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<th>Title</th>
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On a New Existence Result for Cone Saddle Point Problems

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1 Introduction

Studies on vector-valued minimax theorems or vector saddle point problems have been extended widely; see [6] and references cited therein. Existence results for cone saddle points are based on some fixed point theorems or scalar minimax theorems; see [5]. Recently, this kind of problems is solved by a different approach in [3], in which a vector variational inequality problem is treated in a finite dimensional vector space. In this paper, we consider its generalization to vector problems involving the concept of moving cone in the general setting of a normed space.

2 Problem Formulation and Existence Result

Let $K$ and $E$ be nonempty subsets of a normed space $X$ and a topological vector space $Y$, respectively, and let $Z$ be a normed space.

Given a vector-valued function $L : K \times E \to Z$ and a pointed convex cone $C$ on $Z$ with int$C \neq \phi$, Vector Saddle Point Problem (in short, VSPP) is to find $x_0 \in X$ and $y_0 \in Y$ such that

$$L(x_0, y_0) - L(x, y_0) \notin \text{int} C, \quad \forall x \in K,$$

$$L(x_0, y) - L(x_0, y_0) \notin \text{int} C, \quad \forall y \in E.$$ 

A solution $(x_0, y_0)$ of (VSPP) is called a weak $C$-saddle point of the function $L$.

On the other hand, Vector Variational Inequality Problem (in short, VVIP) is to find $x_0 \in K$ and $y_0 \in T(x_0)$ such that

$$
\langle L'(x_0, y_0), x - x_0 \rangle \notin \text{int} C, \quad \forall x \in K,
$$

where $T : X \to Y$ is a multifunction defined by

$$
T(x) := \{ y \in C \mid L(x, v) - L(x, y) \notin \text{int} C, \quad \forall v \in E \},
$$

and $L'(x_0, y_0)$ denotes the Fréchet derivative of $L$ with respect to the first argument at $(x_0, y_0)$. 

...
Definition 2.1 A function \( f : K \rightarrow Z \), where \( K \) is convex set, is called \( C \)-convex if for each \( x, y \in K \) and \( \lambda \in [0,1] \),

\[
\lambda f(x) + (1 - \lambda) f(y) - f(\lambda x + (1 - \lambda)y) \in C.
\]

Definition 2.2 A function \( f : K \rightarrow Z \) is called Fréchet differentiable if for every \( x \in K \) and \( \varepsilon > 0 \), there exists \( f'_x \in L(K, Z) \) and \( \delta > 0 \) such that

\[
\| f(x+h) - f(x) - f'_x(h) \| < \varepsilon \text{ for all } h \in K; \| h \| < \delta,
\]

where \( L(K, Z) \) is the space of all linear continuous operators from \( K \) into \( Z \).

First we show an equivalence condition between (VSPP) and (VPIP).

Theorem 2.1 Suppose that \( K \) is convex and \( L \) is \( C \)-convex and Fréchet differentiable in the first argument. Then problems (VSPP) and (VPIP) have the same solution set.

Proof. Assume that \( (x_0, y_0) \in K \times E \) is a solution of (VSPP). Then

\[
L(x_0, y_0) - L(x, y_0) \notin \text{int } C,
\]

for all \( x \in K \).

\[
L(x_0, y) - L(x_0, y_0) \notin \text{int } C,
\]

for all \( y \in E \). Since \( K \) is convex, We have

\[
x_0 + \alpha (x - x_0) \in K,
\]

for all \( x \in K \) and \( \alpha \in [0,1] \). Hence condition(1) implies

\[
\alpha^{-1} [L(x_0 + \alpha (x - x_0), y_0) - L(x_0, y_0)] \notin \text{int } C,
\]

for all \( x \in K \) and \( \alpha \in (0,1] \). Since \( Z \backslash \text{int } C \) is closed and \( L \) is Fréchet differentiable in the first argument, it follows that

\[
\langle L'(x_0, y_0), x - x_0 \rangle \notin \text{int } C,
\]

for all \( x \in K \). \( y_0 \in T(x_0) \) follows from (2).

