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The Cauchy problem for the nonlinear integro-partial differential equation that describes the time evolution of sociodynamic quantities

神戸大学工学部応用数学教室 田畑稔 (Minoru Tabata)
Department of Applied Mathematics, Faculty of Engineering, Kobe Univ.
大分医科大学医療情報学教室 江島伸興 (Nobuoki Eshima)
Department of Medical Information Analysis, Oita Medical Univ.

Abstract. *The master equation* is a nonlinear integro-partial differential equation, which describes the evolution of various quantities in *quantitative sociodynamics.* For example, the master equation can describe interregional migration. The purpose of this paper is to obtain asymptotic estimates for solutions to the Cauchy problem for the equation.

1. Introduction. Large free economic unions such as EU and NAFTA have been established recently. In such free trade unions, goods are traded freely, but interregional labor mobility is restricted at a certain level of rigidity. However, there is now a move to abolish the restriction entirely. If no restriction is imposed on the regional labor mobility, and if there exists regional economic disparity, then workers will move so as to achieve a higher income. This phenomenon is called *interregional migration motivated by regional economic disparity*, and it is known in [3-4] and [11-12] that *the master equation* can describe such a phenomenon (see, e.g., [1-2], [5], and [13-14] for the theory of interregional migration). The equation plays very important roles in *quantitative sociodynamics* (see, e.g., [4]). Furthermore, the master equation approach is taken also in *nonlinear evolutionary economics* (see, e.g., [10]).

The master equation is a nonlinear integro-partial differential equation, which has the form:

$$(1.1) \qquad \partial v(t,x)/\partial t = -w(t,x)v(t,x) + \int_{y\in D} W(t;x|y)v(t,y)dy,$$

where D is a bounded Lebesgue measurable set included in the 2-dimensional Euclidean space. By $v = v(t,x)$ we denote the unknown function which represents the density of population at time $t \geq 0$ and at a point $x \in D$. By $W = W(t;x|y)$ we denote the *transition rate* at time $t \geq 0$ and from a point $y \in D$ to a point $x \in D$. The coefficient $w = w(t,x)$ is defined from the transition rate as follows: $w = w(t,x) := \int_{y \in D} W(t;y|x)dy$. The master equation has its origin in statistical physics, and has been fully studied in mathematical physics. However, the transition rate of the master equation studied in quantitative sociodynamics is completely different from that treated in statistical physics. Hence we cannot apply various methods developed in statistical physics to the master equation studied in quantitative sociodynamics. There have been few studies on the master equation treated in quantitative sociodynamics except for [6-8]. Therefore it is important to investigate the master equation treated in quantitative sociodynamics (we simply call the master equation studied in quantitative sociodynamics "the master equation").

In the same way as [4, pp. 137-138] and [12, pp. 81-100], we will impose the following assumption on the transition rate $W = W(t;x|y)$ in this paper:

Assumption 1.1. The transition rate $W = W(t;x|y)$ has the following form: $W(t;x|y) = \theta(t)\exp\{U(t,x) - U(t,y) - E(x,y)\}$, where $\theta = \theta(t)$ denotes the *flexibility* at time $t \geq 0$, $U = U(t,x)$ is the *utility* at time $t \geq 0$ and at a point $x \in D$, and $E = E(x,y)$ denotes the *effort* from a point $y \in D$ to a point $x \in D$.

See, e.g., [4, p. 137] for the sociodynamic definitions of flexibility, utility, and effort. In the same way as [8], in this paper we make the following assumption (see [8] for the reasons for making this assumption):

Assumption 1.2. The flexibility $\theta = \theta(t)$ and the effort $E = E(x,y)$ are identically equal to positive constants.

