数理解析研究所講究録 1303 巻 2003 年 46-55

ON THE COINCIDENCE OF SMALL AND LARGE INDUCTIVE DIMENSIONS

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1. On questions of Engelking

All spaces are assumed to be normal and Hausdorff.

We shall consider the question: What conditions do we need for the coincidence of ind $=$ Ind?

It is one of the most important and fundamental facts in dimension theory is the coincidence of the three fundamental dimensions ind, Ind and dim for separable metrizable spaces. Furthermore, as is well known that the coincidence of Ind $=$ dim holds for metrizable spaces (Katětov (1950) and Morita (1954), see [E]). On the other hand, we have the famous Roy's example of a completely metrizable space X with ind $X = 0 < \operatorname{Ind} X$ ([R], 1963). Kulesza ([K1], 1990) succeeded to simplify the example. Recently, Mrowka [Mu1], [Mu2] and Kulesza [K2] get the metrizable spaces X which show the gap of $\operatorname{Ind} X - \operatorname{ind} X$ can be arbitrarily high under some set-theoretic assumption. These examples show that the metrizablity does not work for the conincidence of ind $=$ Ind.

On the other hand, it is known that the equality ind $=$ Ind holds for the following classes of spaces.

- Strongly paracompact, metrizable spaces (Morita, 1950)
- Order totally paracompact, metrizable spaces (Fitzpatrick and Ford, 1967)
- σ-totally paracompact, totally normal spaces (Nagami, 1969)
- Closure totally paracompact totally norma spaces (French, 1976)
- Order totally paracompact, totally normal spaces (Mizokami, 1979)

- σ-totally paracompact, strongly hereditarily normal spaces (Engelking, 1995)

Let us recall from [FF] that a space X is called *order totally paracompact* (shortly, OTP) if for every open base $\mathcal{B}$ of X there exists a linearly ordered open cover $(\mathcal{V}, <)$ of X satisfying:

(1) for every $V \in \mathcal{V}$, there exists $U \in \mathcal{B}$ such that $V \subset U$ and $\mathrm{Bd}\, V \subset \mathrm{Bd}\, U$, where $\mathrm{Bd}\, A$ denotes the boundary of A in X, and

(2) $(\mathcal{V}, <)$ is *order locally finite*, i. e. for every $V \in \mathcal{V}$, $\{V' \in \mathcal{V} : V' < V\}$ is locally finite at each point in V.

We notice the following fact:

(a) The class of order totally paracompact spaces is hereditary with respect to closed subspaces.

We also recall that a space X is said to be *strongly hereditarily normal* ([E]) if for every separated sets A and B of X there are disjoint open sets U and V such that $A \subset U$, $B \subset V$ and both U and V are unions of point finite families of open F_σ-sets of X. We notice that every totally normal space is strongly hereditarily normal, and the countable sum theorem, locally finite sum theorem and subspace thereom for large inductive dimension Ind holds for every strongly hereditarily normal space. In [E, Remark on page 165], Engelking asked the following questions:

Question 1. For every order totally paracompact space X, are the conditions $\mathrm{ind}\, X = 1$ and $\mathrm{Ind}\, X = 1$ equivalent?

Question 2. For every order totally paracompact, strongly hereditarily normal space X, does the equality $\mathrm{ind}\, X = \mathrm{Ind}\, X$ hold?

We shall answer the questions positively and we have general results in this direction.

It is known that

(b) the conditions $\mathrm{ind}\, X = 0$ and $\mathrm{Ind}\, X = 0$ are equivalent for every order totally paracompact space X ([E], Problem 2.4.D (b)), and

(c) the conditions $\mathrm{ind} X = 1$ and $\mathrm{Ind} X = 1$ are equivalent for σ-totally paracompact space X ([E], Problem 2.4.C (a)).

In the proof of the following main lemma, we use some Mizokami's ideas from [M].

Main lemma *Let X be an order totally paracompact space and $\mathcal{B}$ be a base of X. Then for every pair A, B of disjoint closed subsets of X*

there exist a partition C between A and B, a locally finite family $\mathcal{F}$ of closed subsets of X which satisfying the following conditions.

