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Stochastic Unit Commitment Problem

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Abstract The unit commitment problem is an important problem for electric power utilities. The unit commitment problem is to determine the schedule of power generating units and the generating level of each unit. The decisions are which units to commit at each time period and at what level to generate power meeting the electricity demand. In this paper we propose a new algorithm that is based on the Dantzig-Wolfe reformulation and column generation approach to solve the stochastic unit commitment problem. The algorithm continues adding schedules from the dual solution of the restricted linear master program until the algorithm cannot generate new schedules. The schedule generation problem is solved by the calculation of dynamic programming on the scenario tree.

1. Introduction

The economic operation and planning of electric power generation occupy an important position in electric power industry. Wood and Wollenberg [17] offered brief overview and many applications of operations research methods to real electric power problems. The electric power utilities have to maintain sufficient capacity to meet electricity demand during the peak load periods. The unit commitment problem is to determine the schedule of power generating units and the generating level of each unit. The decisions are which units to commit at each time period and at what level to generate power meeting the electricity demand. The objective function is to minimize the operational cost which is the sum of the fuel cost and the start up cost. The problem is a typical scheduling problem of electric power system. This problem becomes a multi stage nonlinear integer programming problem because the fuel cost function is assumed to be a convex quadratic function.

Many types of optimization technique have been applied to the unit commitment problem. Delson and Shahidehpour [7] illustrated how linear and integer programming had been applied to power system engineering such as generation scheduling, allocation of reactive power supply or planning of capital investment in generation equipment. Sheble and Fafd [12] is a survey in this field for the period from the late 1960's to the early 1990's. They classified the techniques into exhaustive enumeration, priority list, dynamic programming, integer and mixed-integer programming, branch-and-bound, linear programming, network flow programming, Lagrangian relaxation, and expert systems/artificial neural networks. In these approaches, the Lagrangian relaxation technique seems to be the most promising because it decompose the original problem into smaller subproblems. Muckstadt and Koenig [9] used this approach by relaxing the demand constraints. Bard [1] used the Lagrangian relaxation to disaggregate the problem by generator into separate subproblems that were solved by a dynamic programming.

In these studies, the electricity demand at any time period is known in advance. However for many actual problems, such assumption is often unjustified. These data contain uncertainty and are represented as random variables since the data represent information about the future. Takriti, Birge and Long [13] is a first paper that deals with the stochastic programming approach. Stochastic programming (Birge [3], Birge and Louveaux [4]) is a method that deals with optimization problem under uncertainty. They developed the technique used in the traditional deterministic unit commitment problem. The uncertainty in demand is modeled by introducing a set of scenarios. The problem is decomposed and solved by using a Lagrangian relaxation type method, progressive hedging algorithm [11]. For each scenario, the relaxed deterministic problem is then decomposed into single generator subproblem by Lagrangian relaxation. It can be solved efficiently by dynamic programming. Takriti and Birge [15] generalized this approach and showed that the duality gap of the relaxation is bounded by a certain constant. Carpentier et al. [5] applied the augmented Lagrangian technique to the problem. Nowak [10] used the stochastic Lagrangian relaxation method which led to a decomposition to into stochastic single unit subproblems. Takriti and Birge [14] developed a technique for refining the solution obtained from solving the Lagrangian relaxation problem. Their approach is to select a schedule among the feasible solutions sought up to the latest iteration, and to combine them so that the demand constraint can be met. The suggested model is the mixed-integer programming problem and is solved by branch-and-bound method. Though the numerical results indicated

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improvements, the approach is no more than heuristic. The efficient method to seek the feasible schedules and to refine them is requested. Takriti, Krasenbrink and Wu [16] presented a stochastic programming model that incorporates power trading and uncertainty in electricity demand and spot prices.

In this paper we propose a new algorithm that is based on the Dantzig-Wolfe reformulation(Dantzig and Wolfe [6]) and column generation approach(Barnhart et al. [2]) to solve the stochastic unit commitment problem. Our method refines the approach of Takriti and Birge [14]. It can deal with the case that the number of the units in operation at the same time is restricted. Therefore it can be applied to the general scheduling problems that have more complicated constraints.

