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SECTIONAL KNOTS IN SEIFERT FIBERED 3-MANIFOLDS

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はじめに

本稿では, 2001 年 12 月に京都大学数理解析研究所で行われた研究集会「双曲空間及び離散群の研究 II」において,「Drilling surfaces and surface-automorphisms」という題目で発表した共同研究の, もう一つの側面であるザイフェルト多様体内の双曲結び目に関する結果を報告する. 尚, 研究集会で発表した, 曲面に穴をあけるという操作による曲面上の自己同相写像の Nielsen-Thurston 分類型の変化に関する結果については, プレプリント [7] を参照のこと.

1. Introduction

The aim of this note is to take another side view of the result of [7]. The preprint [7] mainly concerns Nielsen-Thurston type of a surface-automorphism and its behavior under drilling a surface. In this note, as an application of [7], we consider the hyperbolicity of a knot appearing as a section in a surface bundle over the circle S^1 admitting a Seifert fibration.

We begin with recalling some fundamental definitions and results.

A compact, orientable 3-manifold is called *Seifert fibered* if it admits a foliation by circles called *Seifert fibers*. A Seifert fibered 3-manifold can be regarded as a fiber bundle over a 2-orbifold with circular fiber. Every Seifert fibered 3-manifold with non-empty boundary and some of closed ones also admits a fibration over S^1 with surface fiber. In this note, we will mainly deal with such Seifert fibered 3-manifolds. About Seifert fibered 3-manifolds, see [13] for a survey.

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As usual, a *knot* will mean an embedding of S^1 or its image in a 3-manifold. We empirically know that 'most' knots are *hyperbolic*, that is, have the complements with a complete hyperbolic metric of finite volume, even in a non-hyperbolic 3-manifold. About hyperbolic knots, see [2] for a survey.

In a Seifert fibered 3-manifold which also fibers over S^1, we consider a knot appearing as a section of the fibration, which we call a *sectional knot*, and ask the next question.

Question. *In a Seifert fibered 3-manifold fibering over S^1, which sectional knot is hyperbolic?*

Although no Seifert fibered 3-manifold is hyperbolic, it is conjectured that there exist plenty of hyperbolic sectional knots. We will actually confirm this in some cases by describing when such a knot is hyperbolic in terms of the *projection* of a knot.

To state our theorems, we prepare some notations, which will be used throughout the article. Let F be a closed, connected, orientable surface and f an orientation preserving automorphism of F. Let M_f be a *mapping torus with the gluing map* f, meaning that $M_f = (F \times I)/\{(x,0) = (f(x),1)\}$, where I denotes the unit interval $[0,1]$. This M_f is obviously regarded as a fiber bundle over the circle S^1 with fiber F. Note tha if f is periodic, i.e., some power of f is the identity map of F, then M_f is foliated by circles. This gives the unique Seifert fibration of M_f up to isotopy when the genus of the surface F is greater than one [9].

Let $p : F \times I \to F$ be a natural projection and $q : F \times I \to M_f$ a natural quotient map. For a sectional knot K in M_f, we call the curve appearing as $p \circ q^{-1}(K)$ on F a *projection* of K. This definition, unlike the usual knot theory, yields a projection which is not a closed curve. To avoid this, if necessary, we isotope f to have at least one fixed point x_0 and isotope a sectional knot to run through the point $q(x_0 \times \{0\})$ in M_f. Under this setting, every projection of a sectional knot is a (not necessarily simple) closed curve on F containing x_0.

When the genus of the surface F is less than two, it is shown that no sectional knots are hyperbolic in M_f. A brief observation about this will be given in the next section. Henceforth, except for Section 2, we will always assume that the genus of F is greater than one.

The first theorem, which was essentially obtained by Kra [10], concerns the simplest case that f is the identity map. In this case, M_f is homeomorphic to $F \times S^1$.

Theorem 1. *A sectional knot in $F \times S^1$ is hyperbolic if and only if its projection is stably filling on F.*

We say that a closed curve on F is *stably filling* if any curve freely homotopic to it intersects every nontrivial embedded loop on F. The proof we will give is based on 3-manifold topology, and so it is quite different from the one induced from [10].

