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Finite regular semigroups which are amalgamation bases for finite semigroups

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Let $\mathcal{A}$ be a class of semigroups. A triple of semigroups $S, T, U$ with $U = S \cap T$ being a subsemigroup of $S$ and $T$ with a core $U$ in $\mathcal{A}$ and denoted by $[S, T; U]$. An amalgama $[S, T; U]$ of $\mathcal{A}$ is weakly embedable in $\mathcal{A}$ if there exist a semigroup $K$ belonging to $\mathcal{A}$ and monomorphisms $\xi_1 : S \to K$, $\xi_2 : T \to K$ such that the restrictions to $U$ of $\xi_1$ and $\xi_2$ are equal to each other (that is, $\xi_1(S) \cap \xi_2(T) \supseteq \xi_1(U)$). In the case that $\xi_1(S) \cap \xi_2(T) = \xi_1(U)$, we say that an amalgama $[S, T; U]$ of $\mathcal{A}$ is strongly embeddable in $\mathcal{A}$. A semigroup $U$ in $\mathcal{A}$ is amalgamation base [resp. weak amalgamation base] if any amalgam with a core $U$ in $\mathcal{A}$ is strongly embeddable [resp. weakly embeddable] in $\mathcal{A}$. In this paper, we restrict ourselves to the cases that $\mathcal{A}$ is the class of all semigroups or the class of all finite semigroups. We will use the terms “amalgamation base for semigroups” or “weak amalgamation base for finite semigroups” in the former case or the latter.

Okiniki and Putcha [8] proved that any finite semigroup $U$ is an amalgamation base for all finite semigroups if the $J$-classes of $U$ is linearly ordered and the semigroup algebra $\mathbb{C}[U]$ over $\mathbb{C}$ has a zero Jacobson radical. As a by-products they obtained that any finite inverse semigroup $U$ is an amalgamation base for all finite semigroups. In the paper [9] we gave a proof of the result by using representations of semigroups only. The purpose of this paper is to show that the same method enable to extend the the result from inverse semigroup to regular semigroups whose Rees factors satisfy the the conditions, $\text{Ann}_l$ and $\text{Ann}_r$.

Let $U$ be a semigroup with zero, 0, and $a, b \in S$.

The set $\{s \in U \mid sa = 0\}$ is called the left annihilator of $a$ in $S$ and is denoted by $\text{ann}_l(a)$.

In this case, we say that $U$ satisfies the condition $\text{Ann}_l$ if $\text{ann}_l(a) = \text{ann}_l(b)$ implies $aU = bU$. 

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The right annihilator and the condition $Ann_r$ are defined by left-right duality.

**Theorem 1.** Let $U$ be a finite regular semigroup with a chain of ideals such that $U_i$ is a maximal subgroup and each $U_i/U_{i+1}$ is a completely 0-simple semigroups satisfying the conditions $Ann_i$ and $Ann_r$ ($1 \leq i \leq n - 1$). Then $U$ is an amalgamation base for finite semigroups.

Now we can present an example of a semigroup which does not satisfy the right annihilator condition but is an amalgamation base for finite semigroups.

Let $S_n$ be the symmetric group of degree $n$ on the set $X = \{1, 2, \cdots, n\}$ and $R_n$ the right zero semigroup $X$ with multiplication $ij = j$ ($i, j \in X$).

Let $U = S_n \cup R_n$ with multiplication of $U$:

$x \cdot \alpha =$ the image of $x$ by $\alpha$ and $\alpha \cdot x = x$ ($x \in R_n$ and $\alpha \in S_n$).

Then $U$ has the representation extension property and the free representation extension property. Consequently, $U$ is an amalgamation base for semigroups (see [3]).

**Theorem 2.** The semigroup $U = S_n \cup R_n$ with multiplication of $U$:

$x \cdot \alpha =$ the image of $x$ by $\alpha$ and $\alpha \cdot x = x$ ($x \in R_n$ and $\alpha \in S_n$). $U$ is an amalgamation base for finite semigroups or not.

**References**


