Certain associated graded rings of 3-dimensional regular local rings are regular (Newton polyhedrons and Singularities)

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Citation
数理解析研究所講究録 (2001), 1233: 95-101

Issue Date
2001-10

URL
http://hdl.handle.net/2433/41498

Type
Departmental Bulletin Paper

Textversion
publisher

Kyoto University
Certain associated graded rings
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This note is a preliminary version.

Introduction. The study of various blowing-ups is very important in the theory of singularities. In many cases some blowing-up appears as the blowing-down of divisors of algebraic varieties, and is understood naturally as a filtered blowing-up. From this point of view, one of the most interesting results in this field is M. Kawakita's classification of a special divisorial contraction of dimension three [2]. In [2], Kawakita proved that every divisorial contraction to a smooth 3-dimensional point is a weighted blowing-up induced by certain weightings on a regular system of parameters of 3-dimensional regular local rings. It is natural to study his theorem from the theory of filtered blowing-ups, and this is my motivation for this talk.

In this paper, I will discuss the filtered blowing-up of singularities, and, by using special equi-singular deformation induced from a filtration on local ring, I show the following simple assertion.

Theorem 1. Let $A \cong \mathbb{C}\{x_1, x_2, x_3\}$ and $F = \{F^k\}_{k \geq 0}$ be a filtration on $A$ such that $gr_F A = \bigoplus_{k \geq 0} F^k/F^{k+1}$ is an integral domain with isolated singularity. Then $gr_F A$ is regular, i.e., $gr_F A \cong \mathbb{C}[y_1, y_2, y_3]$.

In this paper, a filtration $F$ on the local ring $(A, m)$ is; $F = \{F^k\}$; a decreasing sequence of ideals $F^k \subset A$ such that $F^0(A) = A, m \supset F^1, F^k = A(k \leq -1), F^kF^l \subset F^{k+l}(\forall k, l)$ and $R = \bigoplus_{k \geq 0} F^kT^k \subset A[T]$ is a finitely generated $A$-algebra, where $T$ is an indeterminate. There is an integer $N$ such that the relation $F^{kN} = F^N \cdots F^N$ for all $k \geq 0$, and we assume that $F^N$ is $m$-primary. We denote $G = gr_F(A)$ and remark that $G = R'/T^{-1}R'$, where $R' = \bigoplus_{k \in \mathbb{Z}} F^kT^k$ is the extended Rees algebra.

Theorem 1 is shown as a special case of the following more general results.

Theorem 2. Let $(V, p)$ be a normal d-dimensional isolated terminal singularity of index $r$ (resp. canonical, resp. log terminal, resp. log canonical), and $F = \{F^k\}$ be a filtration on $A = O_{V, p}$ such that $G = gr_F A$ is an integral domain with isolated singularity. Then

(1) $G$ is normal and terminal singularity of index $r$ (resp. canonical, resp. log terminal, resp. log canonical).

(2) There is a filtration $F_B = \{F^k_B\}$ on the canonical cover (the index one cover) $B = \bigoplus_{m=0}^{k-1} [m]_A$ such that $G_B = gr_{F_B} B \cong$ the canonical cover of $G$ and there exists an integer $M \geq 1$ such that the relations $F^k_B \cap A = F^k \subset A$ for $k \geq 0$ and $(gr_{F_B \cap A}(A))(M) = gr_F(A)$ hold.

(3) If $d = 3$ and $(V, p)$ is terminal, then the relation $e(m_B, B) = e((G_B)_+, G_B) (= 1, 2)$ holds.
We have a corollary as follows:

**Corollary 3.** $(V,p)$: 3-dimensional cyclic terminal and $F$: as above, then $gr_F(A)^\wedge \cong A^\wedge$.

As the case of index one, we obtain Theorem 1 from Corollary 3. Here recall that every isolated quotient singularity of dimension not less than three is rigid.

In general, if we consider a filtration induced from a divisorial contraction, the associated graded ring is not necessary an integral domain with isolated singularity ([1,3]).

§1. Sketch of proof of Theorem 2.

We assume that there is no $N \geq 2$ such that $G^{(N)} = G$, where $G^{(N)}$ is defined by $G^{(N)} = \oplus_{k \geq 0} G_{kN} \subset G$.