Next, we consider the case that M_f is a *small* Seifert fibered 3-manifold. A Seifert fibered 3-manifold is called *small* if it is a circle bundle over a 2-orbifold whose underlying space is the 2-sphere S^2 and whose singular set consists of at most three cone points. Note that M_f is small if and only if the gluing map f is irreducible and periodic in the sense of Nielsen-Thurston. In this case, we have the following.

Theorem 2. *Let M_f be a small Seifert fibered 3-manifold fibering over S^1.*

(1) *Suppose that M_f has no sectional Seifert fiber. Then every sectional knot in M_f is hyperbolic.*

(2) *Suppose that M_f has sectional Seifert fibers $t_0, t_1, \dots, t_n$. Let x_i be the point $p \circ q^{-1}(t_i)$ for $0 \leq i \leq n$. Set the point x_0 as the base point of projections of sectional knots. Then a sectional knot K in M_f is hyperbolic if and only if no projection of K represents an element of $\pi_1(F, x_0)$ which has the form $[\bar{\gamma} * (f \circ \gamma)]$ for a path γ from some x_i to x_0.*

In the statement above, $\bar{\gamma}$ denotes the path obtained from a path γ by inverting its orientation. The product of two paths γ_1 and γ_2 is denoted by $\gamma_1 * \gamma_2$. For a closed curve c with a base point x_0, $[c]$ denotes the element of $\pi_1(F, x_0)$ represented by c.

Note that for a sectional Seifert fiber t in M_f, $p \circ q^{-1}(t)$ is a fixed point of f, and so the case (1) corresponds to the case that f is irreducible and periodic without fixed points.

In the special case that M_f has only one sectional Seifert fiber t_0, we immediately have the following corollary. In the following, f_* denotes the automorphism of $\pi_1(F, x_0)$ induced from f.

Corollary 1. *Let M_f be a small Seifert fibered 3-manifold which fibers over S^1 and contains single sectional Seifert fiber t_0. Let x_0 be the point $p \circ q^{-1}(t_0)$ and set x_0 as the base point of projections of sectional knots. Then a sectional knot K in M_f is hyperbolic if and only if no projection of K represents an element of $\pi_1(F, x_0)$ which has the form $[c']^{-1} f_*([c'])$ for any closed curve c' with base point x_0.* □

Also note that this corresponds to the case that f is irreducible and periodic with single fixed point.

2. Small genera case

In this section, we give brief observations about the small genera cases.

Suppose that the genus of F equals 0, that is, F is homeomorphic to S^2. Then there exists only one M_f, that is, $S^2 \times S^1$. It is well-known that every sectional knot in $S^2 \times S^1$ is isotopic to be vertical, that is, the form $\{*\} \times S^1$. This means that every sectional knot in $S^2 \times S^1$ is isotopic to a sectional Seifert fiber and so no one is hyperbolic.

Next, suppose that the genus of F equals 1, that is, F is the torus. In this case, it is known that there exist just five M_f's which are Seifert fibered ([6, Examples 12.3.]). All of these have sectional Seifert fibers. Moreover it is verified that every sectional knot is equivalent to be vertical; there is a self-homeomorphism of the ambient manifold which sends the given sectional knot to a sectional Seifert fiber. Again, in this case, no sectional knot is hyperbolic.

3. Sectional knots in $F \times S^1$

3.1. **Outline of the proof of Theorem 1.** Throughout this subsection, let M be the mapping torus with trivial gluing map; $(F \times I)/\{(x,0)=(x,1)\}$, K a sectional knot in M, c a projection of K on F and $E(K)$ the exterior $M - \mathrm{Int}N(K)$.

Let us first show that 'only if' part. For a contradiction, suppose that K is hyperbolic and c is freely homotopic to a closed curve c' which avoids some nontrivial simple loop ε. There is the sectional knot K' in M which has c' as a projection. In $M - \mathrm{Int}N(K')$, there is a vertical torus $T_\varepsilon = q(\varepsilon \times I)$. This T_ε is essential (i.e., incompressible and not boundary parallel) since ε is nontrivial on F. Note that the free homotopy between c and c' implies that an isotopy between K and K', and it extends to an ambient isotopy of M which moves K to K'. In particular, $E(K)$ is homeomorphic to $M - \mathrm{Int}N(K')$. Therefore we find an essential torus in $E(K)$. This contradicts that K is hyperbolic.