2. Uncertainty in Electricity Demand

We assume that the duration of the planning horizon in T time periods. Since the electric demand at any point in time period may be uncertain, we have to model the unit commitment problem as a stochastic programming problem.

To model uncertainty, we define the total demand for electricity during period t as a random variable $\tilde{d}_t (\geq 0)$. It is assumed that $\tilde{d}_t$ is defined on a known probability space and has a finite discrete distribution. Let d_t be the realization of random variable $\tilde{d}_t$. The sequence of the realization of electricity demand $d = (d_1, \ldots, d_T)$ is called scenario. It is assumed that we have a set of S scenarios, $d^s, s = 1, \ldots, S$. We associate a probability p_s with each scenario $s, s = 1, \ldots, S$.

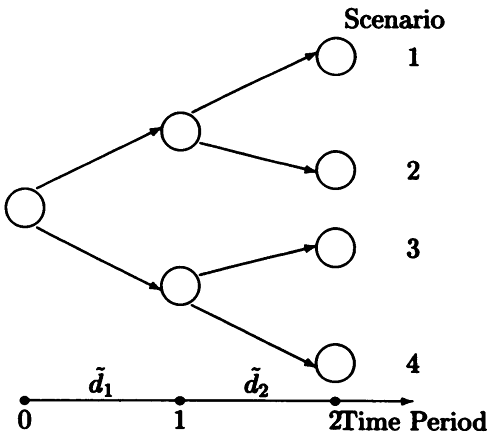

Figure 1: Scenario tree

If two scenarios s_1, s_2, $(s_1 \neq s_2)$ satisfy the condition $(d_1^{s_1}, \ldots, d_t^{s_1}) = (d_1^{s_2}, \ldots, d_t^{s_2})$, they are indistinguishable up to period t. The decisions made for scenario s_1 up to period t must be the same as those made for s_2 up to period t. Two scenarios s_1 and s_2 are said to be included in the same bundle at time t. The scenario sets $\{1, \ldots, S\}$ at each time can be partitioned into disjoint subsets which represent scenario bundles. We define $B(s,t)$ to be the bundle in which scenario s is member at time period t. This type of constraint is called a nonanticipativity constraint or a bundle constraint.

If $B(s',t) = B(s,t)$ and $B(s',t+1) \neq B(s',t+1), s' < s$, the time period $t+1$ is a point when scenario s splits from other scenario s'. The scenario s' is called a predecessor of scenario s. If there are multiple predecessors for s, we define the scenario with the lowest index as the predecessor of s. The predecessor of scenario s is denoted by $\mathcal{P}(s)$. The time period $\mathcal{T}(s)$ is defined to be the first period in which a scenario s does not share a bundle with another scenario $s' < s$. For scenario 1, we define $\mathcal{T}(1) = 1$.

To store $d_t^s, t = 1, \ldots, T, s = 1, \ldots, S$, we adopt the special data structure based on the method of Takriti, Krasenbrink and Wu [16]. The demand data are stored in a list. For each scenario s, it is sufficient to store only $d_{\mathcal{T}(s)}^s, \ldots, d_T^s$ to save memory space. We define $\mathcal{B}(s)$ be the address in which $d_{\mathcal{T}(s)}$ is stored so that $d_{\mathcal{T}(s)}, \ldots, d_T$ are stored in the space with the address $\mathcal{B}(s), \ldots, \mathcal{B}(s+1) - 1$. When we take out $d_t^s, t = 1, \ldots, T$, we trace backward from $\mathcal{B}(s+1) - 1$ to $\mathcal{B}(s)$ to obtain $d_T^s, \ldots, d_{\mathcal{T}(s)}^s$. Then, we trace backward from $\mathcal{B}(\mathcal{P}(s) + 1) - 1 - \{\mathcal{B}(s+1) - \mathcal{B}(s)\}$ to $\mathcal{B}(\mathcal{P}(s))$ to obtain $d_{\mathcal{T}(s)-1}^s, \ldots, d_{\mathcal{T}(\mathcal{P}(s))}^s$ for the scenario $\mathcal{P}(s)$.