**Step 1.** Let $\psi : X = \text{Proj}(R) \rightarrow V = \text{Spec} A$ be the filtered blowing-up by $F$ with $E = \text{Proj}(G)$. We obtain the relation $F^k = \phi_*(O_X(-kE))$ for $k \in \mathbb{Z}$. (cf [6, §2]).

**Proof.** Since $G$ is an integral domain and $V = \text{Spec} A$ is normal, we can easily see that $R' = \oplus_{k \in \mathbb{Z}} F^kT^k \subset A[T, T^{-1}]$ is a normal domain.

This claim is shown as follows: We have $G = R'/uR'$, where $u = T^{-1} \not\in R_{\leq 1}$. If $P \in V(u) \subset \text{Spec}(R')$, then $G_P \cong R'_P/uR'_P$ satisfies the conditions $R_0$ and $S_1$, hence $R'_P$ is normal. Further, if $P \not\in V(u)$, then we obtain the relations $R'_P = (R'_{f,P}) = A[T, T^{-1}]_P$ which is normal.

By the assumption that $R$ is a finitely generated $A$-algebra, there is a positive integer $N > 0$ such that $F^{kN} = F^N \cdots F^N$, for $k \geq 0$, i.e., $R^{(N)} = A[F^NT^N]$. Here $\psi$ is the blowing-up with center $F^N$ and $F^{kN} = \psi_*(F^{kN}O_X)$. Since $Q(G)$ has a homogeneous element of degree 1, we have $O_X(k) = (O_X(1)^\otimes k)^{**}$ for $k \in \mathbb{Z}$. We have $O_X(1) = O_X(-E)$, hence $O_X(1) = O_X(-NE)$. Since $G$ is an integral domain, $\{F^k\}$ defines a valuation $V$ on $Q(A)$ such that $F^k = \{x \in Q(A) \mid V(x) \geq k\}$. Further $\{F^{kN}\}$ defines the valuation $V'$ on $Q(A)$ as $F^{kN} = \{x \in Q(A) \mid V_E(x) \geq kN\}$ where $V_E(x) = ord_E(x)$ on $X$. Therefore $F^k = \{x \in Q(A) \mid V_E(x) \geq k\}$ for $k \in \mathbb{Z}$.

**Step 2.** $X$ has only cyclic quotient singularities, in particular $X$ has only log terminal singularities.

**Proof.** (cf §5 [6]). For $P \in E = \text{Proj}(G) \subset X = \text{Proj}(R)$, there exists $f \in F^d - F^{d+1}$, with $P \in V_+(f^*)$, where $f^* = fT^d \in R_d$. Here we denotes $fT^d \in G_d$. Now $R_{f^*} = \oplus_{k \in \mathbb{Z}} (R_{f^*})_k$ is a regular ring. This is shown as follows: We see that $(R_{f^*})_{(T^{-1})^{-1}} = A_{f[T, T^{-1}]}$ is regular and that $R_{f^*}/T^{-1}R_{f^*} = R_{f^*}/T^{-1}R_{f^*} = G_f$ is regular. Hence so is $R_{f^*}$.

Now let $B = (R_{f^*})_P = \oplus_{k \in \mathbb{Z}} ((R_{f^*})_P)_k$ and $t \in B$ be a homogeneous unit of the minimal degree $N(P)$. Let $C = B/t - 1$. Then, by [6,§5], $C$ is a regular local ring. Here $((R_{f^*})_P)_0$ is a finite direct summand of $C$.

**Step 3.** (The log canonical condition of $A$ implies that ) $G$ is normal.

**Proof.** Let $\omega_0 \in \omega_A^r$ be a generator at $p$ as $\omega_A^r = A \cdot \omega_0$. We define the integer
a' by the relation $\text{div}_X(\omega_0) = -(r + a')E$ on $X$. That is $\omega_X^{[r]} \cong O_X(-(r + a')E)$ or $K_X = \psi^*(K_V) - (1 + \frac{a'}{r})E$. Since $A$ is log canonical, we have $a' \leq 0$. We will show the following.

Claim. $R^1\psi_*(O_X(-mE)) = 0$ for $m \geq 1$, $(m \in \mathbb{Z})$.

