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Behaviors of synoptic eddies in the atmosphere

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1. Observational analysis

Synoptic eddies and the background pressure fields are separated in weather charts at 500hPa to investigate the characteristics of propagation of disturbances produced by the baroclinic instability. The geopotential height data used in this analysis are objectively analysed data set from 1985 to 1991 provided by ECMWF. The weather charts for synoptic eddies and the background pressure fields were made with filtered data of periods from 2.5 to 6.5 days and of periods longer than 7 days, respectively.

Spatial correlation of synoptic eddies were calculated at a base grid point at 45N. The correlation pattern depends upon the choice of the base grid point. We calculated 36 correlation patterns by changing the longitude of the base grid points by 10 degrees (at 45N). Fig.1 is the result made by composite of all correlation patterns so that all base points are the center of the figure to eliminate longitudinal effects as done by Randel (1988). Since the synoptic eddies are produced by the baroclinic instability of the westerly, it is not surprising to get a wave train of a low and high pressure system propagating to the east direction along the latitude circle. However, there is a slight tendency for the wave to propagate equatorward.

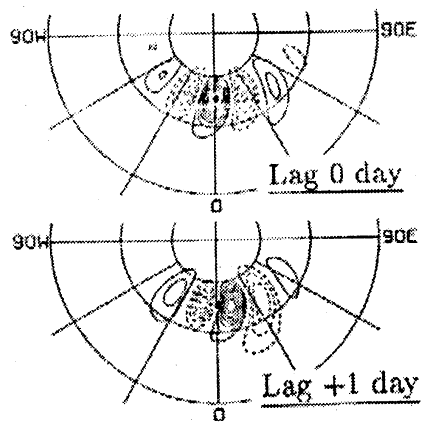

Fig.1 One-point correlation map at 45N in winter season 500hPa. Contour interval is 0.1m.

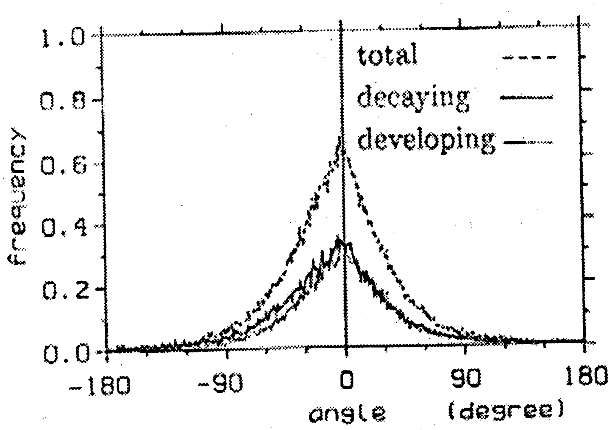

Fig.2 Frequency distributions of angle with zonal direction, normalized by total frequency divided by 50.

This tendency was confirmed by calculating the propagation-velocity vectors of the center of individual pressure anomalies (both positive and negative) by comparing two weather charts with 12-hour interval. Fig.2 shows frequency distributions of angle between each propagation-velocity vectors and the latitude line (negative values mean equatorward propagation). In this figure we separate eddies in the developing stage from the decaying stage. Note that the distribution is not symmetric: The frequency of the equatorward propagation is greater than that of the poleward propagation. Besides, the deviation from the symmetric distribution is greater for the eddies in the decaying stage than for the eddies in the developing stage. This result implies that the equatorward propagation is caused by the barotropic process, because the baroclinic eddies turn out to be barotropic in the decaying stage. Encouraged by this fact, we try to explain this tendency by means of the non-divergent barotropic vorticity equation.

2. Propagation of wave-packet and phase tilt

The linearized non-divergent barotropic vorticity equation with a zonal flow $\bar{u}$ on a sphere is written as

$$\frac{\partial \zeta'}{\partial t} = -\frac{\bar{u}}{\cos\theta}\frac{\partial \zeta'}{\partial \lambda} - v'\frac{d}{d\theta}\left(f + \bar{\zeta}\right) \tag{1}$$

where all variables are non-dimensional, time being scaled by Ω^{-1} and length by the radius of the earth. λ is longitude, θ is latitude and f is the Coriolis parameter, $2\Omega \sin\theta$. Let us introduce the stream function to express $u = -\frac{\partial \varphi}{\partial \theta}$, $v = \frac{1}{\cos\theta}\frac{\partial \varphi}{\partial \lambda}$ and $\zeta = \nabla^2 \varphi$. If Eq.(1) is multiplied by $\zeta' \cos\theta$ and averaged in the zonal direction,