3. Stochastic Unit Commitment Problem

We assume that there are I generating units. The status of unit i at period t under scenario s is represented by the 0-1 variable u_{it}^s. Unit i is on at time period t under scenario s, if $u_{it}^s = 1$, and off if $u_{it}^s = 0$. When

Scenario 1					Scenario s			Scenario $s+1$			
1	2	$\cdots$	T	$\cdots$	$\mathcal{B}(s)$	$\cdots$	$\mathcal{B}(s+1)-1$	$\mathcal{B}(s+1)$	$\cdots$	$\mathcal{B}(s+2)-1$	$\cdots$
d_1^1	d_2^1	$\cdots$	d_T^1	$\cdots$	$d^s_{\mathcal{T}(s)}$	$\cdots$	d_T^s	$d^{s+1}_{\mathcal{T}(s+1)}$	$\cdots$	d_T^{s+1}	$\cdots$

Figure 2: Data structure to store demand data

unit i is switched on, it must continue to run at least for a certain periods L_i. These minimum up-time constraints are described in (3.1).

$$u_{it}^s - u_{i,t-1}^s \leq u_{i\tau}^s, \tau = t+1, \dots, \min\{t + L_i - 1, T\}, t = 2, \dots, T \tag{3.1}$$

Similarly, when unit i is switched off, it must continue to be off at least l_i periods. These constraints are called minimum down-time constraints (3.2).

$$u_{i,t-1}^s - u_{it}^s \leq 1 - u_{i\tau}^s, \tau = t+1, \dots, \min\{t + l_i - 1, T\}, t = 2, \dots, T \tag{3.2}$$

The power generating level of the unit i at period t under scenario s is $x_{it}^s \geq 0$. Let $[q_i, Q_i]$ be an operating range of the generating unit i. The unit is operated within the range so that x_{it}^s has to satisfy the following constraints (3.3).

$$q_i u_{it}^s \leq x_{it}^s \leq Q_i u_{it}^s, i = 1, \dots, I, t = 1, \dots, T, s = 1, \dots, S \tag{3.3}$$

The fuel cost function $f_i(x_{it}^s)$ is a convex quadratic function of x_{it}^s. The start up cost function $g_i(u_{i,t-1}, u_{it})$ satisfies the condition $g_i(0,1) > 0, g_i(0,0) = 0, g_i(1,0) = 0, g_i(1,1) = 0$. The mathematical formulation of the stochastic unit commitment problem is described as follows.

$$\left(\begin{array}{c}\text{Stochastic}\\ \text{Unit}\\ \text{Commitment}\\ \text{Problem}\end{array}\right) \left|\begin{array}{rl}
\min & \displaystyle\sum_{s=1}^{S} p_s \sum_{i=1}^{I}\sum_{t=1}^{T}\{f_i(x_{it}^s)u_{it}^s + g_i(u_{i,t-1}^s, u_{i,t}^s)\} \\
\text{subject to} & \displaystyle\sum_{i=1}^{I} x_{it}^s \geq d_t^s, t = 1, \dots, T, s = 1, \dots, S \\
& u_{it}^s - u_{i,t-1}^s \leq u_{i\tau}^s, \tau = t+1, \dots, \min\{t + L_i - 1, T\}, \\
& \quad i = 1, \dots, I, t = 2, \dots, T, s = 1, \dots, S \\
& u_{i,t-1}^s - u_{it}^s \leq 1 - u_{i\tau}^s, \tau = t+1, \dots, \min\{t + l_i - 1, T\}, \\
& \quad i = 1, \dots, I, t = 2, \dots, T, s = 1, \dots, S \\
& u_{it}^s \in \{0,1\}, i = 1, \dots, I, t = 1, \dots, T, s = 1, \dots, S \\
& q_i u_{it}^s \leq x_{it}^s \leq Q_i u_{it}^s, i = 1, \dots, I, t = 1, \dots, T, s = 1, \dots, S \\
& u_{it}^{s_1} = u_{it}^{s_2}, i = 1, \dots, I, t = 1, \dots, T, \\
& \quad \forall s_1, s_2 \in \{1, \dots, S\}, s_1 \neq s_2, \mathcal{B}(s_1, t) = \mathcal{B}(s_2, t)
\end{array}\right.$$

The problem results in a large scale mixed integer quadratic programming problem that combines S deterministic unit commitment problems. The objective function is to minimize the expected operational cost over all possible scenarios.