Proof of the claim. We have the relation

$$O_X(-mE) \cong \omega_X^r((r-1)K_X + (r+a')E-mE) \cong \omega_X^r((r-1)(K_X+E)-(m-1-a')E).$$

Further $(r-1)(K_X+E)-(m-1-a')E$ is relatively numerically equivalent to $-\frac{r-1}{r}a'E-(m-1-a')E = -(m-1-\frac{a'}{r})E$ with respect to $\psi$. Since $-E$ is relatively $\psi$-ample, $(r-1)(K_X+E)-(m-1-a')E$ is $\psi$-nef. Hence by the vanishing theorem of Grauert-Riemenschneider,Kawamata-Viehweg, we obtain the claim.

Here we have the exact sequence

$$0 \rightarrow O_X(-(k+1)E) \rightarrow O_X(-kE) \rightarrow O_E(k) \rightarrow 0$$

for $k \in \mathbb{Z}$. By the claim, we obtain the following exact sequence

$$0 \rightarrow F^{k+1} = \psi_*(O_X(-(k+1)E)) \rightarrow F^{k} = \psi_*(O_X(-kE)) \rightarrow H^0(O_E(k)) \rightarrow R^1\psi_*(O_X(-(k+1)E)) = 0$$

for $k \geq 0$.

We have

$$0 \rightarrow H^0_{G_+}(G) \rightarrow G \rightarrow \oplus_{k \in \mathbb{Z}} H^0(O_E(k)) \rightarrow H^1_{G_+}(G) \rightarrow 0.$$

Since $G$ is an integral domain, $H^0_{G_+}(G) = 0$. Further $\oplus_{k \in \mathbb{Z}} H^0(O_E(k)) = \Gamma_*(G)$ is normal. This is shown as follows: Let $\bar{G}$ be the normalization of $G$ in $Q(G)$. Since $G$ has only isolated singularity, $\bar{G}/G$ has finite length. Hence on $E = \text{Proj}(G)$, we have the relation $\bar{G}(k) = G(k)$. By Demazure, with $T \in Q(\bar{G})_1$, there exists $D \in \text{Div}(E)\otimes \mathbb{Q}$ as follows; $\bar{G}(k) = O_E(kD)T^k$, for $k \in \mathbb{Z}$. Hence $\Gamma_*(G) = R(E, D) = \bar{G}$.

Therefore, we obtain the relation $H^1_{G_+}(G) = 0$ for $k \leq -1$. And the relation $H^1_{G_+}(G) = 0$ follows.

Step 4. We will discuss the log terminal property of $G = R(E, D)$ under the assumption that $A$ is log terminal of index $r$.

We have the following.

Lemma [8]. Let us assume the conditions that $G$ is an integral domain where $\text{Spec}(G) - V(G_+)\mathbb{Z}$ is normal Gorenstein and that $\text{Spec}(A) - V(m)$ is Gorenstein. Then the following relations hold.

$$\frac{\omega_X^m(mE - \alpha E)}{\omega_X^m(mE - (\alpha+1)E)} \cong \omega_E^m(mD' + \alpha D) \text{ for } m, \alpha \in \mathbb{Z}.\)$$

Here $O_E(k) = O_E(kD)T^k$ as before, with $D = \sum_{V \in \text{Irr}^1(X)} \frac{p_V}{q_V} V$ with $(p_V, q_V) = 1$, $\text{Spec}(G) - V(m)$. 

Hence by the vanishing theorem of Grauert-Riemenschneider,Kawamata-Viehweg, we obtain the claim.
\( q_V \geq 1 \) and \( D' = \sum_{V \in \text{Irr}^1(X)} \frac{q_V - 1}{q_V} V \).

By the relation
\[ \omega_X^{[r]}(rE - \alpha E) \cong O_X(-(a' + \alpha)E), \text{ for all } \alpha \in \mathbb{Z} \]
we obtain
\[ \omega_E^{[r]}(rD' + \alpha D) \cong O_E((\alpha + a')D), \text{ for all } \alpha \in \mathbb{Z} \]
by Lemma. Hence \( K_{R}^{[r]} = R(a') \) follows.

