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THE NORMAL OPEN SET ALMOST EQUAL TO A REGULAR OMEGA LANGUAGE

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Abstract. A regular omega language is the omega language which is recognised by a Buchi automaton. A normal open set is an open set which is equal to the internal kernel of its closure. This paper shows that for each regular language, there exists a normal open set which is equal to the regular language except for the area of a null set as the error, and that the measure of the boundary of the normal open set is zero.

Keywords: Regular language, Regular omega language, measure, topology.

1 Introduction

A regular omega language is the omega language which is recognised by a Buchi automaton. After one of the earliest studies was made by Büchi as in the literature [B], there have been many studies on regular omega languages, which appear in the literatures [S97, T].

A normal open set is an open set which is equal to the internal kernel of its closure. This property is natural for the shapes of real material objects.

The main result of this paper is the following: for each regular language, there exists a normal open set which is equal to the regular language except for the area of a null set, and that the measure of the boundary is 0.

This paper shows two ways of the proofs of the main theorem. One proves the theorem directly, and the other proves a little extended theorem, which is the theorem for finite-state omega languages. All the regular omega languages are finite-state, although not all finite-state omega languages are regular.

In both proofs, we will use decomposition lemmata. One is Lemma 5.2 for regular omega languages, and it appears in the literature [S98]. The other is Lemma 6.8 for finite-state languages, and it is proved for the first time in this paper.

2 Regular omega languages

Notation 2.1 Let Σ be a finite set of characters.

The empty word is written as e. For $w \in \Sigma^*$ and $x \in \Sigma^* \cup \Sigma^\omega$, $w \sqsubset x$ iff $w \neq x$ and $x = w \cdot y$ for some $y \in \Sigma^* \cup \Sigma^\omega$. For $W \subset \Sigma^*$, the set W^ω is the set of all the ω-words which have the form $w_1 w_2 ...$ where $w_i \in W - \{e\}$ for each i.

We use the notations U and $U(w)$ such as $U = \Sigma^\omega$ and $U(w) = w \cdot \Sigma^\omega \subset \Sigma^\omega$ for $w \in \Sigma^*$.

Definition 2.2 (Regular omega languages) A set $X \subset U$ is *regular* iff it is recognised by some Buchi automaton.

Notation 2.3 A subset of $U = \Sigma^\omega$ is usually called an *omega language*, and a regular subset of Σ^ω is called an *regular omega language*. We call an omega language a set in this paper. Thus, a regular omega language is called a regular set.

Notation 2.4 We call a subset of Σ^* a *language*. We use the word of *regular languages* as the ordinary definition, which is defined with ordinary finite automata.

Proposition 2.5 (Büchi '60) *Let $X, Y \subset U$ be regular sets. Then $X \cup Y$, $X \cap Y$ and $X - Y$ are also regular.*

Definition 2.6 (Measure) Let n be the number of symbols in Σ. Then for a set $X \subset U$, the measure $\mu(X)$ is defined as:

$$\mu(X) = \inf \left\{ \sum_{i \in I} n^{-\text{length}(w_i)} \;\middle|\; X \subset \bigcup_{i \in I} U(w_i) \right\}.$$

Remark 2.7 This $\mu(X)$ is usually called the outer measure.

Definition 2.8 (Measurability) A set X is *measurable* iff $\mu(X) + \mu(U - X) = 1$.

Remark 2.9 If $X \subset U$ is a $F\sigma\delta$-set, then X is measurable. All the regular sets are $F\sigma\delta$-sets. Therefore, a regular set is measurable.

3 Normal sets

Notation 3.1 Let X be a subset of U. The notation $\mathcal{G}X$ is written for the internal kernel of X, that is, $\mathcal{G}X = \bigcup\{U(w) \mid w \in \Sigma^*, U(w) \subset X\}$. The notation $\mathcal{F}X$ is written for the closure of X, that is, $\mathcal{F}X = U - \mathcal{G}(U - X)$. The notation ∂X is written for $\mathcal{F}X - \mathcal{G}X$, which is the boundary of X.

Remark 3.2 For $X \subset U$ and $x \in U$, $x \in \mathcal{F}X$ iff for each $w \sqsubset x$, there is $y \in U$ such that $w \sqsubset y \in X$.