(1) $C \subset \cup\mathcal{F}$,

(2) For every $F \in \mathcal{F}$ there exists $U \in \mathcal{B}$ such that $F \subset \operatorname{Bd} U$

Proof. Consider two pairs (G_1, G_2) and (H_1, H_2) of disjoint open subsets of X such that $A \subset G_1 \subset \overline{G}_1 \subset H_1$, $B \subset G_2 \subset \overline{G}_2 \subset H_2$ and, $\overline{H}_1 \cap \overline{H}_2 = \emptyset$. We put $F_1 = \overline{G}_1$ and $F_2 = \overline{G}_2$. One can suppose that for every $U \in \mathcal{B}$ we have $U \cap \overline{H}_1 = \emptyset$ or $U \cap \overline{H}_2 = \emptyset$. By the definition of order totally paracompact spaces, there exists a linearly ordered open cover $(\mathcal{V}, <)$ of X satisfying:

(1) for every $V \in \mathcal{V}$, there exists $U \in \mathcal{B}$ such that $V \subset U$ and $\operatorname{Bd} V \subset \operatorname{Bd} U$, and

(2) $(\mathcal{V}, <)$ is order locally finite.

For each $V \in \mathcal{V}$, we put $P(V) = \bigcup\{V' \in \mathcal{V} : V' < V\}$ and $W(V) = V \setminus \overline{P(V)} \subset V$. Then, it follows from [M, Lemma 2] that

(*) the family $\{\operatorname{Bd} W(V) : V \in \mathcal{V}\}$ is locally finite in X,

(**) $X \setminus \bigcup\{W(V) : V \in \mathcal{V}\} \subset \bigcup\{\operatorname{Bd} W(V) : V \in \mathcal{V}\}$, and

(***) for every $V \in \mathcal{V}$ we have $\operatorname{Bd} W(V) \subset (\operatorname{Bd} V \setminus P(V)) \cup \bigcup\{(\operatorname{Bd} V') \cap V : V' < V\}$.

Claim 1 *For every $V \in \mathcal{V}$ we have $\operatorname{Bd} W(V) \subset \operatorname{Bd} V \cup \bigcup\{\operatorname{Bd} V' \cap \operatorname{Bd} W(V) : V' < V\}$.*

Proof. By use of (***), we get $\operatorname{Bd} W(V) \subset \operatorname{Bd} V \cup \bigcup\{\operatorname{Bd} V' : V' < V\}$. Now it is easy to see that the inclusion $\operatorname{Bd} W(V) \subset \operatorname{Bd} V \cup \bigcup\{\operatorname{Bd} V' \cap \operatorname{Bd} W(V) : V' < V\}$ is also valid.

Claim 2 *For every $V \in \mathcal{V}$ the family $\{\operatorname{Bd} V' \cap \operatorname{Bd} W(V) : V' < V\}$ is locally finite in X.*

Proof. Consider a point $x \in X$. There exists $V_0 \in \mathcal{V}$ such that $x \in V_0$. We shall check three cases.

Case 1: We asume that $V_0 = V$. Recall that the system $\{V' \in \mathcal{V} : V' < V\}$ is locally finite in V. So there is a nbd Ox of x which meets only finitely many of sets V' with $V' < V$. Hence Ox meets only finitely many of sets $\operatorname{Bd} V'$ with $V' < V$.

Case 2: We assume that $V_0 > V$. It is clear that $\{V' : V' < V\} \subset \{V' : V' < V_0\}$ and there is a nbd Ox of x which meets only finitely many of sets $\operatorname{Bd} V'$ with $V' < V_0$. Hence Ox meets only finitely many of sets $\operatorname{Bd} V'$ with $V' < V$.

Case 3: Finally we shall consider the case of $V_0 < V$. Recall that $x \in V_0 \subset P(V)$ and $W(V) \cap P(V) = \emptyset$. Hence $V_0 \cap W(V) = \emptyset$ and $V_0 \cap \operatorname{Bd} W(V) = \emptyset$.

Now, we put $\mathcal{V}_1 = \{V \in \mathcal{V} : V \cap \overline{H}_2 = \emptyset\}$ and $\mathcal{V}_2 = \mathcal{V} \setminus \mathcal{V}_1$.

