<table>
<thead>
<tr>
<th>Title</th>
<th>ON THE CONSTRUCTION OF CONFORMAL MEASURES FOR PIECEWISE $C^0$-INVERTIBLE SYSTEMS (Studies on complex dynamics and related topics)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>Yuri, Michiko</td>
</tr>
<tr>
<td>Citation</td>
<td>数理解析研究所講究録 数理解析研究所講究録</td>
</tr>
<tr>
<td>Issue Date</td>
<td>2001-07</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/2433/41289">http://hdl.handle.net/2433/41289</a></td>
</tr>
<tr>
<td>Type</td>
<td>Departmental Bulletin Paper</td>
</tr>
<tr>
<td>Textversion</td>
<td>publisher</td>
</tr>
</tbody>
</table>

Kyoto University
ON THE CONSTRUCTION OF CONFORMAL MEASURES
FOR PIECEWISE $C^0$-INVERTIBLE SYSTEMS

MICHIKO YURI

ABSTRACT. We present a new method for the construction of conformal measures $\nu$ for infinite to one piecewise $C^0$-invertible Markov systems. We direct our attention to potentials $\phi$ which may fail both summable variations and bounded distortion but satisfy the weak bounded variation. Our results apply to higher-dimensional maps which are not necessarily conformal and admit certain nonhyperbolic periodic orbits.

§0 Introduction

Let $(T, X, Q = \{X_i\}_{i \in I})$ be a piecewise $C^0$-invertible system i.e., $X$ is a compact metric space with metric $d$, $T : X \to X$ is a noninvertible map which is not necessarily continuous, and $Q = \{X_i\}_{i \in I}$ is a countable disjoint partition $Q = \{X_i\}_{i \in I}$ of $X$ such that $\bigcup_{i \in I} \text{int}X_i$ is dense in $X$ and satisfy the following properties.

(01) For each $i \in I$ with $\text{int}X_i \neq \emptyset$, $T|_{\text{int}X_i} : \text{int}X_i \to T(\text{int}X_i)$ is a homeomorphism and $(T|_{\text{int}X_i})^{-1}$ extends to a homeomorphism $v_i$ on $\text{cl}(T(\text{int}X_i))$.

(02) $T(\bigcup_{\text{int}X_i = \emptyset} X_i) \subset \bigcup_{\text{int}X_i = \emptyset} X_i$.

(03) $\{X_i\}_{i \in I}$ generates $\mathcal{F}$, the sigma algebra of Borel subsets of $X$.

Let $i = (i_1 \ldots i_n) \in I^n$ satisfy $\text{int}(X_{i_1} \cap T^{-1}X_{i_2} \cap \ldots T^{-(n-1)}X_{i_n}) \neq \emptyset$. Then we define $X_i := X_{i_1} \cap T^{-1}X_{i_2} \cap \ldots T^{-(n-1)}X_{i_n}$ which is called a cylinder of rank $n$ and write $|i| = n$. By (01), $T^n|_{\text{int}X_{i_1 \ldots i_n}} : \text{int}X_{i_1 \ldots i_n} \to T^n(\text{int}X_{i_1 \ldots i_n})$ is a homeomorphism and $(T^n|_{\text{int}X_{i_1 \ldots i_n}})^{-1}$ extends to a homeomorphism $v_{i_1} \circ v_{i_2} \circ \ldots \circ v_{i_n} = v_{i_1 \ldots i_n}$.

We impose on $(T, X, Q)$ the next condition which gives a nice countable states symbolic dynamics similar to sofic shifts (cf. [5]):

(Finite Range Structure). $\mathcal{U} = \{\text{int}(T^nX_{i_1 \ldots i_n}) : \forall X_{i_1 \ldots i_n}, \forall n > 0\}$ consists of finitely many open subsets $U_1 \ldots U_N$ of $X$.

In particular, we say that $(T, X, Q)$ satisfies Bernoulli property if $\text{cl}(T(\text{int}X_i)) = X(\forall i \in I)$ so that $\mathcal{U} = \{\text{int}X\}$ and that $(T, X, Q)$ satisfies Markov property if $\text{int}(\text{cl}(\text{int}X_i) \cap \text{cl}(\text{int}T^iX_j)) \neq \emptyset$ implies $\text{cl}(\text{int}TX_i) \supset \text{cl}(\text{int}X_i)$. $(T, X, Q)$ satisfying Bernoulli (Markov) property is called a piecewise $C^0$-invertible Bernoulli (Markov) system respectively.