Conversely, assume that \( (x_0, y_0) \in K \times E \) is a solution of (VPIP). Then we have

\[
\langle L'(x_0, y_0), x - x_0 \rangle \notin \text{int } C,
\]

for all \( x \in K \) and

\[
L(x_0, y) - L(x_0, y_0) \notin \text{int } C,
\]

for all \( y \in E \). Since \( L \) is \( C \)-convex with respect to the first argument, we have

\[
\alpha L(x, y_0) + (1 - \alpha) L(x_0, y_0) - L(x_0 + \alpha (x - x_0), y_0) \in C,
\]

for all \( x \in K \) and \( \alpha \in (0,1) \), and since \( C \) is cone, we have

\[
L(x, y_0) - L(x_0, y_0) - \frac{L(x_0 + \alpha (x - x_0), y_0) - L(x_0, y_0)}{\alpha} \in C,
\]
for all $x \in K$ and $\alpha \in (0,1)$. Since $L$ is Fréchet differentiable with respect to the first argument, if $\alpha$ converge to 0, then we have
\[
L(x, y_0) - L(x_0, y_0) - \langle L'(x_0, y_0), x - x_0 \rangle \in C,
\]
for all $x \in K$. From condition(3), it follows
\[
L(x_0, y_0) - L(x, y_0) \notin \text{int } C
\]
for all $x \in K$. Hence $(x_0, y_0) \in K \times E$ is also a solution of (VSPP).

Now, we introduce Fan-KKM theorem, which is important in the field related to (VVIP), for theorem 2.3.

**Theorem 2.2** (Fan-KKM Theorem see[4]) Let $X$ be a subset of a topological vector space. For each $x \in X$, let a closed set $F(x)$ in $X$ be given such that $F(x)$ is compact for at least one $x \in X$. If the convex hull of every finite subset $\{x_1, \ldots, x_n\}$ of $X$ is contained in the corresponding union $\bigcup_{x \in X} F(x)$, then $\bigcap_{x \in X} F(x) \neq \phi$.

Next we show an existence result of (VSPP) by using (VVIP).

**Theorem 2.3** Let $K$ and $E$ be a nonempty closed convex subset of a normed space $X$ and a nonempty compact subset of a topological vector space $Y$, respectively. Assume that the vector-valued function $L$ is continuously differentiable and $C$-convex in the first argument and $L'$ is continuous in both $x$ and $y$, and let $T : K \to E$ be the multifunction defined by

\[
T(x) := \{ y \in E \mid L(x, v) - L(x, y) \notin \text{int } C, \quad \forall v \in E \}.
\]

If there exists a nonempty compact subset $B$ of $X$ and $x \in B \cap K$ such that for any $x \in K \setminus B$ and $y \in T(x),$

\[
\langle L'(x, y), x_0 - x \rangle \in -\text{int } C,
\]
then problem (VSPP) has at least one solution.

**Proof.** In order to proof the theorem, it is sufficient to show that (VVIP) has at least one solution $x_0 \in K$, $y_0 \in T(x_0)$. Define a multifunction $F : K \to K$ by

\[
F(u) = \{ x \in K \mid \langle L'(x, y), u - x \rangle \notin -\text{int } C, \quad \text{for some } y \in T(x) \}, \quad u \in K.
\]

First, we prove that the convex hull of every finite subset $\{x_1, x_2, \ldots, x_n\}$ of $K$ is contained in the corresponding union $\bigcup_{i=1}^{m} F(x_i)$, that is, $\text{Co}\{x_1, x_2, \ldots, x_m\} \subset \bigcup_{i=1}^{m} F(x_i)$. Suppose to the contrary that there exist $x_1, x_2, \ldots, x_m$ and $\alpha_1, \alpha_2, \ldots, \alpha_m$ such that

\[
\hat{x} := \sum_{i=1}^{m} \alpha_i x_i \notin \bigcup_{i=1}^{m} F(x_i), \quad \sum_{i=1}^{m} \alpha_i = 1.
\]

Then, $\hat{x} \notin F(x_i)$ for all $i = 1, \ldots, n$, and hence for any $y \in T(\hat{x}),$

\[
\langle L' \hat{x}, y \rangle, x_i - \hat{x} \rangle \in \text{int } C,
\]
for all $i = 1, \ldots, m$. Since $\text{int} C$ is convex, we have
\begin{equation}
\sum_{i=1}^{m} \alpha_i \langle L'(\hat{x}, y), x_i - \hat{x} \rangle \in -\text{int} C.
\end{equation}