Claim 3 *The sets* $U_1 = G_1 \cup \bigcup\{W(V) : V \in \mathcal{V}_1\}$ *and* $U_2 = G_2 \cup \cup\{W(V) : V \in \mathcal{V}_2\}$ *are disjoint open nbds of* A *and* B *respectively. Moreover, we have* $C = X \setminus (U_1 \cup U_2) \subset \cup\{Bd\, W(V) : V \in \mathcal{V}\}$.

Proof. It is clear that $A \subset U_1$ and $B \subset U_2$. Now we shall check that $U_1 \cap U_2 = \emptyset$. In fact, we have the following equalities. The first one is $G_1 \cap G_2 = \emptyset$ and it is evident. The second one is $G_1 \cap (\bigcup\{W(V) : V \in \mathcal{V}_2\}) = \emptyset$ because for every $V \in \mathcal{V}_2$ we have $V \cap \overline{H}_2 \neq \emptyset$ hence $V \cap \overline{H}_1 = \emptyset$ (recall that $G_1 \subset H_1, W(V) \subset V$). The third one is $G_2 \cap (\bigcup\{W(V) : V \in \mathcal{V}_1\}) = \emptyset$ because for every $V \in \mathcal{V}_1$ we have $V \cap \overline{H}_2 = \emptyset$ and $W(V) \subset V$, $G_2 \subset H_2$. The fourth one is $(\bigcup\{W(V) : V \in \mathcal{V}_1\}) \cap (\bigcup\{W(V) : V \in \mathcal{V}_2\}) = \emptyset$. If we consider a pair $W(V_1)$ and $W(V_2)$, where $V_1 \in \mathcal{V}_1$ and $V_2 \in \mathcal{V}_2$ then we have $V_1 < V_2$ or $V_1 > V_2$. Let $V_1 < V_2$. Recall that $P(V_2) \cap W(V_2) = \emptyset$, $V_1 \subset P(V_2)$ and $W(V_1) \subset V_1$. The same with the case $V_1 > V_2$. It follows from (**) that the inclusion $C \subset \cup\{\mathrm{Bd}\, W(V) : V \in \mathcal{V}\}$ is valid.

Now we put the family $\{\mathrm{Bd}\, V' \cap \mathrm{Bd}\, W(V) \cap C : V' < V, V \in \mathcal{V}\}$ as $\mathcal{F}$. Since $\{\mathrm{Bd}\, W(V) : V \in \mathcal{V}\}$ is locally finite (see (*)), $\mathcal{F}$ is desired (recall also Claim 2). The Main lemma is proved.

Main lemma motivates the following definition.

Definition 1. A space X is said to have *the property (#)* if for any base $\mathcal{B}$ of X and any pair A, B of disjoint closed subsets of X there exist a partition C between A and B in X and a locally finite family $\mathcal{F}$ of closed subsets of X satisfying the condition mentioned in the main lemma.

Now, we have the following simple facts.
(d) Every normal space X with $\mathrm{Ind}\, X = 0$ satisfied the condition (#) and for every space X having (#) the conditions $\mathrm{ind}\, X = 0$ and $\mathrm{Ind}\, X = 0$ are equivalent.
(e) Every order totally paracompact space has the property (#) (see Main lemma).

Now, we can answer Question 1.

Theorem 1. *For every order totally paracompact space* X *the conditions* $\mathrm{ind}\, X = 1$ *and* $\mathrm{Ind}\, X = 1$ *are equivalent.*

Proof. It suffices to show that if $\mathrm{ind}\, X = 1$ then $\mathrm{Ind}\, X \leq 1$. Consider a base $\mathcal{B}$ such that for every $U \in \mathcal{B}$, we have $\mathrm{ind}\, \mathrm{Bd}\, U \leq 0$. By facts (a) and (b) we have $\mathrm{Ind}\, \mathrm{Bd}\, U \leq 0$ for every $U \in \mathcal{B}$. By the main lemma and locally finite sum theorem for strongly zero-dimensional spaces, we can show that $\operatorname{Ind} X \leq 1$.