$$\frac{\partial V}{\partial t} = -2\gamma \cdot \overline{v'\zeta'}\cos\theta \tag{2}$$

$$= +2\gamma \cdot \frac{d}{d\mu}\left(\overline{u'v'}\cos^2\theta\right), \tag{3}$$

where μ is sine of latitude, $V = \overline{\zeta'^2}$ and $\gamma = \frac{d}{d\mu}\left(f + \bar{\zeta}\right)$. The positive value of γ makes the mean flow free from barotropic instability (Baines, 1976). The zonaly averaged enstrophy, V, represents the meridional distribution of amplitude of eddies when they are composed of only one zonal wavenumber or an isolated one. We are interested in the propagation of eddies rather than the wave activity discussed in Held and Hoskins (1985).

3. Meridional phase tilt

Let the stream function with single zonal wavenumber m be written as

$$\begin{aligned}\varphi(\lambda,\theta) &= C_m(\theta)\cos(m\lambda) + S_m(\theta)\sin(m\lambda) \\ &= \sqrt{C_m^2 + S_m^2}\,\cos\{m\lambda - m\Xi(\theta)\},\end{aligned} \tag{4}$$

where $m\Xi$ is the longitudinal phase angle at latitude θ, i.e., $m\Xi = \tan^{-1}\frac{Sm}{Cm}$. Then, the meridional gradient of phase or the phase tilt is expressed as

$$\frac{\partial \Xi}{\partial \theta} = \frac{1}{m^2}\frac{\overline{u'v'}\cos\theta}{\overline{\varphi'^2}}. \tag{5}$$

When a vorticity anomaly is isolated, let (λ_0, θ_0) be a point where $\frac{\partial \varphi'}{\partial \lambda} = 0$, i.e., the location of the maximum amplitude. If the latitude is increased by infinitesimal amount $\delta\theta$, the longitudinal location of the maximum amplitude will be shifted by $\delta\lambda$.

$$\frac{\partial \varphi'}{\partial \lambda}(\lambda_0+\delta\lambda,\theta_0+\delta\theta)=\frac{\partial \varphi'}{\partial \lambda}+\frac{\partial^2 \varphi'}{\partial \lambda^2}\cdot\delta\lambda+\frac{\partial^2 \varphi'}{\partial \lambda \partial \theta}\cdot\delta\theta \equiv 0, \tag{6}$$

where the first term in rhs vanishes by definition, and the diffentiation in rhs is taken at (λ_0,θ_0). If Eq.(6) is multiplied by φ' and averaged, we get

$$\frac{\delta\lambda}{\delta\theta}\equiv\frac{\partial \Xi}{\partial \theta}=\frac{\overline{u'v'}\cos\theta}{\overline{v'^2}\cos^2\theta}. \tag{7}$$

When the stream function is represented by a single wavenumber m, Eq.(7) is identical with Eq.(5).

In the Eq.(5) and (7), the NE-SW phase tilt is defined as positive and NW-SE is as negative. Then, the Eq.(3) can be used as a prognostic equation on the wave-packet propagation in the meridional direction. Suppose that a wave-packet of zonal wavenumber m whose phase tilt is exactly NE-SW, i.e., contant with latitude, is located in mid-latitude. In this case the meridional gradient of $\overline{u'v'}\cos\theta$ is positive to the south and negative to the north of the center of it. Form Eq.(3) this means that the wave-packet will propagate into low latitudes. Therefore, the wave-packet with NE-SW(NW-SE) phase tilt is expected to propagate into low latitude (high latitude).

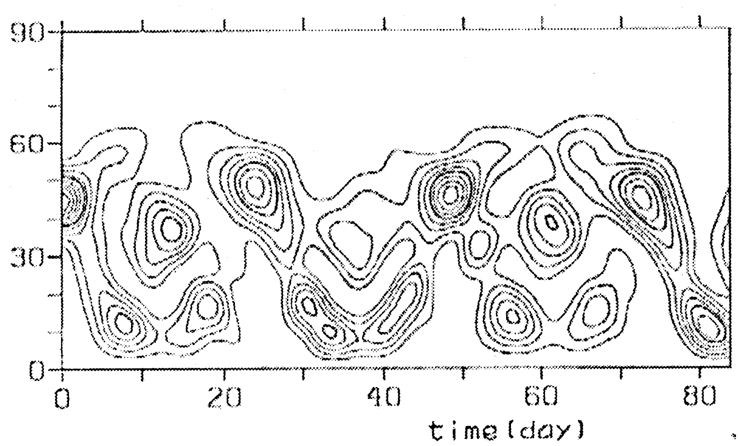

Fig.3 Time evolution of V. Contour interval is 1/10 of the maximum of day 0.