4. Reformulation of Unit Commitment

The unit commitment problem is reformulated as the integer programming master problem. In this problem demand constraints are relaxed. This reformulation is called Dantzig-Wolfe reformulation (Dantzig and Wolfe [6]). It is assumed that for each unit a set of feasible schedule over all scenarios is already given. The number of given feasible schedules for unit i is K_i.

$$\left(\begin{array}{c}\text{Integer}\\ \text{Programming}\\ \text{Master}\\ \text{Problem}\end{array}\right) \left|\begin{array}{rl}
\min & \displaystyle\sum_{i=1}^{I}\sum_{k=1}^{K_i}\sum_{s=1}^{S} p_s\{\sum_{t=1}^{T}(f_i(x_{it}^{sk})u_{it}^{sk} + g_i(u_{i,t-1}^{sk}, u_{it}^{sk}))\}v_i^k \\
\text{subject to} & \displaystyle\sum_{i=1}^{I}\sum_{k=1}^{K_i} x_{it}^{sk} v_i^k \geq d_t^s, t = 1, \dots, T, s = 1, \dots, S \\
& \displaystyle\sum_{k=1}^{K_i} v_i^k = 1, i = 1, \dots, I \\
& v_i^k \in \{0,1\}, i = 1, \dots, I, k = 1, \dots, K_i
\end{array}\right.$$

In this formulation, $x_{it}^{sk}, u_{it}^{sk}, i = 1,\ldots,I, t = 1,\ldots,T, s = 1,\ldots,S, k = 1,\ldots,K_i$ are given parameters. They satisfy the following minimum up-time constraints (4.1), the minimum down-time constraints (4.2), the 0-1 constraints (4.3), the generating level constraints (4.4) and the bundle constraints (4.5).

$$
\begin{aligned}
u_{it}^{sk} - u_{i,t-1}^{sk} \le u_{i\tau}^{sk}, \quad & \tau = t+1,\ldots,\min\{t+L_i-1,T\}, k = 1,\ldots,K_i, && (4.1)\\
& i = 1,\ldots,I, t = 2,\ldots,T, s = 1,\ldots,S \\
u_{i,t-1}^{sk} - u_{it}^{sk} \le 1 - u_{i\tau}^{sk}, \quad & \tau = t+1,\ldots,\min\{t+l_i-1,T\}, k = 1,\ldots,K_i, && (4.2)\\
& i = 1,\ldots,I, t = 2,\ldots,T, s = 1,\ldots,S \\
u_{it}^{sk} \in \{0,1\}, \quad & k = 1,\ldots,K_i, i = 1,\ldots,I, t = 1,\ldots,T, s = 1,\ldots,S && (4.3)\\
q_i u_{it}^{sk} \le x_{it}^{sk} \le Q_i u_{it}^{sk}, \quad & k = 1,\ldots,K_i, i = 1,\ldots,I, t = 1,\ldots,T, s = 1,\ldots,S && (4.4)\\
u_{it}^{s_1k} = u_{it}^{s_2k}, \quad & k = 1,\ldots,K_i, i = 1,\ldots,I, t = 1,\ldots,T, && (4.5)\\
& \forall s_1, s_2 \in \{1,\ldots,S\}, s_1 \ne s_2, B(s_1,t) = B(s_2,t)
\end{aligned}
$$

We cannot enumerate a set of feasible schedules in advance. The schedules are often described implicitly. The column generation approach is applied to generate feasible schedules. Then we solve the following restricted linear programming master problem. We start from where only the subset of all feasible solutions is given.