Since $L'(\hat{x}, y)$ is a linear operator, we have
\begin{equation}
\langle L'(\hat{x}, y), \sum_{i=1}^{m} \alpha_i x_i \rangle - \sum_{i=1}^{m} \alpha_i \langle L'(\hat{x}, y), \hat{x} \rangle \in -\text{int} C.
\end{equation}

Hence
\begin{equation}
\langle L'(\hat{x}, y), \hat{x} \rangle - \langle L'(\hat{x}, y), \hat{x} \rangle = 0 \in -\text{int} C,
\end{equation}

which is inconsistent. Thus, we deduce that
\[ \text{Co}\{x_1, x_2, \ldots, x_m\} \subset \bigcup_{i=1}^{m} F(x_i). \]

Next, we show the multifunction $T$ satisfied Hogan’s upper semi-continuity. Let $\{x_n\}$ be a sequence in $K$ such that $x_n \to x \in K$ and let $\{y_n\}$ be a sequence such that $y_n \in T(x_n)$. Since $y_n \in T(x_n)$, we have
\[ L(x_n, v) - L(x_n, y_n) \notin \text{int} C, \]
for all $v \in E$. Since $\{y_n\} \subset E$ and $E$ is compact we can assume that there exists $y \in E$ such that $y_n \to y$, without loss of generality. Now the continuity of $L$ and the closedness of $(Z \setminus \text{int} C)$ gives that
\[ L(x, v) - L(x, y) \in (Z \setminus \text{int} C) \]
for all $v \in E$, which implies that $y \in T(x)$. Thus the multifunction $T$ is upper semicontinuous.

Next, we show that $F(u)$ is a closed set for each $u \in K$. Let $\{x_n\} \subset F(u)$ such that $x_n \to x \in K$. Since $x_n \in F(u)$ for all $n$, there exists $y_n \in T(x_n)$ such that
\[ \langle L'(x_n, y_n), u - x_n \rangle \notin (Z \setminus \text{int} C) \]
for all $u \in K$. As $\{y_n\} \subset E$, without loss of generality, we can assume that there exists $y \in E$ such that $y_n \to y$. Since $L'$ is continuous, $T$ is upper semicontinuous and $(Z \setminus \text{int} C)$ is closed, we have
\[ \langle L'(x_n, y_n), u - x_n \rangle \to \langle L'(x, y), u - x \rangle \in (Z \setminus \text{int} C). \]

Hence $x \in F(u)$.

Finally, we prove that for $\hat{x} \in B \cap K$, $F(\hat{x})$ is compact. Since $F(\hat{u})$ is closed and $B$ is compact, it is sufficient to show that $F(\hat{u}) \subset B$. Suppose to the contrary that there exists $\hat{x} \in F(\hat{u})$ such that $\hat{x} \notin B$. Since $\hat{x} \in F(\hat{u})$, there exists $\hat{y} \in T(\hat{x})$ such that
\[ \langle L'(\hat{x}, \hat{y}), \hat{u} - \hat{x} \rangle \notin \text{int} C. \]

Since $\hat{x} \notin B$, by the hypothesis, for any $y \in T(\hat{x})$,
\[ \langle L'(\hat{x}, y), \hat{u} - \hat{x} \rangle \in \text{int} C, \]
which contradicts condition (5). Hence $F(\hat{x}) \subset B$. Since $B$ is compact and $F(\hat{x})$ is also closed, $F(\hat{x})$ is compact. Consequently by Theorem 2.2, it follows that $\bigcap_{x \in K} F(x) \neq \emptyset$. Thus, there exists $x_0 \in K$ and $y_0 \in T(y_0)$ such that
\[ \langle L'(x_0, y_0), x - x_0 \rangle \notin \text{int} C, \]
for all $x \in K$.  \[\square\]
3 An Extension based on Moving Cone

We can extension concepts (VSPP) and (VVIP) by considering a moving cone. To begin with, we introduce some parameterized concepts for the extension. Assume that the multifunction $C : X \rightarrow 2^Z$ has solid pointed convex cone values.