Here \( \text{Spec}(R) - V(R_+) = \text{Spec}(G) - V(G_+) \) is regular, \( G = R(E, D) \) is log terminal (resp. log canonical) if and only if \( a' < 0 \) (resp. \( a' \leq 0 \)) by Theorem (2.5) and Theorem (2.8) of [7].

We will discuss the index of \( R \). By Lemma, we have the following exact sequence.
\[
0 \to \frac{T^m \omega_{R}^{[m]}}{T^{m-1} \omega_{R}^{[m]}}, \to K_{R(E,D)}^{[m]} \to \\
\oplus_{k \in \mathbb{Z}} \text{Ker} \left\{ H^1(\omega_X^{[m]}(mE - (k+1)E)) \to H^1(\omega_X^{[m]}(mE - kE)) \right\} \to 0 \text{ for } m \in \mathbb{Z}.
\]

If there exist \( r' > 1 \) where the relation \( K_{R(E,D)}^{[r']} = R(a'') \) is satisfied for some integer \( a'' \in \mathbb{Z} \), we have the relation \( \frac{a''}{r'} = \frac{a'}{r} \). We obtain \( a'' < 0 \).

Here \( (K_{R}^{[r']})_k = R_{k+a''}, \) hence \( (K_{R}^{[r']})_k = 0 \) if \( k \leq -1 \).

For \( k \geq 0 \), we set \( m = r' \geq 1 \) and obtain the relations
\[ \omega_X^{[m]}(mE - (k + 1)E) = \omega_X((m - 1)(K_X + E) - kE), \]
and
\[ (m - 1)(K_X + E) - kE \equiv - \left(-\frac{m-1}{r}a' + k \right)E. \]

This is \( \psi \)-nef, hence the following vanishing hold
\[ H^1(\omega_X^{[m]}(mE - (k+1)E)) = 0 \text{ for } k \geq 0. \]

Hence \( \frac{T^m \omega_{R}^{[m]}}{T^{m-1} \omega_{R}^{[m]}}, \cong K_{R(E,D)}^{[m]} \text{ with } m = r' \). Hence \( T^m \omega_{R}^{[m]} \) is locally principal along \( V(T^{-1}) = \text{Spec}(R(E, D)) \subset \text{Spec}(\mathcal{R}) \).

For \( c \neq 0 \in \text{Spec}(\mathbb{C}[T^{-1}]), \) it follows that \( \omega_{\mathcal{R}}^{[m]}/(T^{-1} - c) \omega_{\mathcal{R}}^{[m]} = \cup_{k \in \mathbb{Z}} \psi_*(\omega_{X}^{[m]}(-kE)) = \omega_{A}^{[m]} \) is a principal \( \mathcal{R}'/(T^{-1} - c) \mathcal{R}' = A \)-module for same \( c \).

**Step 5.** We will show: The condition that \( A \) is a canonical (resp. terminal) singularity implies that \( G \) is also a canonical (resp. terminal) singularity.

**Proof.** Let \( \omega : \mathcal{V} = \text{Spec}(\mathcal{R}') \to \text{Spec}(\mathbb{C}[T^{-1}] \cong \mathbb{C} \text{ with } V_0 = \text{Spec}(G), \) and \( V_c \cong V \) for \( c \neq 0 \). Let us introduce the filtration of ideals \( \{ F^l(\mathcal{R}') \} \) on \( \mathcal{R}' \) by the following way: \( F^l(\mathcal{R}') = \mathcal{R}' |_{\text{Spec}(\mathcal{R}') \subset \mathcal{R}'}, \) where \( \mathcal{R}' = \oplus_{k \geq l} F^l T^l \subset \mathcal{R}' \) for \( l \in \mathbb{Z} \). As is shown in [6]§5, we obtain the following diagram after the blowing-up of \( \mathcal{V} = \text{Spec}(\mathcal{R}') \) by this
\[ Y'' = \Proj(\mathcal{R}(\mathcal{R}')) \xrightarrow{\xi} \Spec(\mathcal{R}') = \mathcal{V} \]

\[ \omega'' \xrightarrow{\psi} \omega \]

\[ \Spec\mathbb{C}[T^{-1}] \]