$$
\left(\begin{array}{c}\text{Restricted}\\ \text{Linear}\\ \text{Programming}\\ \text{Master}\\ \text{Problem}\end{array}\right)
\left|\begin{array}{rl}
\min & \displaystyle\sum_{i=1}^{I}\sum_{k\in K_i'}\sum_{s=1}^{S} p_s\{\sum_{t=1}^{T}(f_i(x_{it}^{sk})u_{it}^{sk} + g_i(u_{i,t-1}^{sk}, u_{it}^{sk}))\}v_i^k \\
\text{subject to} & \displaystyle\sum_{i=1}^{I}\sum_{k\in K_i'} x_{it}^{sk} v_i^k \ge d_t^s, t = 1,\ldots,T, s = 1,\ldots,S \\
& \displaystyle\sum_{k\in K_i'} v_i^k = 1, i = 1,\ldots,I \\
& v_i^k \ge 0, i = 1,\ldots,I, k \in K_i', K_i' \subseteq \{1,\ldots,K_i\}
\end{array}\right.
$$

Solving the restricted linear programming master problem gives an optimal primal solution $v_i^{*k}, i = 1,\ldots,I, k \in K_i'$ and an optimal dual solution $\pi_t^{*s}, t = 1,\ldots,T, s = 1,\ldots,S$, $\mu_i^*, i = 1,\ldots,I$. We need to check whether (π^*, μ^*) is dual feasible for the linear programming relaxation problem of the original integer programming master problem. The dual problem for the linear programming master problem is described as follows.

$$
\left(\begin{array}{c}\text{Dual}\\ \text{Problem}\\ \text{for}\\ \text{Linear}\\ \text{Programming}\\ \text{Master}\\ \text{Problem}\end{array}\right)
\left|\begin{array}{rl}
\max & \displaystyle\sum_{s=1}^{S}\sum_{t=1}^{T} d_t^s \pi_t^s + \sum_{i=1}^{I}\mu_i \\
\text{subject to} & \displaystyle\sum_{s=1}^{S}\sum_{t=1}^{T} x_{it}^{sk}\pi_t^s + \mu_i \le \sum_{s=1}^{S} p_s\{\sum_{t=1}^{T}(f_i(x_{it}^{sk})u_{it}^{sk} + g_i(u_{i,t-1}^{sk}, u_{it}^{sk}))\}, \\
& i = 1,\ldots,I, k = 1,\ldots,K_i \\
& \pi_t^s \ge 0, t = 1,\ldots,T, s = 1,\ldots,S
\end{array}\right.
$$

If $\sum_{s=1}^{S}\sum_{t=1}^{T} x_{it}^{sk}\pi_t^{*s} + \mu_i^* \le \sum_{s=1}^{S} p_s\{\sum_{t=1}^{T}(f_i(x_{it}^{sk})u_{it}^{sk} g_i(u_{i,t-1}^{sk}, u_{it}^{sk}))\}, i = 1,\ldots,I, k = 1,\ldots,K_i$, (π^*, μ^*) is dual feasible for the linear programming master problem and the optimal solution of the linear programming master problem is obtained. Rather than examining each schedule, we can treat all schedules implicitly by solving the following schedule generation problem. In the schedule generation problem for unit i, we regard the parameters $x_{it}^{sk}, u_{it}^{sk}, t = 1,\ldots,T, s = 1,\ldots,S$ as variables and seek to minimize the objective function. If the optimal value of the objective function ζ_i is greater than or equal to 0 for $i = 1,\ldots,I$, the optimal dual variable (π^*, μ^*) is feasible for the dual problem of the original linear programming master problem. If the optimal objective function value is less than 0 for some i, we can adopt the optimal

$x_{it}^{sk}, u_{it}^{sk}, t = 1, \ldots, T, s = 1, \ldots, S$ as a feasible schedule and set $|K_i'| = |K_i' + 1|$.