Proposition 3.3 *For a regular set $X \subset U$, the internal kernel $\mathcal{G}X$ and the closure $\mathcal{F}X$ are also regular.*

Definition 3.4 (Normality) A set $X \subset U$ is an normal open set iff $X = \mathcal{G}\mathcal{F}X$. A set $X \subset U$ is a normal closed set iff $X = \mathcal{F}\mathcal{G}X$.

Remark 3.5 For each set $X \subset U$, the set $\mathcal{G}\mathcal{F}X$ is a normal open set and the set $\mathcal{F}\mathcal{G}X$ is a normal closed set.

Definition 3.6 (Almost equality) For sets $X, Y \subset U$, the set Y is *almost equal* to X iff $\mu(X - Y) = \mu(Y - X) = 0$.

Remark 3.7 The almost equality is a equivalence relation. Thus, if X is almost equal to Y, and Y is almost equal to Z, then X is almost equal to Z.

Proposition 3.8 *Let X, Y be subsets of U. If X is open and almost equal to Y, then $X \subset \mathcal{F}Y$.*

Proof. $X - \mathcal{F}Y$ is open and $\mu(X - \mathcal{F}Y) \leq \mu(X - Y) = 0$. Therefore $X - \mathcal{F}Y = \emptyset$. ∎

Lemma 3.9 (Uniqueness) *Let X, Y, and Z be subsets of U. Suppose that both Y and Z are normal open sets both of which are almost equal to X. Then $Y = Z$.*

Proof. By Proposition 3.8, we have $Y \subset \mathcal{F}Z$, thus $Y = \mathcal{G}Y \subset \mathcal{G}\mathcal{F}Z = Z$, and vice visa. ∎

Proposition 3.10 *Let $X \subset U$ be a closed set such that $\mu(\partial X) > 0$. Then there are no normal open set $Y \subset U$ which is almost equal to X.*

Proof. Suppose that a normal open set Y is almost equal to X. Then, by Proposition 3.8, we have $Y \subset \mathcal{F}X = X$, Hence $Y = \mathcal{G}Y \subset \mathcal{G}X = X - \partial X$, therefore $\mu(X - Y) \geq \mu(\partial X) > 0$. ∎

Example 3.11 There is a normal closed set X such that $\mu(\partial X) > 0$.

Let Σ consist of two symbols a and b. Put $A_1 \subset U$ as $A_1 = \Sigma \cdot a \cdot U$, and $B_1 \subset U$ as $B_1 = \Sigma^4 \cdot b^2 \cdot U - A_1$. The sets A_i and B_i are defined as the following for $i \geq 2$.

$$A_i = \Sigma^{(2i-1)^2} \cdot a^{2i-1} \cdot U - \left(\bigcup_{j \leq i-1} A_j \right) - \left(\bigcup_{j \leq i-1} B_j \right)$$

$$B_i = \Sigma^{(2i)^2} \cdot b^{2i} \cdot U - \left(\bigcup_{j \leq i} A_j \right) - \left(\bigcup_{j \leq i-1} B_j \right)$$

The sets A, B, C and X is defined as: $A := \cup A_i$, $B := \cup B_i$, $C := U - A - B$, $X := A \cup C = U - B$. Then $\mathcal{G}X = A$ and $\mathcal{F}A = X$, thus X is a normal closed set. It holds that $\partial X = C$ and $\mu(C) = \prod_i (1 - 1/2^i) > 1/8$.

Therefore, it does not hold for each measurable set $X \subset U$ that there is a normal open set Y which is almost equal to X.

Remark 3.12 Let $X \subset U$ be a measurable set. Put F and G as:

$$F = \bigcap \{Y \subset U \mid Y \text{ is closed}, \mu(X - Y) = 0\},$$

$$G = \bigcup \{Y \subset U \mid Y \text{ is open}, \mu(Y - X) = 0\}.$$

Then, the set F is the least closed set such that $\mu(X - F) = 0$, and the set G is the greatest open set such that $\mu(G - X) = 0$. It holds that $\mathcal{G}X \subset G \subset F \subset \mathcal{F}X$.

If there are open sets which are almost equal to X, then G is the greatest of them, and it holds that $F = \mathcal{F}G$ and $G = \mathcal{G}F$, thus G is a normal open set.