Let us discuss the utility. In a real world we often observe that the utility increases with the population density. If such a phenomenon is observed, then we say that *imitative process* works. In order to assume that imitative process works in interregional migration, in [8] we impose the following assumption on the utility (by this assumption, in [8] we fully investigate the asymptotic behavior of solutions to the Cauchy problem for the master equation):

Assumption 1.3. The utility $U = U(t,x)$ has the form $U(t,x) = c_{1.1}\underline{v}(t,x) + c_{1.2}$, where $\underline{v} = \underline{v}(t,x) := v(t,x)/\|v(t,\cdot)\|_{L^1(D)}$ (we denote the norm of $L^1(D)$ by $\|\cdot\|_{L^1(D)}$), $c_{1.1}$ is *a* positive constant, and $c_{1.2}$ is a real constant.

It is plausible to assume that imitative process works at a certain degree. However, in a real world, we observe that if the density of population is sufficiently large, then the utility does not increases with the population density, and moreover we find that over population makes the utility decrease. If such a phenomenon is observed, then we say that *avoidance process* works. In [8] we assume that only imitative process works, but in this paper we take not only imitative process but also avoidance process into account. Hence, for example, we need to assume that the utility $U=U(t,x)$ is a strictly concave function of $\underline{v}=\underline{v}(t,x)$ which monotonously increases (decreases, respectively) with $\underline{v}\in[0,k)$ ($\underline{v}\in(k,+\infty)$, respectively), where k is a positive constant. Therefore in the present paper we will make the following assumption in place of Assumption 1.3:

Assumption 1.4. The utility has the form $U(t,x):=-(\alpha_1\underline{v}(t,x)-\alpha)^2+\alpha_2$, where α and α_1 are positive constants, and α_2 is a real constant.

We will impose Assumptions 1.1-2 and Assumption 1.4 on this paper. In the same way as [6-8], we can prove that the Cauchy problem for the master equation has a unique positive-valued local solution (Proposition 2.1). Combining this result and a priori estimates for solutions (Lemma 4.1), we can demonstrate that the Cauchy problem has a unique positive-valued global solution (Theorem 4.2). The purpose of this paper is to prove that if certain assumptions are made, then each global solution to the Cauchy problem converges to a stationary solution (Theorem 4.3). This paper has 6 sections in addition to this section. In Section 2 we give preliminaries. In Section 3 we obtain all the stationary solutions of the master equation. In Section 4 we present the main result, which will be proved in Sections 5-7.

Remark 1.5. (i) In [12, pp. 92-96] Assumption 1.4 is proved in the sociodynamic level of rigor. See [12, (4.15-19)].

(ii) We can apply the results of this paper and [8] to economics. This subject will be discussed in [9].

2. Preliminaries. Integrating both sides of (1.1) with respect to $x\in D$, in the same way as [6-8] we obtain the conservation law of total population, $\|v(t,\cdot)\|_{L^1(D)}=\|v(0,\cdot)\|_{L^1(D)}$ for each $t\geq 0$. Hence, $\underline{v}(t,x)=v(t,x)/\|v(0,\cdot)\|_{L^1(D)}$ (see Assumption 1.3 for $\|\cdot\|_{L^1(D)}$ and $\underline{v}=\underline{v}(t,x)$). Assumptions 1.1-2 and Assumption 1.4 give

$$W(t;x|y)=\alpha_3\exp\{-(\alpha_1\underline{v}(t,x)-\alpha)^2+(\alpha_1\underline{v}(t,y)-\alpha)^2\}, \tag{2.1}$$

where α_3 is a positive constant. Let us rewrite (1.1) with (2.1) by introducing the new unknown function $u=u(t,x):=\alpha_1\underline{v}(t/\alpha_3|D|,|D|^{1/2}x)$ in place of $v=v(t,x)$, where we denote the Lebesgue measure of a subset $d \subseteq \mathbb{R}\times\mathbb{R}$ by $|d|$. In exactly the same way as [8, p. 82], we obtain the new integro-partial differential equation,

$$(2.2) \qquad \partial u(t,x)/\partial t = -a(u(t,\cdot))u(t,x)e^{(u(t,x)-\alpha)^2} + b(u(t,\cdot))e^{-(u(t,x)-\alpha)^2},$$

where $a(u(t,\cdot)) := \int_{y\in\Omega} e^{-(u(t,y)-\alpha)^2}dy$, $b(u(t,\cdot)) := \int_{y\in\Omega} u(t,y)e^{(u(t,y)-\alpha)^2}dy$, and $\Omega :=$ $\{x = |D|^{-1/2}z;\, z\in D\}$. Hence,

$$(2.3) \qquad |\Omega| = 1.$$

We denote the norm of $L^1(\Omega)$ by $\|\cdot\|_1$.