$$\left(\begin{array}{c}\text{Schedule}\\ \text{Generation}\\ \text{Problem}\\ \text{for}\\ \text{Unit } i\end{array}\right) \left| \begin{array}{ll} \zeta_i = \min & \sum_{s=1}^{S} p_s\{\sum_{t=1}^{T}(f_i(x_{it}^{sk})u_{it}^{sk} + g_i(u_{i,t-1}^{sk}, u_{it}^{sk}))\} - \sum_{s=1}^{S}\sum_{t=1}^{T}\pi_t^{*s}x_{it}^{sk} - \mu_i^* \\ \text{subject to} & u_{it}^{sk} - u_{i,t-1}^{sk} \le u_{i\tau}^{sk}, \tau = t+1, \ldots, \min\{t + L_i - 1, T\}, \\ & \quad i = 1, \ldots, I, t = 2, \ldots, T, s = 1, \ldots, S \\ & u_{i,t-1}^{sk} - u_{it}^{sk} \le 1 - u_{i\tau}^{sk}, \tau = t+1, \ldots, \min\{t + l_i - 1, T\}, \\ & \quad i = 1, \ldots, I, t = 2, \ldots, T, s = 1, \ldots, S \\ & u_{it}^{sk} \in \{0, 1\}, i = 1, \ldots, I, t = 1, \ldots, T, s = 1, \ldots, S \\ & q_i u_{it}^{sk} \le x_{it}^{sk} \le Q_i u_{it}^{sk}, i = 1, \ldots, I, \\ & \quad t = 1, \ldots, T, s = 1, \ldots, S \\ & u_{it}^{s_1 k} = u_{it}^{s_2 k}, i = 1, \ldots, I, t = 1, \ldots, T, \\ & \quad \forall s_1, s_2 \in \{1, \ldots, S\}, s_1 \ne s_2, B(s_1, t) = B(s_2, t) \end{array}\right.$$

Let ζ_i^* be the optimal objective value for the schedule generation problem for unit i, then $(\pi^*, \mu^* + \zeta^*)$ is a feasible solution for the dual problem of the linear programming master problem since $\zeta_i^* \le \sum_{s=1}^{S} p_s\{\sum_{t=1}^{T}(f_i(x_{it}^{sk})u_{it}^{sk} + g_i(u_{i,t-1}^{sk}, u_{it}^{sk}))\} - \sum_{s=1}^{S}\sum_{t=1}^{T}\pi_t^{*s}x_{it}^{sk} - \mu_i^*$. The lower bound for the linear programming master problem is given as $\sum_{s=1}^{S}\sum_{t=1}^{T} d_t^s \pi_t^{*s} + \sum_{i=1}^{I}(\mu_i^* + \zeta_i^*)$ from the duality theory of linear programming. The lower bound for the optimal value of the original stochastic unit commitment problem is calcurated as 4.6 from the Lagrange duality.

$$\sum_{i=1}^{I}\sum_{k=1}^{K_i}\sum_{s=1}^{S} p_s\{\sum_{t=1}^{T}(f_i(x_{it}^{sk})u_{it}^{sk} + g_i(u_{i,t-1}^{sk}, u_{it}^{sk}))\}v_i^{*k} - \sum_{i=1}^{I}\sum_{k=1}^{K_i}\pi_t^{*s}(x_{it}^{sk}v_i^{*k} - d_t^s) \tag{4.6}$$

The schedule generation problem can be solved by calculating dynamic programming on the scenario tree. First, we solve the next generation level decision problem to seek optimal $x_{it}^{sk}, t = 1, \ldots, T, s = 1, \ldots, S$. The problem is a convex quadratic programming problem that we can solve easily.

$$\left(\begin{array}{c}\text{Generation Level}\\ \text{Decision Problem}\\ \text{for Unit } i\\ \text{at Period } t\\ \text{under Scenario } s\end{array}\right) \left| \begin{array}{ll} \min & f_i(x_{it}^{sk}) - \frac{\pi_t^{*s}}{p_s}x_{it}^{sk} \\ \text{subject to} & q_i \le x_{it}^{sk} \le Q_i \end{array}\right.$$