If $\mu(\partial X) = 0$, then $G = \mathcal{GF}X$ and G is almost equal to X.

4 Finite-state sets and strongly connected sets

Definition 4.1 (States) For $X \subset U$ and $w \in \Sigma^*$, we write X/w for $\{x \in U \mid w \cdot x \in X\}$, and $\mathcal{S}(X)$ for $\{Y \subset U \mid w \in \Sigma^*, Y = X/w\}$. We call a set in $\mathcal{S}(X)$ *States* of X.

Definition 4.2 (Finite-state sets) A set $X \subset U$ is *finite-state* iff $\mathcal{S}(X)$ is finite.

Proposition 4.3 *Each regular set is finite-state.*

Proposition 4.4 *Let $X \subset U$ be finite-state and $Y \in \mathcal{S}(X)$. Then $\{w \in \Sigma^* \mid Y = X/w\}$ is a regular language.*

Definition 4.5 (Nowhere-denseness) A set X is *nowhere dense* iff $\emptyset \in \mathcal{S}(Y)$ for each $Y \in \mathcal{S}(X)$

Remark 4.6 A set $X \subset U$ is nowhere dense iff $\mathcal{GF}X = \emptyset$.

Definition 4.7 (Strong connectedness) A set $X \in U$ is *strongly connected* iff either $X \in \mathcal{S}(Y)$ or $Y = \emptyset$ for each $Y \in \mathcal{S}(X)$

Remark 4.8 This notion appears in the literature [S83].

Proposition 4.9 *If a set X is strongly connected and $\emptyset \notin \mathcal{S}(X)$, then X is dense in U. If a set X is strongly connected and $\emptyset \in \mathcal{S}(X)$, then X is nowhere-dense. Thus, a strongly connected set is either dense in U or nowhere dense.*

Proof. First we suppose that X is strongly connected and $\emptyset \notin \mathcal{S}(X)$. For each $w \in \Sigma^*$, we have $X/w \neq \emptyset$, therefore $X \cap U(w) \neq \emptyset$. Thus X is dense. Next we suppose that X is strongly connected and $\emptyset \in \mathcal{S}(X)$. For each $Y \in \mathcal{S}(X)$, it holds either $Y = \emptyset$ or $Y \neq \emptyset$. If $Y \neq \emptyset$, then $X \in \mathcal{S}(Y)$. And also $\emptyset \in \mathcal{S}(X)$. Therefore $\emptyset \in \mathcal{S}(Y)$. Thus X is nowhere dense. ∎

Proposition 4.10 *Let X be a subset of U. If $\mu(X) = 0$ then $\mu(Y) = 0$ for each $Y \in \mathcal{S}(X)$. If $\mu(X) = 1$ then $\mu(Y) = 1$ for each $Y \in \mathcal{S}(X)$.*

Lemma 4.11 *For each finite-state set $X \subset U$, either $\mu(X) = 0$ or there is a $Y \in \mathcal{S}(X)$ such that $\mu(Y) = 1$.*

Proof. Corollary 13 in [MS]. ∎

Proposition 4.12 *For each strongly connected finite-state set X, $\mu(X) = 1$ or $\mu(X) = 0$.*

Proof. By Proposition 4.10 and Lemma 4.11. ∎

Proposition 4.13 *The measure of a nowhere dense finite-state set is* 0.

Proof. By Proposition 4.10 and Lemma 4.11. ∎

5 Main theorem

Proposition 5.1 *If a set X is regular and open, then $\mu(\partial X) = 0$.*

Proof. By Propositions 3.3 and 2.5, the $\partial X = \mathcal{F}X - \mathcal{G}X$ is regular. As the definition, $\mathcal{F}\partial X = \partial X$. Because X is open, $\partial X = \mathcal{F}X - \mathcal{G}X = \mathcal{F}X - X$. Hence $\mathcal{G}\mathcal{F}\partial X = \mathcal{G}(\mathcal{F}X - X) = \mathcal{G}\mathcal{F}X - \mathcal{F}X = \emptyset$. By Remark 4.6, ∂X is nowhere dense. Thus we have $\mu(\partial X) = 0$ by Proposition 4.13 ∎