**Definition 3.1 (Parameterized Cone Convexity)**
A vector valued function $f : K \rightarrow Z$ is said to be $C(x)$-convex if

$$\alpha f(x_1) + (1 - \alpha)f(x_2) - f(\alpha x_1 + (1 - \alpha)x_2) \in C(\alpha x_1 + (1 - \alpha)x_2),$$

for all $x_1, x_2 \in K$ and $\alpha \in [0, 1]$.

**Definition 3.2 Parameterized Vector Saddle Point Problem**
The Parameterized Vector Saddle Point Problem, (PVSPP) for short, is to find $x_0 \in K$ and $y_0 \in T(x_0)$ such that

$$(L(x_0, y_0) - L(x, y_0), x - x_0) \notin -\text{int} C(x), \forall x \in K,$$

A solution $(x_0, y_0) \in K \times E$ of (PVSPP) is called a weak $C(x)$-saddle point of function $L$.

**Definition 3.3 Parameterized Vector Variational Inequality Problem**
The Parameterized Vector Variational Inequality Problem, (PVVIP) for short, is to find $x_0 \in K$ and $y_0 \in T(x_0)$ such that

$$\langle L'(x_0, y_0), x - x_0 \rangle \notin -\text{int} C(x), \forall x \in K,$$

where $T: X \rightarrow 2^Y$ is a multifunction defined by

$$T(x) := \{y \in C \mid L(x, v) - L(x, y) \notin \text{int} C(x), \forall v \in E \}.$$  

**Definition 3.4** A multifunction $F : K \rightarrow 2^Z$ is called upper-semicontinuous if for every $x \in K$ and $U_x \subset Z$; neighborhood of $F(x)$ there exists $V_x \subset K$; neighborhood of $x$ such that $F(y) \subset U_x$ for all $y \in V_x$.

**Definition 3.5** A multifunction $F : K \rightarrow 2^Z$ is called lower-semicontinuous if for every $x \in K$ there exists $V_x \subset K$; neighborhood of $x$ such that $F(y) \cap V_x \neq \emptyset$ for all $V_x \subset Z$, where $V_x$ is an open set satisfying $F(x) \cap V_x \neq \emptyset$.

**Definition 3.6** A multifunction $F : K \rightarrow 2^Z$ is called continuous if $F$ satisfy upper-semicontinuous and lower-semicontinuous.

**Definition 3.7** A multifunction $F : K \rightarrow 2^Z$ is called closed if $\{x_n\} \subset K$ converging to $x$, and $\{z_n\} \subset Z$, with $z_n \in F(x_n)$, converging to $z$, implies $z \in F(x)$.

**Lemma 3.1** Assume that the multifuncion $C : K \rightarrow 2^Z$ is continuous. Then the multifunction $C$ and $W$ are closed, where $W : K \rightarrow 2^Z$ is a multifunction defined by

$$W(x) := Z \setminus \text{int} C(x)$$

Now, we extend the results of Section 2 by using these concepts.
Theorem 3.1 Let $K$ and $E$ be a convex subset of a normed space $X$ and an arbitrary subset of a topological vector space $Y$. Assume that the multifunction $C : X \to 2^Z$ has solid pointed convex cone values and it is continuous, and $L$ is $C(x)$-convex and Fréchet differentiable in the first argument. Then problems (PVSP) and (PVVIP) have the same solution set.

Proof. Assume that $(x_0, y_0) \in K \times E$ is a solution of (PVSP). Then

$$L(x_0, y_0) - L(x, y_0) \notin \text{int} C(x_0),$$

for all $x \in K$.

$$L(x_0, y) - L(x_0, y_0) \notin \text{int} C(x_0),$$

for all $y \in E$. Since $K$ is convex, We have

$$x_0 + \alpha(x - x_0) \in K,$$

for all $x \in K$ and $\alpha \in [0, 1]$. Hence condition (6) implies

$$\alpha^{-1}[L(x_0 + \alpha(x - x_0), y_0) - L(x_0, y_0)] \notin -\text{int} C(x_0 + \alpha(x - x_0)),$$

for all $x \in K$ and $\alpha \in (0, 1]$. Since $Z(-\text{int} C(x))$ is continuous and $L$ is Fréchet differentiable in the first argument, it follows that

$$\langle L'(x_0, y_0), x - x_0 \rangle \notin -\text{int} C(x_0),$$

for all $x \in K$. $y_0 \in T(x_0)$ follows from (7).