Then the binary decisions $u_{it}^{sk}, t = 1, \ldots, T, s = 1, \ldots, S$ are made. Because each scenario s does not duplicate with other scenario for the period from $\mathcal{T}(s)$ to T, the calculation of dynamic programming is done by the following recursive equations. A unit i must be in one of $L_i + l_i$ states. The first L_i states mean that the unit i is on, and the last l_i states mean that the unit i is off. Let $C_i(s, t, k)$ is the optimal cost of unit i under the scenario s from stage t to the end of the horizon, if unit i is in state k at stage t. The recursive equations are defined as follows.

$$C_i(s,t,k) = \begin{cases} C_i(s,t+1,k+1) + p_s\{f_i(x_{it}^{sk}) + g_i(0,1)\} - \pi_t^{*s}x_{it}^{sk} & \text{if } k = 1 \\ C_i(s,t+1,k+1) + p_s f_i(x_{it}^{sk}) - \pi_t^{*s}x_{it}^{sk} & \text{if } 1 < k < L_i \\ \min\{C_i(s,t+1,k) + p_s f_i(x_{it}^{sk}) - \pi_t^{*s}x_{it}^{sk}, & \\ \quad C_i(s,t+1,k+1)\} & \text{if } k = L_i \\ C_i(s,t+1,k+1) & \text{if } L_i < k < L_i + l_i \\ \min\{C_i(s,t+1,k), C_i(s,t+1,1)\} & \text{if } k = L_i + l_i \end{cases}$$

But in the period before $t = \mathcal{T}(s) - 1$, the decisions made for the scenario that belongs to the same scenario bundle must be same to satisfy the nonanticipativity constraints. If a bundle $B(s, t)$ is composed of a set $\{s_{t_1}, \ldots, s_{t_n}\}$, we define s_{min} to be the scenario with the lowest index in the same bundle such that $s_{min} = \min\{s_{t_j} : s_{t_j} \in B(s, t), j = 1, \ldots, n\}$. We let the scenario s_{min} represent the bundle $B(s_{min}, t)$ in which s_{min} is a member. The scenario set $\{s_{t_1}, \ldots, s_{t_n}\}$ is unified into the representative scenario s_{min} and it is redefined that $p_{s_{min}} = \sum_{s_{t_j} \in B(s_{min}, t)} p_{s_{t_j}}$. The dynamic programming calculation is done by taking

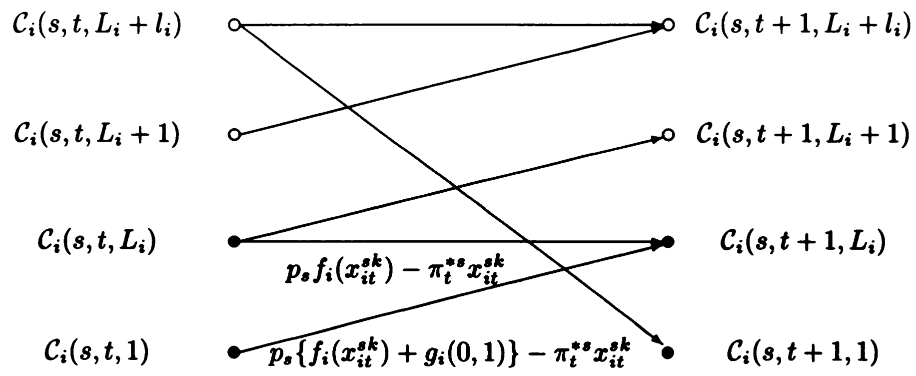

Figure 3: Recursive equation of dynamic programming($L_i = l_i = 2$)

the mathematical expectation as (4.7).

$$C_i(s_{min}, \mathcal{T}(s), k) = \sum_{s_{t_j} \in B(s_{min}, \mathcal{T}(s)-1)} p_{s_{t_j}} C_i(s_{t_j}, \mathcal{T}(s), k) \tag{4.7}$$

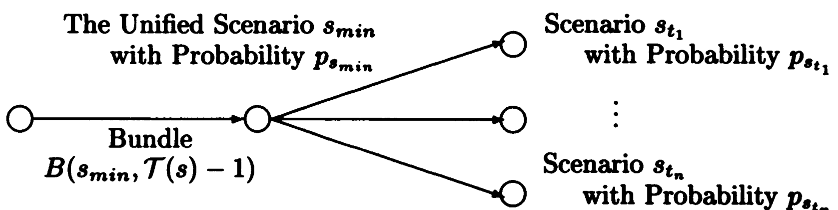

Figure 4: Unification of scenario

The algorithm of dynamic programming on the scenario tree is shown as Figure 5.