Next, let us consider the 'if' part. Suppose that c is stably filling on F with the base point x_0. By the Thurston's Uniformization Theorem [14], it suffices to show that (1) $E(K)$ contains no essential tori and (2) $E(K)$ is not Seifert fibered.

Suppose for a contradiction to (1) that $E(K)$ contains an essential torus T.

Let F_t be the surface $q(F \times \{t\})$ in M for $t \in I$ and $\check{F}_t$ the surface $F_t \cap E(K)$ in $E(K)$. Since $\check{F}_0 (= \check{F}_1)$ is incompressible in $E(K)$, by an isotopy of T in $E(K)$, we assume that the intersection $T \cap F_0$ consists of non-empty nontrivial loops in both T and F_0. Also we assume that the number of components of $T \cap F_0$ is minimal. The preimage $q^{-1}(T)$ of T are the disjoint union of annuli $A_1, \dots, A_n \subset F \times I$.

Claim 1. *For each A_i $(i = 1, \ldots, n)$, one boundary component is in $F \times \{0\}$ and the other is in $F \times \{1\}$.*

Proof. Remark that these annuli is incompressible in $F \times I - q^{-1}(K)$ since T is essential. If the boundary of one of them is entirely contained in $F \times \{0\}$ or $F \times \{1\}$, then by [15, Corollary 3.2] it is boundary parallel, and so it contradicts the minimality of the number of components of $T \cap F_0$. □

The following lemma therefore implies that each annulus A_i $(i = 1, \ldots, n)$ is also incompressible in $F \times I$.

Lemma 3.1.1. *Let F be a closed orientable surface and K' an monotone arc in $F \times I$ connecting $(x_0, 0)$ and $(x_0, 1)$ for some point $x_0 \in F$. Let A be an incompressible annulus in $E(K') = F \times I - intN(K')$ with one boundary component in $F \times \{0\}$ and the other in $F \times \{1\}$. If A is compressible in $F \times I$, then A is parallel to the frontier of $N(K')$ in $F \times I$.* □

Moreover the following holds in this case.

Claim 2. *$p(A_1 \cap (F_0)) = p(A_1 \cap (F_1))$ holds. In particular, the number n of annuli is equal to 1.* □

Let c_1 denote the curve $p(\partial A_1)$ for this single annulus A_1.

Claim 3. *There is an isotopy of $F \times I$ such that A_1 is moved to the vertical annulus $c_1 \times I$ and it is identity on the surfaces $F \times \{0\}$ and $F \times \{1\}$.* □

For proofs of these claims, see [7].

Under the isotopy above, the arc $q^{-1}(K)$ is moved to an arc k keeping the endpoints fixed. Thus there is a homotopy between $c = p(q^{-1}(K))$ and $p(k)$ on F. Since the original A_1 is disjoint from $q^{-1}(K)$, the annulus $c_1 \times I$ does not intersect k, and hence $p(k) \cap c_1 = \emptyset$. However this contradicts that c is stably filling on F.

Suppose for a contradiction to (2) that $E(K)$ is Seifert fibered. Then there exists a Seifert fibration of M in which K is a fiber by the the next lemma.

Lemma 3.1.2. *Let M be an irreducible manifold and k is a knot in M. If $M - \mathrm{Int}N(k)$ is Seifert fibered, then M admits a Seifert fibration in which k is a fiber.* □

Here each Seifert fiber of $M = F \times S^1$, up to free isotopy, is the form $\{*\} \times S^1$, and so it is sectional.

Finally we use the following:

Lemma 3.1.3. *Let M_f denote the mapping torus $(F \times I)/\{(x,0) = (f(x),1)\}$ with the periodic gluing map f which has a non-empty fixed point set $\{x_0, \dots, x_n\}$. Let K be a sectional knot K in M_f and c a projection of K. Suppose that K is isotopic to a sectional Seifert fiber of the form $(\{x_i\} \times I)/\{(x_i,0) = (x_i,1)\}$ if and only if $[c] = [\bar{\gamma} * (f \circ \gamma)]$ in $\pi_1(F, x_0)$ for some path γ from x_i to x_0, where $*$ denotes the product of (possibly non-closed) paths.* □

In our case, the gluing map f is the identity map, and so we have

$$[c] = \alpha^{-1} f_*(\alpha) = \alpha^{-1}\alpha = 1$$

for some $\alpha \in \pi_1(F, x_0)$. However, since a stably filling curve c is nontrivial, it is absurd.