Lemma 5.2 (Staiger '98) *Let X be a regular set. Then there is a finite index set I and regular languages V_i and prefix-free regular languages W_i for each $i \in I$ such that the following hold.*

- $X = \bigcup_{i \in I} V_i \cdot W_i^\omega$
- $V_i \cdot W_i^\omega \cap V_j \cdot W_j^\omega = \emptyset$ *for* $i \neq j$
- $v \cdot W_i^\omega \cap v' \cdot W_i^\omega = \emptyset$ *for* $v, v' \in V_i$ *such that* $v \neq v'$

Proof. In [S98]. ∎

Proposition 5.3 *Let $V \subset \Sigma^*$ be prefix-free and regular. Then, $\mu(\mathcal{F}V^\omega) = \mu(V^\omega)$.*

Proof. Theorem 6 in [MS]. ∎

Lemma 5.4 *Let X be a regular set. Then there is a regular language V such that X is almost equal to $V \cdot U$.*

Proof. Put I, V_i's and W_i's as in Lemma 5.2 for X. Put $J \subset I$ and $Y \subset U$ as $J = \{i \in I \mid \mu(W_i \cdot U) = 1\}$ and $Y = \bigcup_{i \in J} V_i \cdot W_i^\omega$. It is obvious that $\mu(W_i^\omega) = 1$ if $i \in J$ and $\mu(W_i^\omega) = 0$ if $i \notin J$. Then, $Y \subset X$ and $\mu(X - Y) = \sum_{i \in I - J} \mu(V_i \cdot W_i^\omega) = 0$. Therefore, Y is almost equal to X.

Next, put $V \subset \Sigma^*$ as $V = \bigcup_{i \in J} V_i$. The language V is regular. Then, $V \cdot U \supset Y$, and $\mu(Y) = \sum_{i \in J} \mu(V_i \cdot U) = \mu(V \cdot U)$. Therefore, $V \cdot U$ is almost equal to Y. ∎

Theorem 5.5 (Main theorem for regular sets) *Let $X \subset U$ be a regular set. Then, there is a normal open set Y such that Y is almost equal to X and $\mu(\partial Y) = 0$.*

Proof. Put $V \subset \Sigma^*$ as in Lemma 5.4 for X. Then, X is almost equal to $V \cdot U$. By Proposition 5.1, $\mu(\partial V \cdot U) = 0$. We have $V \cdot U \subset \mathcal{GF}(V \cdot U) \subset \mathcal{F}(V \cdot U)$, hence $\mu(V \cdot U) = \mu(\partial \mathcal{GF}(V \cdot U)) = 0$. Therefore, $V \cdot U$ is almost equal to a normal open regular set $\mathcal{GF}(V \cdot U)$. ∎

6 Main theorem for finite-state sets

Definition 6.1 (Connected part) For $X \subset U$, the *connected part* $\mathcal{C}(X)$ is defined as: $\mathcal{C}(X) = \{x \in X \mid \forall w \sqsubset x. X \in \mathcal{S}(X/w)\}$.

Remark 6.2 This notion appears in the literature [S83]. The definition here is equivalent to that in [S83] as the next proposition.

Proposition 6.3 *Let X be a subet in U. Put $W \subset \Sigma^*$ such as $w \in W$ iff*

$$w \neq e \text{ and } X = X/w \text{ and for each } v \sqsubset w, \text{ either } v = e \text{ or } X \neq X/v.$$

Then $\mathcal{C}(X) = X \cap \mathcal{F}(W^\omega)$.

Proof. As Remark 3.2, we point out that $x \in \mathcal{F}(W^\omega)$ iff for each $v \sqsubset x$, there is $y \in U$ such that $v \cdot y \in W^\omega$, which is equivalent to $v \cdot w \in W^*$ for some $w \in \Sigma^*$. Moreover, $v \cdot w \in W^*$ is equivalent to $X = X/vw = (X/v)/w$. Hence $v \cdot w \in W^*$ for some w iff $X \in \mathcal{S}(X/v)$. Therefore we have that $x \in \mathcal{F}(W^\omega)$ iff $X \in \mathcal{S}(X/v)$ for each $v \sqsubset x$. Thus, the asserted equation is obtained immediately from the definition of $\mathcal{C}(X)$. ∎