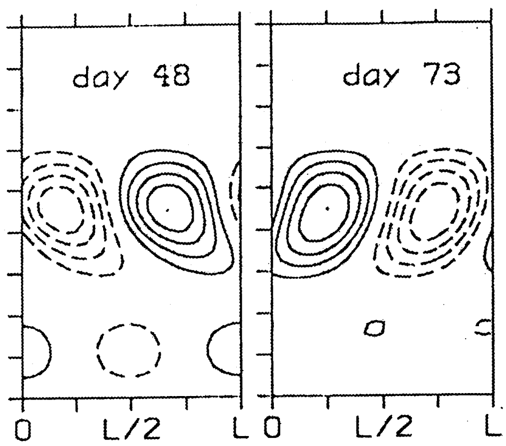

Fig.4 Two phase configurations, NW-SE and NE-SW. CI is 1/5 of the maximum value of the initial condition.

4. Phase tilt and wave-packet propagation with $\bar{u}=0, m=6$

Eq.(1) is represented by the spectral method with a truncation N and time integrated with an appropriate initial condition. We consider an initial vorticity field given by

$$\zeta = C\frac{\cos\theta}{\cos\theta_0}\cos(m\lambda)\exp\left\{-\left(\frac{\theta-\theta_0}{10^\circ}\right)^2\right\}, \tag{8}$$

with $m=6$, $\theta_0=45^\circ$ and $C=0.1$. This vorticity field is symmetric with respect to the equator. Notice that this initial eddy field has no phase tilt. Fig.3 shows time evolution of V. The wave-packet initially located in mid-latitude propagates into low or high latitudes, continuously changing the phase tilt. The initial vorticity field is dispersed by the dispersion relation of each modes of Rossby-Haurwitsz waves; $\sigma_n^m=-\frac{2m}{n(n+1)}$. The local maximum of the eddy amplitude propagates into high or low latitudes, by the interference of Rossby-Haurwitsz waves. Around the day 73 the wave-packet shows an apparent tendency to propagate toward low latitude, while around the day 48 the wave-packet propagates toward high latitude. Vorticity fields of day 48 and 73 are shown in Fig.4. The phase tilt of them is NW-SE and NE-SW, respectively.

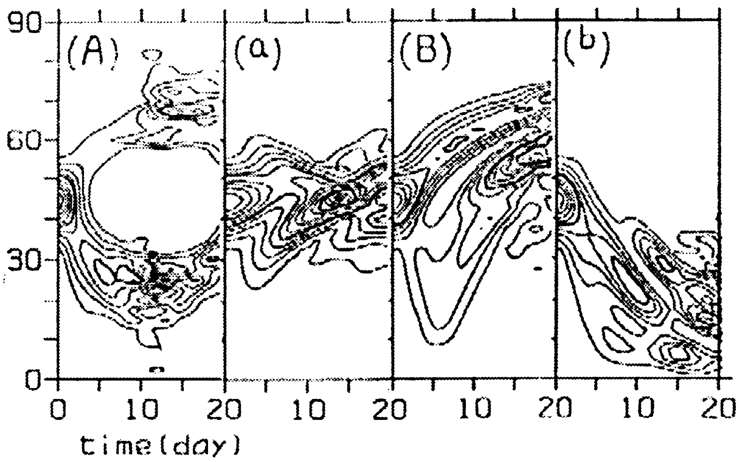

Fig.5 Time evolution of V. The zonal flow type is written on each panel.

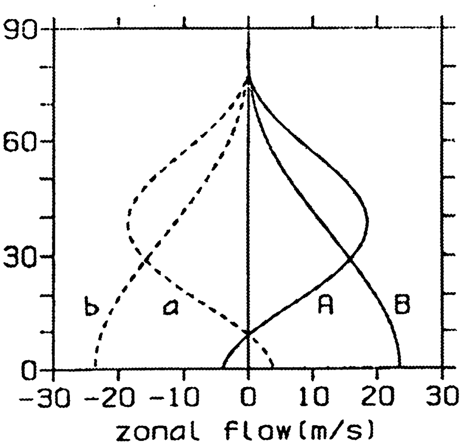

Fig.6 Zonal flows used here.