If for every pair A, B of disjoint closed subsets of a normal space X there exists a partition C between A and B such that $\dim C \leq n-1$, then $\dim X \leq n$ (cf. [E, Lemma 3.1.27]). Hence, by a similar argument above, we have the following.

Theorem 2. *For every order totally paracompact space X we have $\dim X \leq \mathit{ind} X$.*

One can show that every closed subspace of a hereditarily normal space having the property (#) has the property (#). Hence, by the induction, we can prove the following theorem.

Theorem 3. *For every strongly hereditarily normal space X which has the property (#), we have $\mathit{ind} X = \mathit{Ind} X$.*

Now, by the main lemma, we answer Question 2 as a corollary to the theroem above.

Corollary 1. *For every order totally paracompact, strongly hereditarily normal space X, we have $\mathit{ind} X = \mathit{Ind} X$.*

2. On perfectly κ-normal spaces

Recall from Ščepin [Sc1] that a space X is called *perfectly κ-normal* if $\overline{U}$ is a G_δ-set in X for every open set U of X.

Recall from Fedorchuk [Fe1] that a space X is called *hereditarily perfectly κ-normal* if every closed G_δ-set of X is perfectly κ-normal.

Theorem 4 (Fe1). *Let X be a completely paracompact hereditarily perfectly κ-normal space. Then $\mathit{ind} X = \mathit{Ind} X$.*

As a corollary from this fact, Fedorchuk showed that the dimensions ind and Ind coincide for κ-metrizable compact spaces, in particular for Miljutin spaces and Dugundji spaces (because every κ-metrizable compact space is hereditarily perfectly κ-normal [Ščpin [Sc2]). Other examples of hereditarily perfectly κ-normal completely paracompact spaces were found by Shakhmatov [Sh]. He showed that every Lindelöf Σ-space, which is a retract of a G_δ-set in a topological group, is hereditarily perfectly κ-normal.

Fedorchuk [Fe2] asked about a generalization of the theorem above.

Problem (Fedorchuk). Is the equality $\operatorname{ind} X = \operatorname{Ind} X$ valid for any completely paracompact (compact) perfectly κ-normal space?

We shall propose a generalization of the theorem above in another direction.

Theorem 5. *Let X be an order totally paracompact hereditarily perfectly κ-normal space. Then $\operatorname{ind} X = \operatorname{Ind} X$.*

To prove the theorem, we need a dimension functions ind_0 and Ind_0 introduced by Filippov [Fil].

Definition 2. Let X be a space. By induction one defines $\mathrm{Ind}_0 X$ as follows:

(i) $\mathrm{Ind}_0 X = -1$ iff $X = \emptyset$,

(ii) $\mathrm{Ind}_0 X \leq n$ iff for any two closed disjoint subsets A and B of X there is a partition C which is a G_δ-set in X and $\mathrm{Ind}_0 C \leq n-1$,

(iii) $\mathrm{Ind}_0 X = n$ iff $\mathrm{Ind}_0 X \leq n$ and the inequality $\mathrm{Ind}_0 X \leq n-1$ does not hold,

(iv) $\mathrm{Ind}_0 X = \infty$ iff the inequality $\mathrm{Ind}_0 X \leq n$ does not hold for any n.

Analogously, one defines the dimension ind_0. In this case the subset A is a point.

It is evident that $\mathrm{Ind}_0 X \geq \mathrm{ind}_0 X$, $\mathrm{Ind}_0 X \geq \operatorname{Ind} X$, $\mathrm{ind}_0 X \geq \operatorname{ind} X$ for any space X and $\mathrm{Ind}_0 X = \operatorname{Ind} X$, $\mathrm{ind}_0 X = \operatorname{ind} X$ for any perfectly normal space X.

It is also clear that the dimension ind_0 is monotone with respect to arbitrary subsets of X and the dimension Ind_0 is monotone with respect to closed subsets of X. If X is the free sum $\oplus\{X_\alpha : \alpha \in A\}$ of subspaces $X_\alpha, \alpha \in A$, of X, then $\mathrm{Ind}_0 X \leq \max\{\mathrm{Ind}_0 X_\alpha : \alpha \in A\}$.

At first, we shall consider sum theorems for Ind_0.