By (CP) we denote the Cauchy problem for (2.2) with the initial condition, $u(0,x) = u_0(x)$, where $u_0 = u_0(x)$ is a Lebesgue-measurable given function of $x\in$ Ω such that $0 < u_{0,-} :=$ ess $\inf_{x\in\Omega}u_0(x)$, $u_{0,+} :=$ ess $\sup_{x\in\Omega}u_0(x) < +\infty$. In the same way as [6-8], we can define a solution to (CP) as follows: if $u = u(t,x) \in$ $L^\infty([0,T]_t\times\Omega_x)$, if $u = u(t,x)$ satisfies (2.2) almost everywhere in $[0,T]_t\times\Omega_x$, and if $u = u(t,x)$ satisfies the above initial condition, then we say that $u = u(t,x)$ is a solution to (CP) in $[0,T]$, where T is a positive constant. In the same way as [8, Proposition 2.7], we can prove the following proposition:

PROPOSITION 2.1. *The Cauchy problem* (CP) *has a unique solution* $u = u(t,x)$ *in* $[0,R]$, *where* R *is a positive constant dependent on* $u_{0,+}$ *and* $u_{0,-}$. *If* $u = u(t,x)$ *is a solution to* (CP) *in* $[0,T]$ for some $T>0$, *then the following (i-iv) hold*:

(i) $\partial u(t,x)/\partial t\in L^\infty([0,T]\times\Omega)$, *and* $u = u(t,x)$ *is absolutely continuous with respect to* $t\in[0,T]$ *for a.e.* $x\in\Omega$.

(ii) $0 <$ ess $\inf_{(t,x)\in[0,T]\times\Omega}u(t,x)$, ess $\sup_{(t,x)\in[0,T]\times\Omega}u(t,x) < +\infty$.

(iii) $\|u(t,\cdot)\|_1 = \alpha_1/|D|$ *for each* $t\in[0,T]$.

(iv) If $u(t,x_1) = u(t,x_2)$ *for some* $t\in[0,T]$ *and for some* $x_j\in\Omega$, $j = 1,2$, *then* $u(t,x_1) = u(t,x_2)$ *for each* $t\in[0,T]$. *If* $u(t,x_1) < u(t,x_2)$ *for some* $t\in[0,T]$ *and for some* $x_j\in\Omega$, $j = 1,2$, *then* $u(t,x_1) < u(t,x_2)$ *for each* $t\in[0,T]$.

Remark 2.2. It will be shown that the constant $\alpha_1/|D|$ strongly governs the asymptotic behavior of solutions to the Cauchy problem. We define $A := \alpha_1/|D|$.

3. Stationary solutions. Let us rewrite the equation (2.2) as follows:

$$(3.1) \qquad \partial u(t,x)/\partial t = a(u(t,\cdot))g_\alpha(u(t,x))\{-f_\alpha(u(t,x))+b(u(t,\cdot))/a(u(t,\cdot))\},$$

where $f_\alpha = f_\alpha(z) := ze^{2(z-\alpha)^2}$ and $g_\alpha = g_\alpha(z) := e^{-(z-\alpha)^2}$. See Assumption 1.4 for α. Noting that $a(u(t,\cdot))g_\alpha(u(t,x)) > 0$, we see that the following equation is a sufficient and necessary condition for $u = u(x)$ to be a stationary solution of (2.2):

$$f_\alpha(u(x)) = b(u(\cdot))/a(u(\cdot)). \tag{3.2}$$

We can easily prove the following lemma (hence we omit the proof):

LEMMA 3.1. *(i)* $f_\alpha(0) = 0, f_\alpha(z) > 0$ *for each* $z > 0$, $\lim_{z\to+\infty} f_\alpha(z) = +\infty$.