where \( \omega'' \) gives the filtered blowing-up for each fiber as follows: \( \omega''_0 : Y''_0 \to \Spec(\mathcal{V}_0) \) is nothing but the graded blowing-up of \( \Spec(\mathcal{G}) \) and \( \omega''_c : Y''_c \to \Spec(\mathcal{V}_c) \) is nothing but the blowing-up of \( \Spec(\mathcal{A}) \) by \( F \) for \( c \neq 0 \in \mathbb{C} \). By J. Wahl [9], \( \omega'' \) is a locally trivial family under the assumption that \( \Spec(\mathcal{G}) - V(G_+ \cup G_-) \) is regular. Here \( \mathcal{V} \) is an \( r \)-Gorenstein \( d + 1 \)-dimensional scheme and we have the following relation

\[ K^{[r]}_{\mathcal{R}} \cong \mathcal{R}'(a' + r). \]

There is a meromorphic \( r \)-ple \( d + 1 \)-form \( \tilde{\Omega}_0 \) of \( \mathcal{R}' \) such that \( \mathcal{R}' \to K^{[r]}_{\mathcal{R}}; 1 \to \tilde{\Omega}_0 \) gives an isomorphism. This induces the isomorphism

\[ \omega_{Y''}^{[r]} = O_{Y''}(r + a')\xi^*(\tilde{\Omega}_0), \]

that is, we have the relation \( \mathrm{div}_{Y''}\tilde{\Omega}_0 = -(r + a')E \), where the relation \( \Proj gr_{\mathcal{F}}(\mathcal{R}') = \mathcal{E} \cong E \times \mathbb{C} \). Here \( \mathcal{E} = \Proj(\mathcal{G}) \). Since \( a' \leq -r \), \( \xi^*(\tilde{\Omega}_0) \) is holomorphic on \( Y'' \). Hence \( \Res_{(Y'')_c}(\tilde{\Omega}_0) \) is a holomorphic \( r \)-ple \( d \)-form on \( (Y'')_c \) which does not vanish on \( (Y'')_c - E \). Here \( (Y'')_c = \Proj(\mathcal{R}) \) for \( c \neq 0 \), and \( (Y'')_c = \Proj(G^k) = C(E, D) \) for the case \( c = 0 \). Here \( \Res_{(Y'')_c}(\tilde{\Omega}_0) \) gives a generator of \( \omega_{Y''}^{[r]} \) for \( c \in \mathbb{C} = \Spec(\mathbb{C}[T^{-1}]) \).

We state the following claim.

Claim. There is a resolution of singularities \( \beta : \tilde{Y}'' \to Y'' \) such that the natural induced map \( \tilde{\omega}'' : \tilde{Y}'' \to \mathbb{C} \) is locally trivial along the fiber over \( \{0\} = V(T^{-1}) \):

\[ \tilde{Y}'' \xrightarrow{\tilde{\omega}''} \Spec\mathbb{C}[T^{-1}] \]

Let \( \mathcal{F} \subset \tilde{Y}'' \to \mathbb{C} \) be the horizontal divisor of \( \tilde{Y}'' \) which is exceptional for \( \beta : \tilde{Y}'' \to Y'' \). For \( c \neq 0 \), we have the relation:

\[ \Res_{|\tilde{Y}''_c}(\beta^*(\tilde{\Omega}_0)) = \beta^*(\Res_{|Y''_c}(\tilde{\Omega}_0)). \]

Since \((A, m)\) has only canonical singularities, this is holomorphic. Hence \( \tilde{\Omega}_0 \) is holomorphic on \( \tilde{Y}'' \). Therefore \( \Res_{|\tilde{Y}''_0}(\beta^*(\tilde{\Omega}_0)) \) is holomorphic.

Q.E.D. for the claim.

Step 6. Here we will introduce a filtration \( F_B \) on the local ring \( B = \oplus_{k=0}^{r} \mathcal{R}'_{k} \mathcal{R}_A \) which has the desired properties as is claimed in Theorem 2.

By a tentative way, we set \( F_B^k(\omega_A^{[m]}) \subset \omega_A^{[m]} \) as follows:

\[ F_B^k(\omega_A^{[m]}) = \sum_{ma'+rh \geq k \cdot \gcd(a', r)} \psi_* \left( \omega_X^{[m]}(mE - kE) \right) \subset \omega_A^{[m]}, \]

and

\[ F_B^k(B) = \oplus_{m=0}^{r-1} F_B^k(\omega_A^{[m]}) U^m \subset B = \oplus_{m=0}^{r-1} \omega_A^{[m]} U^m. \]
The main point which we have to check here is the assertion that the associated graded ring of $gr_{F_{B}}B$ is nothing but the graded canonical cover $G = R(E, D)$. We can show this assertion by the following formula about graded cyclic covers which we will recall in the below.

