}(\kappa_{h-1,h})^{(s-r)\sigma+\mu_{h-1,h}} \Gamma((r-s)\sigma - \mu_{h-1,h}) + O\{\Gamma((r-M)\sigma - \Re \mu_{h-1,h})\}$$

provided $1 \leq M < r$. 

\[ \text{(1999)} \]
This theorem can be used to study the structure of divergent power-series solutions to the non-homogeneous differential equations associated to linear ordinary differential equations, for example, Bessel equations, Whittaker equations, Weber equations and so on. In this talk, we will give a refined version of the above theorem. The result will be published as a joint-work with C. J. Howls, and A. B. Olde Daalhuis.

References