Conversely, assume that $(x_0, y_0) \in K \times E$ is a solution of (PVVIP). Then we have

$$\langle L'(x_0, y_0), x - x_0 \rangle \notin -\text{int} C(x_0),$$

for all $x \in K$.

$$L(x_0, y) - L(x_0, y_0) \notin \text{int} C(x_0),$$

for all $y \in E$. Since $L$ is $C$-convex with respect the first argument, we have

$$\alpha L(x, y_0) + (1 - \alpha)L(x_0, y_0) - L(x_0 + \alpha(x - x_0), y_0) \in C(x_0 + \alpha(x - x_0)),$$

for all $x \in K$ and $\alpha \in (0, 1)$, and since $C(x)$ is cone, we have

$$L(x, y_0) - L(x_0, y_0) - \frac{L(x_0 + \alpha(x - x_0), y_0) - L(x_0, y_0)}{\alpha} \in C(x_0),$$

for all $x \in K$ and $\alpha \in (0, 1)$. Since $L$ is Fréchet differentiable with respect to the first argument, if $\alpha$ converges to 0, then we have

$$L(x, y_0) - L(x_0, y_0) - \langle L'(x_0, y_0), x - x_0 \rangle \in C(x_0),$$

for all $x \in K$. From (8), it follows

$$L(x_0, y_0) - L(x, y_0) \notin \text{int} C(x_0),$$

for all $x \in K$. Hence $(x_0, y_0) \in K \times E$ is also a solution of (PVSP).
Theorem 3.2 Let $K$ and $E$ be a nonempty closed convex subset of a normed space $X$ and a nonempty compact subset of a topological vector space $Y$, respectively. Assume that the multifunction $C : X \rightarrow 2^Z$ has solid pointed convex cone values and it is continuous. Assume that the vector valued function $L$ is $C(z)$-convex and Fréchet differentiable in the first argument, $L'$ is a continuous function in both $x$ and $y$, and let $T, K \rightarrow E$ be the multifunction defined by

$$T(x) := \{ y \in E \mid L(x,v) - L(x,y) \notin \text{int} C(x), \forall v \in E \}.$$ 

If there exist a nonempty compact subset $B$ of $X$ and $x_0 \in B \cap K$ such that for any $x \in K \setminus B$, $y \in T(x)$,

$$\langle L'(x,y), x_0 - x \rangle \in -\text{int} C(x),$$

then problem (PVSSP) has at least one solution. 

Proof. It is sufficient to show that the (PVVIP) has at least one solution $x_0 \in K$ and $y_0 \in T(x_0)$. Define a multifunction $F : K \rightarrow K$ by

$$F(u) = \{ x \in K \mid \langle L'(x,y), u - x \rangle \notin -\text{int} C(x), \text{ for some } y \in T(x) \}, \quad u \in K.$$ 

We first prove that the convex hull of any finite subset $\{x_1, x_2, \ldots, x_n\}$ of $K$ is contained in the corresponding union $\bigcup_{i=1}^{m} F(x_i)$, that is, $\text{Co}\{x_1, x_2, \ldots, x_m\} \subset \bigcup_{i=1}^{m} F(x_i)$. Suppose that there exists $x_1, x_2, \ldots, x_m$ and $\alpha_1, \alpha_2, \ldots, \alpha_m$ such that

$$\hat{x} = \sum_{i=1}^{m} \alpha_i x_i \notin \bigcup_{i=1}^{m} F(x_i), \quad \sum_{i=1}^{m} \alpha_i = 1.$$ 

Then for any $y \in T(\hat{x})$,

$$\langle L'(\hat{x},y), x_i - \hat{x} \rangle \in -\text{int} C(\hat{x}),$$

for all $i = 1, \ldots, m$. Since $\text{int} C(x)$ is convex, we have

$$\sum_{i=1}^{m} \alpha_i \langle L'(\hat{x},y), x_i - \hat{x} \rangle \in -\text{int} C(\hat{x}).$$