Proposition 6.4 *If $X \subset U$ is measurable, then so is $\mathcal{C}(X)$.*

Proof. Put $W \subset \Sigma^*$ as in Proposition 6.3. Then both X and $\mathcal{F}(W^\omega)$ are measurable, then so is $\mathcal{C}(X) = X \cap \mathcal{F}(W^\omega)$. ∎

Proposition 6.5 (Linearity) *If $\mathcal{C}(X)/w \neq \emptyset$ then $\mathcal{C}(X/w) = \mathcal{C}(X)/w$.*

Proof. First we will show $\mathcal{C}(X/w) \subset \mathcal{C}(X)/w$. Put $y \in \mathcal{C}(X)/w$, because $\mathcal{C}(X)/w \neq \emptyset$. Then $w \cdot y \in \mathcal{C}(X)$. Therefore $X \in \mathcal{S}(X/w)$. Let x be an arbitrary element of $\mathcal{C}(X/w)$. Then $X/w \in \mathcal{S}(X/wv)$ for each $v \sqsubset x$. Thus $X \in \mathcal{S}(X/w) \subset \mathcal{S}(X/wv)$ for each $v \sqsubset x$. On the other hand, we have that for each $u \sqsubset w \cdot x$, there is v such that $u \sqsubset w \cdot v$. It implies $X \in \mathcal{S}(X/u)$ for each $u \sqsubset w \cdot x$. Thus $w \cdot x \in \mathcal{C}(X)$, that is $x \in \mathcal{C}(X)/w$. Therefore $\mathcal{C}(X/w) \subset \mathcal{C}(X)/w$.

Next we will show $\mathcal{C}(X)/w \subset \mathcal{C}(X/w)$. Let x be an arbitrary element of $\mathcal{C}(X)/w$. Then $w \cdot x \in \mathcal{C}(X)$. Thus $X \in \mathcal{S}(X/wv)$ for each $v \sqsubset x$. Hence $X/w \in \mathcal{S}(X) \subset \mathcal{S}(X/wv)$ for each $v \sqsubset x$. Therefore $\mathcal{C}(X/w) \subset \mathcal{C}(X)/w$. ∎

Proposition 6.6 (Staiger '83) *For $X \subset U$, the connective part $\mathcal{C}(X)$ is strongly connected.*

Proof. Lemma 16 in [S83]. ∎

Lemma 6.7 (Staiger '83) *Let X be a finite-state set. Then for each $x \in X$, there is $w \in \Sigma^*$ and $y \in X/w$ such that $x = w \cdot y$ and $y \in \mathcal{C}(X/w)$.*

Proof. The equation (23) in [S83]. ∎

Lemma 6.8 (Decomposition lemma) *For each measurable finite-state set X, there are a nowhere dense set Z, a finite index set I and indexed families $\{W_i\}_{i\in I}$ and $\{Y_i\}_{i\in I}$ such that*

$$X = \bigcup_{i \in I} W_i \cdot Y_i$$

where each W_i is a prefix-free regular language, each Y_i is a measurable strongly connected finite-state set, and $W_i \cdot Y_i \cap W_j \cdot Y_j = \emptyset$ for any $i \neq j$.

Proof. Put $I = \{1, 2, ..., k\}$ and $Z_1, Z_2, ..., Z_k$ such as $\{Z_1, Z_2, ..., Z_k\} = \{Z \in \mathcal{S}(X) \mid \mathcal{C}(Z) \neq \emptyset\}$ and $Z_i \neq Z_j$ for $i \neq j$. Put Y_i as $Y_i = \mathcal{C}(Z_i)$ for each $i \in I$.

For each $i \in I$, let $W_i \subset \Sigma^*$ be the language such that $w \in W_i$ iff

$$X/w = Z_i \text{ and there are no } v \sqsubset w \text{ such that } X/v \in \mathcal{S}(Z_i).$$

We have that W_i is regular. Moreover it is obvious that W_i is prefix-free.

First we show that $W_i \cdot Y_i \subset X$. It holds because for each $w \in W_i$, we have $X/w = Z_i$, thus $w \cdot Z_i \subset X$, and $Y_i \subset Z_i$.