- **Step 0.** Set time pointer for each scenario $\mathcal{E}_s = T+1$. Set *Scenarioset* $= \{1,\ldots,S\}$. Set $C_i(s,\mathcal{E}_s,k) = 0$ for each scenario $s \in$ *Scenarioset*, $k = 1,\ldots,L_i+l_i$.
- **Step 1.** If *Scenarioset* $= \phi$, stop. Otherwise select a scenario $s = \text{argmin}\{\mathcal{T}(s) : s \in \textit{Scenarioset}\}$.
- **Step 2.** Given $C_i(s,\mathcal{E}_s,k)$, calculate $C_i(s,\mathcal{T}(s),k)$ using backward recursion for $k = 1,\ldots,L_i+l_i$. Given $C_i(\mathcal{P}(s),\mathcal{E}_s,k)$, calculate $C_i(\mathcal{P}(s),\mathcal{T}(s),k)$ using backward recursion for $k = 1,\ldots,L_i+l_i$. Set time pointer $\mathcal{E}_{\mathcal{P}(s)} = \mathcal{T}(s)$.
- **Step 3.** Set $C_i(\mathcal{P}(s),\mathcal{E}_s,k) = C_i(\mathcal{P}(s),\mathcal{E}_s,k) + C_i(s,\mathcal{E}_s,k)$, $p_{\mathcal{P}(s)} = p_{\mathcal{P}(s)} + p_s$. *Scenarioset* $=$ *Scenarioset* $\setminus \{s\}$. Go to Step 1.

Figure 5: Calculaton of Dynamic Programming on Scenario Tree

The algorithm we developed continues adding schedules from the dual solution of the restricted linear program until the algorithm cannot generate new schedules.

5. Concluding Remarks

In this paper we proposed a new algorithm that is based on the Dantzig-Wolfe reformulation and column generation approach to solve the stochastic unit commitment problem. That is an algorithm to continue adding schedules from the dual solution of the restricted linear program until the algorithm cannot generate new schedules. The schedule generation problem is solved by the calculation of dynamic programming on the scenario tree. More research is necessary to make scenario set that reflects real demand. As for application to real power system, the coordination of the operation of hydroelectric generation plants is left as future problem.

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- **Step 0.** Set $z^{tmp} = +\infty$.
- **Step 1.** Solve the restricted linear programming master problem. Let $(\pi^*, \mu^*), z^*$ be the optimal dual solution and the optimal objective value of the restricted linear programming master problem, respectively.
- **Step 2.** Solve the restricted integer programming master problem by branch-and-bound method. A node of search tree is fathomed if its subproblem has an objective that is greater than z^{tmp}. Let z^{master} be the optimal objective value of the integer master problem. If the z^{master} is less than z^{tmp}, set z^{tmp}=z^{master}.
- **Step 3.** Solve the schedule generation problem for unit $i = 1, \ldots, I$. Let ζ_i^* be the optimal objective value of the schedule generation problem for unit i.
- **Step 4.** If $\zeta_i^* \geq 0, i = 1, \ldots, I$, the optimal solution of the linear programming master problem is obtained. And no new schedule is generated. If $\zeta_i^* < 0$ for some i, we adopt the optimal $(u_{it}^{sk}, x_{it}^{sk}), t = 1, \ldots, T$ as a feasible schedule.
- **Step 5.** If no schedule is generated in **Step 4** or the best objective function value found so far fails to decrease in a specified number of iterations, stop. Otherwise, go to **Step 1.**

Figure 6: Algorithm of Column Generation Method

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