(ii) If $0 < \alpha \leq 1$, *then* $f_\alpha = f_\alpha(z)$ *is a strictly monotonously increasing function of* $z \geq 0$. *If* $\alpha > 1$, *then* $f_\alpha = f_\alpha(z)$ *strictly monotonously increases (decreases, respectively) when* $0 \leq z < \beta_2$ or $\beta_3 < z < +\infty$ *(when* $\beta_2 < z < \beta_3$, *respectively), where* $\beta_2 := \{\alpha - (\alpha^2 - 1)^{1/2}\}/2$ *and* $\beta_3 := \{\alpha + (\alpha^2 - 1)^{1/2}\}/2$.

By this lemma we can define positive constants β_1, β_4, γ_1, and γ_2 as follows: $\beta_j \neq \beta_{j+2}$ and $f_\alpha(\beta_j) = f_\alpha(\beta_{j+2}), j = 1,2$, and $\gamma_j := f_\alpha(\beta_j) = f_\alpha(\beta_{j+2})$, $j = 1,2$. By Lemma 3.1 we can easily obtain the following lemma:

LEMMA 3.2. $0 < \gamma_1 < \gamma_2, 0 < \beta_1 < \beta_2 < \beta_3 < \beta_4, \quad \alpha - \beta_3 > 0$.

The right-hand side of (3.2) is a positive constant. Hence we consider the equation,

$$f_\alpha(z) = \gamma, \tag{3.3}$$

where z is an unknown value, and γ is a positive-valued parameter. It follows from Lemma 3.1 that if $0 < \alpha \leq 1$, then (3.3) has only one real solution. Let $\alpha > 1$. By Lemmas 3.1-2, we deduce that if $0 < \gamma < \gamma_1$ or $\gamma_2 < \gamma$, then (3.3) has only one real solution. Taking multiplicity into account, in the same way we see that if $\gamma_1 \leq \gamma \leq \gamma_2$, then (3.3) has only three real solutions. We denote them by $z_j = z_j(\gamma)$, $j = 1,2,3$, $z_1(\gamma) \leq z_2(\gamma) \leq z_3(\gamma)$. Lemmas 3.1-2 give the following lemma:

LEMMA 3.3. *If* $\alpha > 1$, *then the following (i-ii) hold:*

(i) If $\gamma_1 < \gamma < \gamma_2$, *then* $\beta_j < z_j(\gamma) < \beta_{j+1}$, $j = 1,2,3$. $z_1(\gamma_1) = \beta_1$, $z_2(\gamma_1) = z_3(\gamma_1) = \beta_3$, $z_1(\gamma_2) = z_2(\gamma_2) = \beta_2$, *and* $z_3(\gamma_2) = \beta_4$.

(ii) $z_j = z_j(\gamma)$, $j = 1,3$, *are strictly monotonously increasing continuous functions of* $\gamma \in [\gamma_1, \gamma_2]$, *and* $z_2 = z_2(\gamma)$ *is a strictly monotonously decreasing continuous function of* $\gamma \in [\gamma_1, \gamma_2]$.

Let $\gamma \in [\gamma_1, \gamma_2]$. Replace z by $u = u(x)$ in (3.3). We easily see that each solution of the equation thus obtained has the form: $u(x) = U(\gamma, Y_1, Y_2, Y_3; x)$, where

$$\text{(3.4)} \qquad U = U(\gamma, Y_1, Y_2, Y_3; x) := z_j(\gamma) \text{ if } x \in Y_j, j = 1,2,3.$$

Here Y_j, $j = 1,2,3$, are disjoint subsets of Ω such that $\Omega = Y_1 \cup Y_2 \cup Y_3$. For each $\gamma \in [\gamma_1, \gamma_2]$ by $Z = Z(\gamma)$ we denote the set of all (Y_1, Y_2, Y_3) such that Y_j, $j = 1,2,3$, are disjoint subsets of Ω which satisfy the following equalities:

$$\text{(3.5)} \qquad |Y_1| + |Y_2| + |Y_3| = 1, \quad z_1(\gamma)|Y_1| + z_2(\gamma)|Y_2| + z_3(\gamma)|Y_3| = A.$$

See Section 2 for $|\cdot|$ and A.