3.2. **Examples of curves.** The purpose of this subsection is to give concrete examples of stably filling curves on F. Recall that a closed curve c on a closed orientable surface F is called *filling* if every connected component of $F - c$ is an open disk. Note that c is stably filling if and only if every closed curve freely homotopic to c is filling.

In the following, we fix a hyperbolic metric on F. As shown in [1] implicitly, we have:

Lemma 3.2.1. *A filling closed geodesic is stably filling.* □

Let us give examples of stably filling curves on a surface of genus two. Similarly, one can construct such examples for the higher genus case. See [7] for a detail.

By virtue of Lemma 3.2.1, it suffices to find filling geodesics. We start with four copies of a regular truncated triangle, equivalently, a right-angled rectangular hexagon, in $\mathbb{H}^2$. By gluing their edges suitably, we obtain two copies of a pair of pants P_1 and P_2 with equilong boundaries. On P_1, take three geodesic arcs each one of which is the shortest path connecting distinct boundary components. Remark that each boundary component of P_1 is bisected by the endpoints of these arcs. On P_2, take also three geodesic arcs each one of which is the return path of a boundary component. Remark that each boundary component of P_2 again is bisected by the endpoints of these arcs. Then P_1 and P_2 are glued so that the six geodesic arcs form single closed curve c. For these arcs match geodesically as they are all orthogonal to the boundary. It is easily checked that this c is actually filling on the resultant surface of genus two.

We can find a simple closed curve c' on the resultant surface which intersects c exactly once. By performing the m-Dehn twist along c', we obtain infinitely many curves c_n which is also stably filling. These are all mutually non-isotopic. In fact, c_m and c_n $(m \neq n)$ are not homologous. For one can find a simple closed curve c'' intersecting c'

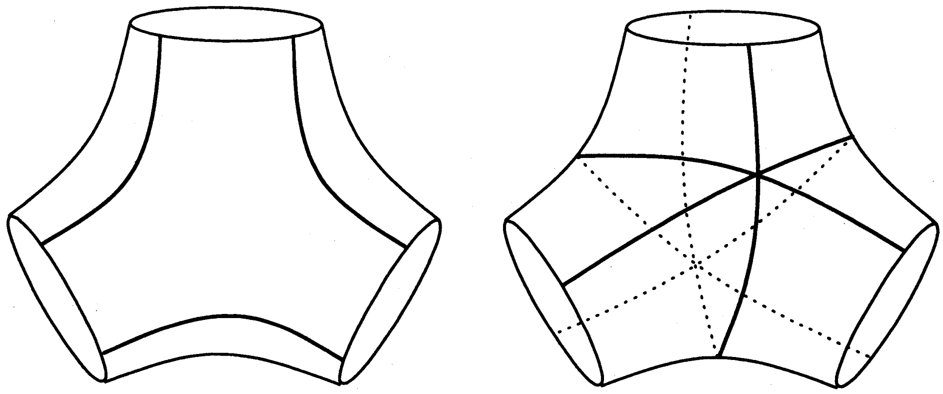

FIGURE 1. geodesic arcs on P_1 and P_2

exactly once, and then the algebraic intersection number of c_m and c'' varies depending only on m.

4. SECTIONAL KNOTS IN A SMALL SEIFERT FIBERED 3-MANIFOLD

4.1. **Outline of the proof of Theorem 2.** Note that Theorem 2 is an immediate corollary of the next proposition together with Lemma 3.1.3.

Proposition 1. *Let K be a sectional knot in a small Seifert fibered 3-manifold which fibers over S^1. Then K is hyperbolic if and only if K is isotopic to none of sectional Seifert fibers.*

Proof. Let M_f denote a small Seifert fibered 3-manifold, equivalently, we assume that f is an irreducible, periodic surface-automorphism. It is known that M_f is *atoroidal*, that is, M_f contains no essential tori [8]. Let K be a sectional knot in M_f, c a projection of K and $E(K)$ the exterior $M_f - \mathrm{Int}N(K)$.