Since $L'(\hat{x},y)$ is a linear operator, we have

$$\langle L'(\hat{x},y), \sum_{i=1}^{m} \alpha_i x_i \rangle - \sum_{i=1}^{m} \alpha_i \langle L'(\hat{x},y), \hat{x} \rangle \in -\text{int} C(\hat{x}).$$

Hence

$$\langle L'(\hat{x},y), \hat{x} \rangle - \langle L'(\hat{x},y), \hat{x} \rangle = 0 \in -\text{int} C(\hat{x}),$$

which is inconsistent. Thus, we deduce that

$$\text{Co}\{x_1, x_2, \ldots, x_m\} \subset \bigcup_{i=1}^{m} F(x_i).$$

Next, we show the multifunction $T$ satisfied Hogan's upper semi-continuity. Let $\{x_n\}$ be a sequence in $K$ such that $x_n \rightarrow x \in K$ and let $\{y_n\}$ be a sequence such that $y_n \in T(x_n)$. Since $y_n \in T(x_n)$, we have

$$L(x_n,v) - L(x_n,y_n) \notin \text{int} C(x_n)$$
for all $v \in E$. Since $\{y_n\} \subset E$ and $E$ is compact we can assume that there exists $y \in E$ such that $y_n \to y$, without loss of generality. Now the continuity of $L$ and the closedness of $(Z \setminus \text{int} C(x))$ gives that

$$L(x,v) - L(x,y) \in (Z \setminus \text{int} C(x))$$

for all $v \in E$, which implies that $y \in T(x)$. Thus the multifunction $T$ is upper semicontinuous.

Next, we show that $F(u)$ is closed for each $u \in K$. Indeed, let $\{x_n\} \subset F(u)$ such that $x_n \to x \in K$. Since $x_n \in F(u)$ for all $n$, there exists $y_n \in T(x_n)$ such that

$$\langle L'(x_n, y_n), u - x_n \rangle \in (Z \setminus \text{int} C(x_n))$$

for all $u \in K$. As $\{y_n\} \subset E$ we can assume that there exists $y \in E$ such that $y_n \to y$, without loss of generality. Since $L'$ is continuous, $T$ is upper semicontinuous and $(Z \setminus \text{int} C(x))$ is closed, we have

$$\langle L'(x_n, y_n), u - x_n \rangle \to \langle L'(x, y), u - x \rangle \in (Z \setminus \text{int} C(x))$$

Hence $x \in F(u)$.

Finally, we prove that for $\tilde{x} \in B \cap K$, $F(\tilde{x})$ is compact. Since $F(\hat{u})$ is closed and $B$ is compact, it is sufficient to show that $F(\hat{u} \subset B)$. Suppose that there exists $\hat{x} \in F(\hat{u})$ such that $\hat{x} \notin B$. Since $\hat{x} \in F(\hat{u})$, there exists $\hat{y} \in T(\hat{x})$ such that

$$\langle L'(\hat{x}, \hat{y}), \hat{u} - \hat{x} \rangle \notin -\text{int} C(\hat{x}).$$

(10)

Since $\hat{x} \notin B$, by hypothesis, for any $y \in T(\hat{x})$,

$$\langle L'(\hat{x}, y), \hat{u} - \hat{x} \rangle \in -\text{int} C(\hat{x}),$$

which contradicts (10). Hence $F(\tilde{x}) \subset B$. Since $B$ is compact and $F(\tilde{x})$ is closed, $F(\tilde{x})$ is compact. By Theorem 2.2, it follows that $\bigcap_{x \in K} F(x) \neq \phi$. Thus, there exists $x_0 \in K$, $y_0 \in T(y_0)$ such that

$$\langle L'(x_0, y_0), x - x_0 \rangle \notin -\text{int} C(x_0),$$

for all $x \in K$.

4 Conclusions

In this paper, we have extended an existence theorem established Kazmi and Khan to a more generalized one. We have also extended the theorem by using a concept of moving cone, which first entered in game theory to cope with turning the purpose of a situation.

参考文献