PROPOSITION 3.4. *(i) If* $0 < \alpha \leq 1$ and $A > 0$, *then the equation* (2.2) *has a unique stationary solution* $u = u(x)$ *such that* $u(x) = A$ *for a.e.* $x \in \Omega$.

(ii) Let $\alpha > 1$. *If* $0 < A \leq \beta_1$ *or* $\beta_4 \leq A$, *then the equation* (2.2) *has a unique stationary solution* $u = u(x)$ *such that* $u(x) = A$ *for a.e.* $x \in \Omega$.

(iii) Let $\alpha > 1$. *If* $\beta_1 < A < \beta_2$ *(*$\beta_2 \leq A \leq \beta_3$, $\beta_3 < A < \beta_4$, *respectively), then the set of all stationary solutions of* (2.2) *is equal to the set of all functions of the form* (3.4) *where* $(Y_1, Y_2, Y_3) \in Z(\gamma)$ *and* $\gamma \in (\gamma_1, f_\alpha(A)]$ *(*$\gamma \in [\gamma_1, \gamma_2]$, $\gamma \in [f_\alpha(A), \gamma_2)$, *respectively).*

Proof. As already mentioned above, we easily see that each stationary solution of (2.2) has the form (3.4). We easily see that each step function of the form (3.4) satisfies (3.2). By (2.3) we see that the first equality of (3.5) is equivalent to the condition that Y_j, $j = 1,2,3$, are disjoint subsets such that $\Omega = Y_1 \cup Y_2 \cup Y_3$. We see that the second equality of (3.5) is equivalent to the equality $\|U(\gamma, Y_1, Y_2, Y_3; \cdot)\|_1 = A$ (see Remark 2.2 and Proposition 2.1, (iii)). Assume that $\beta_1 < A < \beta_2$. By Lemmas 3.1-3 we see that if $\gamma \in (\gamma_1, f_\alpha(A)]$, then $Z(\gamma)$ is not empty, and that if $Z(\gamma)$ is not empty, then $\gamma \in (\gamma_1, f_\alpha(A)]$. Therefore we obtain (iii) when $\beta_1 < A < \beta_2$. (i-ii) and (iii) with $\beta_2 \leq A < \beta_4$ can be proved in the same way. □

4. The main result. Let us prove *a priori* estimates for solutions of (CP).

LEMMA 4.1. *If* (CP) *has a solution* $u = u(t,x)$ *in* $[0,T]$, *where* T *is a positive constant, then the solution satisfies the following (i-ii):*

(i) If $0 < \alpha \leq 1$, *then* $u_{0,-} \leq \text{ess inf}_{(t,x)\in[0,T)\times\Omega} u(t,x)$, $\text{ess sup}_{(t,x)\in[0,T)\times\Omega} u(t,x) \leq u_{0,+}$.

(ii) If $\alpha > 1$, *then* $\min\{u_{0,-}, \beta_1\} \leq \text{ess inf}_{(t,x)\in[0,T)\times\Omega} u(t,x)$,
$\text{ess sup}_{(t,x)\in[0,T)\times\Omega} u(t,x) \leq \max\{u_{0,+}, \beta_4\}$.

Proof. See Assumption 1.4 and Sections 2-3 for α, $u_{0,\pm}$, and β_j, $j = 1,4$. We will prove only the second inequality of (ii), since the other inequalities can be demonstrated in the same way. It follows from Proposition 2.1 that $R = R(t) := b(u(t,\cdot))/a(u(t,\cdot))$ is a continuous function of $t \in [0,T]$ (see (2.2) for $a(\cdot)$ and $b(\cdot)$). We easily obtain

$$f_\alpha(u(t,x)) \leq \text{ess sup}_{x\in\Omega} f_\alpha(u(t,x)), \text{ for each } t \geq 0. \tag{4.1}$$