Ivanov [I] proved the following:

Theorem 6. *([I]) Let X be a space such that $X = \cup_{i=1}^{\infty} X_i$, where X_i is a closed G_δ-set in X with $\mathrm{Ind}_0 X_i \leq n$ for every i. Then $\mathrm{Ind}_0 X \leq n$.*

In connection with this theorem, Ivanov asked

Problem ([I]). Is the countable sum theorem for dimension Ind_0 valid for arbitrary closed subsets?

He answered the problem negatively as follows.

Example 1. ([I]) There is a hereditarily normal compact space X such that $X = X_1 \cup X_2$, where X_i is a closed subset of X with $\mathrm{Ind}_0 X_i = 1$ for $i = 1, 2$, and $\mathrm{Ind}_0 X \geq 2$.

We have the following sum theorems:

Theorem 7. *Let X be a perfectly κ-normal space such that $X = \cup_{i=1}^k X_i$, where X_i is a closed subset of X with $Ind_0 X_i \leq n$ for every i, $k \geq 2$. Then $Ind_0 X \leq n$.*

Theorem 8. *Let X be a perfectly κ-normal paracompact space and $\mathcal{M} = \{M_\alpha : \alpha \in A\}$ be a locally finite closed cover of X such that $Ind_0 M_\alpha \leq n$ for every $\alpha \in A$. Then $Ind_0 X \leq n$.*

We also use the following theorem due to Fedorchuk [Fe1].

Theorem 9. *(Fedorchuk) Let X be a hereditarily perfectly κ-normal space. Then $Ind\, X = Ind_0 X$ and $ind\, X = ind_0 X$.*

We continue with the following.

Lemma 1. *Let X be a perfectly κ-normal space. Then for every open subset U of X the subspace $\overline{U}$ is perfectly κ-normal.*

Proof. Let us observe only that for any open subsets U and V of X we have $\overline{V \cap \overline{U}} = \overline{U \cap V}$.

The proof of Theorem 7. Apply induction on the number k of closed subsets. If $k = 2$, then let us consider the following open subsets of X. Namely, $U_1 = X \setminus X_2$, $U_2 = X \setminus \overline{U_1}$. It is evident that $X = \overline{U_1} \cup \overline{U_2}$. Observe that $\overline{U_i}$ is a G_δ-set in X and $\mathrm{Ind}_0 \overline{U_i} \leq \max\{\mathrm{Ind}_0 X_1, \mathrm{Ind}_0 X_2\} \leq n$ for every i. By Theorem 6, we have $\mathrm{Ind}_0 X \leq n$.

Let now $k \geq 3$. Define $F_1 = \cup_{i=1}^{k-1} X_i$, $F_2 = X_k$, $U_1 = X \setminus F_2$, $U_2 = X \setminus \overline{U_1}$. Observe that $X = \overline{U_1} \cup \overline{U_2}$, $\overline{U_1} \subset \cup_{i=1}^{k-1} X_i$, $\overline{U_2} \subset X_k$ and $\overline{U_i}$ is a G_δ-set in X for every i. By Lemma 1, the subset $\overline{U_1}$ is a perfectly κ-normal space in the subspace topology. Hence, by inductive assumption, we have $\mathrm{Ind}_0 \overline{U_1} \leq \max\{\mathrm{Ind}_0 X_1, ..., \mathrm{Ind}_0 X_{k-1}\} \leq n$. Observe also that $\mathrm{Ind}_0 \overline{U_2} \leq \mathrm{Ind}_0 X_k \leq n$. By Theorem 6, we get $\mathrm{Ind}_0 X \leq n$.

The proof of Theorem 8. Let us choose, for every point $x \in X$, a nbd U_x such that $\overline{U}_x$ meets (and consequently is covered by) only finite number of members of the system $\mathcal{M}$. By Theorem 7, we have $\mathrm{Ind}_0 \overline{U_x} \leq n$. The cover $\{U_x : x \in X\}$ of X has a σ-discrete open refinement $\mathcal{V} = \bigcup_{i=1}^\infty \mathcal{V}_i$ of X, where $\mathcal{V}_i$, $i = 1, 2, ...$, are the discrete subfamilies of $\mathcal{V}$. Define U_i as the union of all elements of subfamily $\mathcal{V}_i$ for every i. Observe that $\overline{U}_i$ is a G_δ-set of X and $\mathrm{Ind}_0 \overline{U}_i \leq n$ for every i. Moreover $X = \cup_{i=1}^\infty \overline{U}_i$. By Theorem 6, we get $\mathrm{Ind}_0 X \leq n$.