If K is isotopic to a sectional Seifert fiber, then $E(K)$ admits a Seifert fibration obviously, and so K is not hyperbolic.

Conversely suppose that K is not hyperbolic. If $E(K)$ contains an essential torus, then it remains incompressible in M_f by Lemma 3.1.1. This contradicts that M_f is small, and so $E(K)$ is atoroidal. Then, by the Thurston's Uniformization Theorem [14], there is a Seifert fibration on $E(K)$. By Lemma 3.1.2, the ambient manifold M_f also admits a Seifert fibration in which K is a fiber. Since the Seifert fibration of such M_f is unique up to isotopy [9], K is isotopic to a Seifert fiber which is sectional. □

4.2. **Examples of maps.** In this subsection, we give examples of irreducible, periodic automorphisms corresponding to Theorem 2 (1) and Corollary 1.

To do this, we give an observation about the covering theory. See [3] for a detail. Let us denote an orbifold with the underlying space S^2 and three cone points b_1, b_2 and b_3 of indices p_1, p_2 and p_3 by $S^2(p_1, p_2, p_3)$. Put $n = \mathrm{lcm}(p_1, p_2, p_3)$. Suppose that a surjective representation $\rho : \pi_1(S^2 - \{b_1, b_2, b_3\}) \to \mathbf{Z}_n$ which sends a small loop around b_i to an element of order p_i is given. Then by taking an n-fold cyclic orbifold covering associated with ρ, we obtain a closed orientable surface F of genus $g = \frac{1}{2}n(1 - \frac{1}{p_1} - \frac{1}{p_2} - \frac{1}{p_3}) + 1$ and a periodic automorphism f so that $F/\langle f \rangle = S^2(p_1, p_2, p_3)$ ([3], [11]). This f so obtained is irreducible by [4, Theorem 3.1]. Algebraic conditions for an existence of such branched coverings (equivalently such representations) was given in [5].

We remark that an irreducible, periodic automorphism $f : F \to F$ of period p fixes at most three points. For if we take a quotient of F by the action $\langle f \rangle$, then we obtain an orbifold $S^2(p_1, p_2, p_3)$ [4, Theorem 3.1]. If f fixes a point $x_i \in F$, then x_i is injectively projected to a cone point b_i of index p. This implies that f fixes at most three points on F.

Example 1. Choose an orbifold $S^2(m_2(m_1 + m_2), m_1(m_1 + m_2), m_1 m_2)$, where m_1 and m_2 are coprime. Take a cyclic orbifold covering associated with a representation $\rho : \pi_1(S^2 - \{b_1, b_2, b_3\}) \to \mathbf{Z}_{m_1 m_2(m_1+m_2)}$ which sends a small loop around b_i to m_i $(i = 1, 2)$. Then we obtain a closed orientable surface F of genus $\frac{1}{2}(m_1 + m_2)(m_1 m_2 - 2) + 1$ and an irreducible periodic automorphism $f : F \to F$ of period $m_1 m_2(m_1 + m_2)$. Since each branching index is strictly smaller than the period, f has no fixed point. This gives an example of Theorem 2 (1).

The simplest case of $(m_1, m_2) = (2, 3)$ was described in [11, Proposition 1]. In this case, the genus of the resultant surface is 11 and the period of the resultant automorphism is 30.

Example 2. Choose an orbifold $S^2(m_2, m_1, m_1 m_2)$, where m_1 and m_2 are coprime. Take a cyclic orbifold covering associated with a representation $\rho : \pi_1(S^2 - \{b_1, b_2, b_3\}) \to \mathbf{Z}_{m_1 m_2}$ which sends a small loop around b_i to m_i $(i = 1, 2)$. Then we obtain a closed orientable surface F of genus $\frac{1}{2}(m_1 - 1)(m_2 - 1)$ and an irreducible periodic automorphism $f : F \to F$ of period $m_1 m_2$. The cone point b_3 has the branching index $m_1 m_2$ which is equal to the covering index. Thus the preimage of b_3 consists of exactly one point and this is a unique fixed point of f. This gives an example of Corollary 1.