Now, $K_{G}$ is a $Q$–Cartier divisor of index $r$ and there exists $\varphi \in k(X)$ such that $rK_{E} - a'D = div_{X}(\varphi)$.

**Corollary (1.7.1) of [7].** Let $S = S(R, K_{R}, \varphi T^{a'})$ be the normal graded cyclic $r$-cover of $R = R(X, D)$ as described in [7]. Then the Pinkham-Demazure construction $S$ with respect to $T = T^{b}u^{\alpha}$ with $\alpha a' + \beta r = s(= (r, a'))$ is given by $S = R(F, D)$ as follows:

1. $F$ is the cyclic cover of $E$ given by
   \[ \rho : F = Spec_{E}(\bigoplus_{l \geq 0} O_{E}(l \left( \frac{r}{s}(K_{X} + D') - \frac{a'}{s}D \right))) \rightarrow E. \]

2. $\tilde{D} = \rho^{*}\{\alpha(K_{X} + D') + \beta D\}$.

3. We obtain the relation $K_{S} = S(\frac{a'}{s})$.

By using Lemma B and the above theorem we can check the assertion. The details are left to the readers.

Further we obtain the following relations;

\[ F_{B}^{k} \cap A = F_{B}^{k}(\omega_{E}^{[0]}) = \sum_{h \geq k^{\text{red}}(a', r)} \psi_{*}(O_{X}(-hE)) = F(k^{\text{red}}(a', r)). \]

**Step 7.** Now we assume that $d = \dim A = 3$ and that $(V, p)$ is a terminal singularity of index $r$. Then so is $gr_{F}(A) = R(E, D)$. Since $gr_{F_{B}}(B)$ is the graded canonical cover of $gr_{F}(A)$, $gr_{F_{B}}(B)$ is a terminal 3-dimensional singularity of index one, hence is regular or compound Du Val singularity. In particular, $gr_{F_{B}}(B)$ is a hypersurface isolated singularity by M. Reid [4].

We have the following results on multiplicities of filtered rings;

**Lemma** [5]. Let $P(G_{B}, \lambda) = \sum_{k>0} l((G_{B})_{k}) \lambda^{k} \in Z[[\lambda]]$ and $x_{1}, \ldots, x_{s} \in (G_{B})_{+}$ be a homogeneous minimal generator with $\deg x_{1} \leq \deg x_{2} \leq \ldots \leq \deg x_{s}$. Then we have the followings.

1. $\deg x_{1} \cdot \deg x_{2} \cdots \deg x_{d} \lim_{\lambda \rightarrow 1} (1 - \lambda)^{d}P(G_{B}, \lambda) \leq e(m_{B}, B) \leq e((G_{B})_{+}, G_{B})$.

   Hence, if $e((G_{B})_{+}, G_{B})$ equals the round up of the rational number $\deg x_{1} \cdot \deg x_{2} \cdots \deg x_{d} \lim_{\lambda \rightarrow 1} (1 - \lambda)^{d}P(G_{B}, \lambda)$, then we have the equality $e(m_{B}, B) = e((G_{B})_{+}, G_{B})$.

2. If $G_{B}$ is a hypersurface isolated singularity which is defined by a quasi-homogeneous polynomial of type $(\deg x_{1}, \ldots, \deg x_{d+1}; h)$, then $\deg x_{1} \cdot \deg x_{2} \cdots \deg x_{d} \lim_{\lambda \rightarrow 1} (1 - \lambda)^{d}P(G_{B}, \lambda) = \frac{h}{\deg x_{d+1}}$ and $e((G_{B})_{+}, G_{B})$ equals to the round up of the rational number $\frac{h}{\deg x_{d+1}}$.

Hence we obtain the relation $e(m_{B}, B) = e((G_{B})_{+}, G_{B})(= 1, or 2).
This completes the proof of Theorem 2.

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