5. Effect of zonal flow

If the initial eddy given by Eq.(8) is located in a zonal flow, the phase is tilted by the meridional shear of it. Fig.5 shows the time evolutions of the initial eddy in the 4 zonal flows shown in Fig.6. The most striking feature in the presence of the zonal flow is that once the wave-packet propagates into high or low latitude, they are trapped there or reflected and eventually reach a certain latitude. These features can be explained qualitatively. The wave-packet 'A' is splitted into two parts by the zonal flow. The northern part whose phase tilt is NW-SE propagates poleward and the southern part whose phase tilt is NE-SW propagates equatorward. During the propagation the phase tilt is steepened more and more; the phase tilt of the northern part tends to be close W-E and the southern E-W. Before reaching the pole region the phase tilt of the northern part becomes NE-SW, the inverse phase tilt of initial stages, which means the turning of the propagation direction. The wave-packet 'a' is splitted into two parts also, but in this case with the reversed phase tilt compared with the wave-packet 'A'. The wave-packet 'a' tends to back to the mid-latitude forming a waveguide. In case of $m = 6$ the shear effect dominates over that of latitudinally varing Coriolis effect. The simple prognostic Eq. (3) is useful in interpreting the propagation of Rossby wave-packet in horizontal shear flow as in Yamagata (1976), where the trajectory of the wave-packet was calculated by the ray path theory.

6. Packet velocity

The usual concept of group velocity cannot be used for the meridional energy propagation in our problem. Instead, we can define a 'packet velocity' on the basis of the basic concept. The latitudinal location of largest amplitude is solely due to the zonal phase propagation of Rossby-Haurwitz waves in the absence of the zonal flow. In the presence of the shear flow, however, the location of largest amplitude is determined by both the shear flow and the dispersion of Rossby-Haurwitz waves due to the Coriolis factor. Which factor will dominate depends on the detailed profile of the zonal flow as well as the zonal wavenumber of the vorticity field. We define the 'packet velocity' as the propagation speed of the location of local maximum. Let the vorticity field be

$$\varsigma' = \sum_{n=m}^{m+N} \hat{\varsigma}_n^m P_n^m(\mu)\cos(m\lambda + \alpha_n^m), \tag{9}$$

where $\alpha_n^m = \sigma_n^m t + \alpha_n^{m0}, \alpha_n^{m0}$ is the initial phase in the longitude. Then, by definition and from Eq. (2)

$$V = \frac{1}{2}\sum_{l,n} \hat{\varsigma}_l^m \hat{\varsigma}_n^m P_l^m(\mu) P_n^m(\mu)\cos(\alpha_l^m - \alpha_n^m), \tag{10}$$

$$\frac{\partial V}{\partial t} = \frac{1}{2}\gamma(\mu)\sum_{l,n} \hat{\varsigma}_l^m \hat{\varsigma}_n^m P_l^m(\mu) P_n^m(\mu)\sigma_l^m \sin(\alpha_l^m - \alpha_n^m) \tag{11}$$

Let μ_0 be where $\frac{\partial v}{\partial \mu} = 0$ at t_0. Then the packet velocity can be written as

$$V_{gv}\cdot\cos\theta = \frac{\delta\mu}{\delta t} = -\frac{\partial^2 V}{\partial\mu\partial t}\left(\frac{\partial^2 V}{\partial\mu^2}\right)^{-1}. \tag{12}$$

The differentiation with respect to μ can be evaluated directly by using the recursion relation of Legendre polynomials. One μ_0 is known, the packet velocity can be calculated for any zonal flow. We show the calculated 'packet velocity' of the vorticity field of day 73 in Fig.7. The presence of the zonal flow alters the packet velocity to a large extent. The zonal flow of 'A' enhances the equatorward propagation of eddies into low latitude, while 'a' does not.

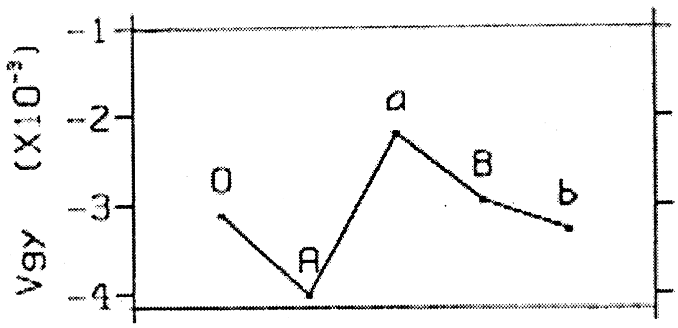

Fig.7 The packet velocity. The letters A, a, B and b represent zonal flow types. O denotes the case without zonal flow.

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