Remark 1. Observe that if for every open subset U of the space X from Theorem 7 (Theorem 8) we have the equality $\mathrm{Ind}_0 \overline{U} = \mathrm{Ind}\, \overline{U}$

(for example if the space X is hereditarily perfectly κ-normal), then in the statement of Theorem 7 (Theorem 8) the dimension Ind_0 can be substituted by dimension Ind.

One can easily check the following two statements.

Lemma 2. *Let X be a hereditarily perfectly κ-normal space and A be a closed G_δ-set in X. Then the subspace A is hereditarily perfectly κ-normal. In particular, $\mathrm{Ind}_0 A = \mathrm{Ind}\, A$.*

Lemma 3. *Let X be a space and C be a partition in X with a pair of open disjoint subsets U, V of X such that $X = C \cup U \cup V$. Then there exists a partition C_1 with a pair of open disjoint subsets U_1, V_1 of X satisfying $X = C_1 \cup U_1 \cup V_1$ such that $C_1 \subset C, U \subset U_1, V \subset V_1$ and $C_1 = \overline{O_1} \cap \overline{O_2}$, where O_1 and O_2 are open subsets of X.*
In particular, C_1 is a closed G_δ-set in X if $\overline{O_1}$ and $\overline{O_2}$ are closed G_δ-sets in X.

Now we are ready to prove the following.

Theorem 10. *Let $\mathcal{K}$ be a subclass of the class of paracompact spaces which satisfies the property (#) and hereditary with respect to closed subspaces and $X \in \mathcal{K}$. If X is also a hereditarily perfectly κ-normal space then $\mathrm{ind}\, X = \mathrm{Ind}\, X$ ($= \mathrm{ind}_0 X = \mathrm{Ind}_0 X$).*

Proof. First we show the equality $\mathrm{ind}_0 X = \mathrm{Ind}_0 X$. Apply induction on $n = \mathrm{ind}_0 X$. For $n = 0$ we have $\mathrm{ind}\, X = 0$ and so the equality $\mathrm{Ind}\, X = 0$ is valid due to (a). It is clear that $\mathrm{Ind}_0 X = 0$.
Let $n \geq 1$ and $\mathrm{ind}_0 X \leq n$. Let us consider a base $\mathcal{B}$ of X such that for every element $U \in \mathcal{B}$ we have $\mathrm{Ind}_0 \mathrm{Bd}\, U \leq n - 1$ (here we use Lemma 2, the inductive assumption and the monotonicity of Ind_0 and the subclass $\mathcal{K}$). By the definition of the property (#), for every pair A, B of disjoint closed subsets of X there exist a partition C between A and B in X and a locally finite family $\mathcal{F}$ of closed subsets of X satisfying;
(i) $C = \bigcup \mathcal{F}$,
(ii) for every $F \in \mathcal{F}$ there exists $U \in \mathcal{B}$ such that $F \subset \mathrm{Bd}\, U$.
Observe also that we can suppose that the partition C is a G_δ-set of X (recall that X is perfectly κ-normal and apply Lemma 3) and hence the subspace C is perfectly κ-normal. By Theorem 8, we get $\mathrm{Ind}_0 C \leq n-1$. Hence $\mathrm{Ind}_0 X \leq n$. The equality $\mathrm{ind}_0 X = \mathrm{Ind}_0 X$ is proved. Now let us recall that by Theorem 9, we have $\mathrm{Ind}\, X = \mathrm{Ind}_0 X$ and $\mathrm{ind}\, X = \mathrm{ind}_0 X$. This completes the proof.

The proof of Theorem 5. Recall that the class of order totally paracompact spaces is a subclass of paracompact spaces which has the property (#) and is hereditary with respect to closed subspaces. Apply now Theorem 10.

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