Next we show that $X \subset W_i \cdot Y_i$. Put x as $x \in X$. As Lemma 6.7, there are some pairs of (w,y)'s such that $x = w \cdot y$ and $y \in \mathcal{C}(X/w)$. Put (w, y) as w is the shortest, that is, there are no $v \in \Sigma^*$ and $z \in U$ such that $v \sqsubset w$, $x = v \cdot z$ and $z \in \mathcal{C}(X/z)$. Put i as $Y_i = X/w$. Then $w \in W_i$ and $y \in Y_i$, thus $x \in W_i \cdot Y_i$.

Lastly we show that $W_i \cdot Y_i \cap W_j \cdot Y_j = \emptyset$ for any $i \neq j$. Suppose that $x \in W_i \cdot Y_i \cap W_j \cdot Y_j$ for some $x \in U$ and $i \neq j$. Then there are $v \in W_i, w \in W_j, y \in Y_i$ and $z \in Y_j$ such that $x = wy = vz$. We have $v \neq w$ because

$X/v = Z_i \neq Z_j = X/w$. Without loss of generousity, we assume that $vu = w$ for some $u \in \Sigma^*$. Then $y = uz \in Y_i = \mathcal{C}(X/v)$, thus $X/vuu' = X/v$ for some $u' \in \Sigma^*$. Hence $(X/w)/u' = X/v$, which implies $X/v \in \mathcal{S}(X/w)$. This contradicts to $w \in W_j$, because $v \sqsubset w$ and $X/v \in \mathcal{S}(Z_j)$, although $w \in W_j$ implies there is no such v. ∎

Proposition 6.9 *Put X, W_i's and Y_i as in the previous lemma. If $i \neq j$ and both Y_i and Y_j are dense, then $W_i \cdot U \cap W_j \cdot U = \emptyset$.*

Proof. We will modify the last part of the proof of the previous lemma a little bit. Suppose $x \in W_i \cdot U \cap W_j \cdot U$. Then there are $v \in W_i$ and $w \in W_j$ such that $v \sqsubset x$ and $w \sqsubset x$. We have $v \neq w$ because $X/v = Z_i \neq Z_j = X/w$. Without loss of generousity, we assume that $vu = w$ for some $u \in \Sigma^*$. We have $V_i \cdot U \subset \mathcal{F}(V_i \cdot Y_i)$ because Y_i is dense i U. Therefore $vu = w \sqsubset x \in \mathcal{F}(V_i \cdot Y_i)$. Then $uz \in Y_i = \mathcal{C}(X/v)$ for some z, thus $X/vuu' = X/v$ for some $u' \in \Sigma^*$. Hence $(X/w)/u' = X/v$, which implies $X/v \in \mathcal{S}(X/w)$. This contradicts to $w \in W_j$, because $v \sqsubset w$ and $X/v \in \mathcal{S}(Z_j)$, although $w \in W_j$ implies there is no such v. ∎

Lemma 6.10 *Let $X \subset U$ be a measurable finite-state set. Then there is a regular language V such that X is almost equal to $V \cdot U$.*

Proof. Put I, W_i's and Y_i's as in Lemma 6.8 for X. By Proposition 6.4, each Y_i is measurable. Put $J \subset I$ and $Y \subset U$ as $J = \{i \in I \mid \mu(Y_i) = 1\}$ and $Y = \bigcup_{i \in J} W_i \cdot Y_i$. Then, $Y \subset X$ and $\mu(X - Y) = \sum_{i \in I - J} \mu(W_i \cdot Y_i) = 0$. Hence Y is almost equal to X.

Next, put $V \subset \Sigma^*$ as $V = \bigcup_{i \in J} W_i$. The language V is regular. Then, $V \supset Y$, and $\mu(Y) = \sum_{i \in J} \mu(V_i \cdot U) = \mu(V \cdot U)$. Therefore, $V \cdot U$ is almost equal to Y. ∎

Theorem 6.11 (Main theorem for finite-state sets) *Let $X \subset U$ be measurable and finite-state. Then, there is a normal open set Y such that Y is almost equal to X and $\mu(\partial Y) = 0$.*

Proof. Similar to Theorem 5.5 with referring Lemma 6.10 instead of Lemma 5.4. ∎

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