In fact, the resultant automorphism is the monodromy of the surface bundle over S^1 obtained by 0-surgery along (m_1, m_2)-torus knot in S^3.

4.3. **Examples of curves.** In this subsection, we see that there are plenty of curves described in Corollary 1.

Let $f : F \to F$ an orientation preserving periodic automorphism satisfying $f(x_0) = x_0$ for some point $x_0 \in F$. We assume that the period p of f is greater than two. For the period two case, please see [7].

Hereafter, fix an $\langle f \rangle$-invariant hyperbolic metric on F; for an existence of such a metric, see [13, Section 2]. Then each element in $\pi_1(F, x_0)$ is represented by a *geodesic closed path* (i.e., a curve $c : I \to F$ such that $c(0) = c(1) = x_0$ and $c|_{(0,1)}$ is geodesic).

Actually we show the following.

Proposition 2. *Let $f : F \to F$ be a periodic automorphism of period $p > 2$. Then there exists a positive constant C_f depending only on f such that an element γ or γ^{-1} in $\pi_1(F, x_0)$ cannot be written as $\alpha^{-1} f_*(\alpha)$ for any element $\alpha \in \pi_1(F, x_0)$ if the length of the geodesic closed path representing γ is greater than C_f.*

By this proposition, the element of $\pi_1(F, x_0)$ which and whose inverse are both represented as $\alpha^{-1} f_*(\alpha)$ with some $\alpha \in \pi_1(F, x_0)$ must have the geodesic closed path with bounded length as a representative. Since the holonomical image of $\pi_1(F, x_0)$ is discrete, there exist only finite number of such elements.

In the following, we outline the proof of Proposition 2. See [7] for a detail.

Let c be a geodesic closed path on F. We denote by c^{-1} the geodesic closed path defined by $c^{-1}(t) = c(1-t)$ for $t \in I$ and by $\theta_{f,c}$ the angle from $\dot{c}(1)$ to $df_{x_0}(\dot{c}(0))$ for a geodesic closed path c, where $-\pi < \theta_{f,c} \leq \pi$.

Set a positive constant Θ_f depending only on f, $0 < \Theta_f < \pi$, as follows.

$$\Theta_f = \begin{cases} \pi - \theta_f & 0 < \theta_f \leq \pi/2 \\ \theta_f & \pi/2 < \theta_f < \pi \\ 2\pi - \theta_f & \pi < \theta_f \leq 3\pi/2 \\ \theta_f - \pi & 3\pi/2 < \theta_f < 2\pi \end{cases}$$

where θ_f denotes the rotation angle ($0 < \theta_f < 2\pi$) of the action of df_{x_0} on $T_{x_0}F$. Remark that $\theta_f \neq \pi$ by the assumption that $p > 2$.

Then the next is proved by careful case-by-case arguments.

Claim 4. *For any geodesic closed path c, we have* $\min\{|\theta_{f,c}|,\ |\theta_{f,c^{-1}}|\} \leq \Theta_f$. □

Moreover we have:

Claim 5. *Suppose that* $|\theta_{f,c}| \leq \Theta_f$ *holds. If*

$$\cosh l_c/2 \geq \frac{1}{\sin(\pi - \Theta_f)/2}$$

holds, then the product $[c][f \circ c] \cdots [f^{p-2} \circ c][f^{p-1} \circ c]$ *is not trivial in* $\pi_1(F, x_0)$, *where* l_c *denotes the length of* c. □

The key of the proof of this claim is to show the preimage of a geodesic closed path in the universal cover $\mathbb{H}^2$ of F is not closed if the assumption satisfied. See [7] for a detail.

Finally, we use the next claim, which is proved by elementary algebraic calculations.

Claim 6. *If* $[c][f \circ c] \cdots [f^{p-2} \circ c][f^{p-1} \circ c] \neq 1$ *in* $\pi_1(F, x_0)$, *then* $[c]$ *cannot be written as* $\alpha^{-1} f_*(\alpha)$ *for any* $\alpha \in \pi_1(F, x_0)$. □

Consequently, we obtain a desired constant $C_f = 2\cosh^{-1}(\frac{1}{\sin(\pi - \Theta_f)/2})$.

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