See Section 3 for $f_\alpha(\cdot)$. Multiply both sides of this inequality by $G_\alpha = G_\alpha(t,x) := g_\alpha(u(t,x))/\int_{y\in\Omega} g_\alpha(u(t,y))dy$. See Section 3 for $g_\alpha(\cdot)$. Integrate both sides of the inequality thus obtained with respect to $x \in \Omega$. Noting that

$$\int_{y\in\Omega} G_\alpha(t,y)dy = 1, \tag{4.2}$$

and recalling the definitions of $a(\cdot)$ and $b(\cdot)$, we see that

$$R(t) \leq \text{ess sup}_{x\in\Omega} f_\alpha(u(t,x)), \text{ for each } t \geq 0. \tag{4.3}$$

Suppose that the equal sign of (4.3) holds at some $t = k \in [0,T]$. We easily deduce that the equal sign of (4.1) holds for a.e. $x \in \Omega$ at $t = k$. From this equality, in the same way as Proof of Proposition 3.4, we see that $u = u(k,x)$ is a stationary solution of (2.2), i.e., that the solution $u = u(t,x)$ is stationary. Hence, by Proposition 3.4, we obtain the second inequality of (ii). Assume that the equal sign of (4.3) does not hold for each $t \in [0,T]$. Applying this inequality to (3.1), and making use of Lemma 3.1 and Proposition 2.1, (iv), we can deduce that if $\text{ess sup}_{x\in\Omega} u(t,x) > \beta_4$, then $\text{ess sup}_{x\in\Omega} u(t,x)$ decreases monotonously with $t \in [0,T]$. Hence we obtain the second inequality of (ii). □

THEOREM 4.2. *The Cauchy problem* (CP) *has a unique global solution, which satisfies the inequalities of (i-ii) of Lemma 4.1 with* $T = +\infty$.

Proof. By Lemma 4.1 and Proposition 2.1, we obtain the theorem. □

Decompose Ω into 3 disjoint subsets as follows for a positive-valued function $p = p(x)$: $\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3$, where $\Omega_1 = \Omega_1(p(\cdot)) := \{x \in \Omega;\, p(x) < \beta_2\}$, $\Omega_2 = \Omega_2(p(\cdot)) := \{x \in \Omega;\, \beta_2 \leq p(x) \leq \beta_3\}$, and $\Omega_3 = \Omega_3(p(\cdot)) := \{x \in \Omega;\, \beta_3 < p(x)\}$.

THEOREM 4.3. *(i) If* $0 < \alpha \leq 1$ *and* $A > 0$, *if* $\alpha > 1$ *and* $0 < A \leq \beta_1$, *or if* $\alpha > 1$ *and* $\beta_4 \leq A$, *then the Cauchy problem* (CP) *has a unique global solution* $u = u(t,x)$, *which converges to* A *as follows*: $|\{x \in \Omega;\, |u(t,x) - A| \geq \delta\}| \rightarrow 0$ *as* $t \rightarrow +\infty$ *for each* $\delta > 0$ (see Section 2 for $|\cdot|$).

(ii) If $\alpha > 1$, $\beta_1 < A < \beta_2$, *and* $u_0 = u_0(x)$ *satisfies that*

$$\beta_1 < u_0(x) < \beta_4, \quad \textit{for a.e. } x \in \Omega, \tag{4.4}$$

$$|\{x \in \Omega_2(u_0(\cdot));\, u_0(x) = \gamma\}| = 0 \quad \textit{for each} \quad \gamma > 0, \tag{4.5}$$

$$A > \beta_1 |\Omega_1(u_0(\cdot))| + \beta_3(|\Omega_2(u_0(\cdot))| + |\Omega_3(u_0(\cdot))|), \tag{4.6}$$

$$0 \leq |\Omega_2(u_0(\cdot))| < c_{4.1}, \tag{4.7}$$

where $c_{4.1}$ *is a sufficiently small positive constant, then the Cauchy problem* (CP) *has a unique global solution* $u = u(t,x)$, *which satisfies the following*:

$$\beta_1 \leq u(t,x) \leq \beta_4, \quad \textit{for a.e. } x \in \Omega \textit{ and each } t \geq 0, \tag{4.8}$$