1991 Mathematics Subject Classification. 28D99, 28D20, 58F11, 58F03, 37A40, 37A30, 37C30, 37D35, 37F10, 37A45.

Typeset by A4AS-TeX
M. YURI

For given a subset $A$ of $X$, let $R_A : A \to \mathbb{N} \cup \infty$ be the first return function over $A$ and we define $D^A_n = \{ x \in A : R_A(x) > n \}$. If we have previously a reasonable measurable dynamics e.g. $(T, X, Q, \mathcal{F}, \nu)$, where $\mathcal{F}$ denotes the $\sigma$- algebra of Borel subsets of $X$ and $\nu$ is a nonsingular ($\nu T^{-1} \sim \nu$) probability measure with $\nu(A) > 0$ satisfying

$$(1) : \lim_{n \to \infty} \nu(D^A_n) = \nu(\bigcap_{n \geq 0} D^A_n) = 0,$$

then the induced map $T_A$ over $A$ can be defined almost everywhere on $A$ and all iterations $\{T^n_A\}_{n \geq 1}$, too. Furthermore, if we can construct a $T_A$-invariant ergodic probability measure $\mu_A$ absolutely continuous with respect to $\nu$, then the integrability of the first return function with respect to $\nu$ (which is equivalent to

$$(2) : \sum_{n \geq 0} \nu(D^A_n) < \infty$$

so that (1) is automatically satisfied) is sufficient for the existence of $T$-invariant ergodic probability measure $\mu$ absolutely continuous with respect to $\nu$ which is given by the well-known Kac formula as follows: for all $f \in L^1(\nu)$, $\frac{1}{\mu(A)} \int_X f d\mu = \int_A f d\mu_A$, where $f_A(x) = \sum_{i=0}^{R_A(x)-1} fT^i(x)$.

Those results and a generalized Thermodynamic Formalism for potential $\phi$ of weak bounded variation were established in [6] for piecewise $C^0$-invertible Bernoulli systems by assuming some regular condition on $T_A$ and on the associated potential $\phi_A$. In particular, this approach works satisfactorily to establish Thermodynamic formalism for piecewise $C^1$- invertible maps with the Bernoulli property admitting certain nonhyperbolic periodic orbits (e.g., indifferent periodic points) and for the natural potential $\phi = -\log |det DT|$. In fact, $A$ can be taken as a hyperbolic region which is away from the nonhyperbolic periodic orbits (see [6] for details) and the absolutely continuous invariant measure $\mu$ with respect to the normalized Lebesgue measure $\nu$ attains the measure theoretical pressure for $\phi = -\log |det DT|$. On the other hand, when $(T, X, Q)$ does not satisfy the Bernoulli property we have no evidence of the existence of nonsingular reference measure $\nu$ even if the Markov property is satisfied. If we restrict our attention to (countable) Markov shifts then we can find some answer to this problem (e.g.,[4]). However, if the systems are not symbolic dynamics the existence problem is still remain open (cf.[1]). In this talk, for infinite to one piecewise $C^0$-invertible transitive Markov systems we shall give a partial answer to this problem. For this purpose, we first clarify properties of topological pressure for $\phi$ and for the associated potential $\phi_A$ defined on a single cylinder $A \in Q$. Then we introduce Schweiger’s jump transformations $T^*$ over cylinders which are mapped onto $X$ under $T$. We shall see a good relation between the topological pressure for $\phi_A$ and the topological pressure for $\phi^*$ associated to $T^*$. This observation allows one to establish the existence of an eigenvalue 1 of the Perron-Frobenius operator associated to $\phi_A$ by using a formula of zeta function for $\phi$ in terms of zeta function for $\phi^*$ obtained in [5]. Finally we can construct a conformal measure $\nu$ by using a result in [1]. We also establish the existence of conformal measures by using jump transformation defined. Again the existance of an eigenvalue 1 of the Perron-Frobenius operator associated to $\phi^*$ plays an important role for the construction of $\nu$.

REFERENCES

1. M. Denker and M. Yuri, A note on the construction of nonsingular Gibbs measures, Collo-
EQUILIBRIUM STATES FOR PIECEWISE INVERTIBLE SYSTEMS


YURI: DEPARTMENT OF BUSINESS ADMINISTRATION, SAPPORO UNIVERSITY, NISHIOKA, TOYOHIRA-KU, SAPPORO 062, JAPAN.
E-mail address: yuri@math.sci.hokudai.ac.jp, yuri@mail-ext.sapporo-u.ac.jp