$$\Omega_j(u(t_1,\cdot)) \subseteq \Omega_j(u(t_2,\cdot)),\, j=1,3,\; \Omega_2(u(t_1,\cdot)) \supseteq \Omega_2(u(t_2,\cdot)), \text{ if } 0 \leq t_1 \leq t_2, \tag{4.9}$$

$$\lim_{t \rightarrow +\infty} |\Omega_2(u(t,\cdot))| = 0, \tag{4.10}$$

$$\lim_{t \rightarrow +\infty} |\{x \in \Omega;\, |u(t,x) - u_\infty(x)| \geq \delta\}| = 0 \textit{ for each} \quad \delta > 0, \tag{4.11}$$

where $u_\infty = u_\infty(x)$ *is a stationary solution of* (2.2) *such that*

$$u_\infty(x) = U(R_\infty, \Omega_{1,\infty}, \phi, \Omega_{3,\infty}; x), \quad (\Omega_{1,\infty}, \phi, \Omega_{3,\infty}) \in Z(R_\infty). \tag{4.12}$$

Here we define $\Omega_{j,\infty} := \bigcup_{t \geq 0} \Omega_j(u(t,\cdot))$, $j = 1,3$, *and* $R_\infty \in (\gamma_1, f_\alpha(A)]$ *is a constant such that* $R_\infty := \lim_{t \rightarrow +\infty} b(u(t,\cdot))/a(u(t,\cdot))$.

(iii) If $\alpha > 1$, $\beta_2 \leq A \leq \beta_3$, *and* $u_0 = u_0(x)$ *satisfies* (4.4-7) *and*

(4.13) $$\beta_2(|\Omega_1(u_0(\cdot))|+|\Omega_2(u_0(\cdot))|)+\beta_4|\Omega_3(u_0(\cdot))|>A,$$

then (CP) *has a unique global solution* $u = u(t,x)$, *which satisfies* (4.8-12) *and*

(4.14) $$R_\infty \in (\gamma_1, \gamma_2).$$

(iv) If $\alpha>1, \beta_3<A<\beta_4$, *and* $u_0 = u_0(x)$ *satisfies* (4.4-5), (4.7), *and* (4.13), *then* (CP) *has a unique global solution* $u = u(t,x)$, *which satisfies* (4.8-12) *and* $R_\infty \in [f_\alpha(A), \gamma_2)$.

Remark 4.4. (i) Applying (4.4) and Theorem 4.2 (see Lemma 4.1), we easily obtain (4.8).

(ii) From (4.5) we see that $\Omega_2(u_0(\cdot))$ is empty or that $u_0 = u_0(x)$ is not constant in $\Omega_2(u_0(\cdot))$.

(iii) It follows from (4.7) that $\Omega_2(u_0(\cdot))$ is empty or sufficiently small. We employ (4.7) in order to prove (7.9).

(iv) By Lemma 3.2 we deduce that if $|\Omega_2(u_0(\cdot))|$ is so small that

$$|\Omega_2(u_0(\cdot))| < \{(\beta_2-\beta_1)|\Omega_1(u_0(\cdot))| + (\beta_4-\beta_3)|\Omega_3(u_0(\cdot))|\}/(\beta_3-\beta_2),$$

then (the left-hand side of (4.13)) > (the right-hand side of (4.6)), i.e., there exists A which satisfies both (4.6) and (4.13). We easily see that there exists an infinite number of α, A, and u_0 which satisfy (4.4-7) and (4.13). We can say that (4.7) restricts the value of $|\Omega_2(u_0(\cdot))|$, and that (4.5) restricts the behavior of the initial function $u_0 = u_0(x)$ in $\Omega_2(u_0(\cdot))$.

(v) Performing calculations similar to those done in showing Theorem 4.3, (iii), we can prove Theorem 4.3, (i), (ii), (iv). Hence we will demonstrate only Theorem 4.3, (iii). In what follows throughout the paper, we will assume the conditions of Theorem 4.3, (iii).

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