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Some Results in Asymptotic Theory on Special Manifolds

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1 Introduction

We may be interested in statistical analysis on special manifolds, e.g., Stiefel, Grassmann and shape manifolds. The analysis of data on these manifolds is required in practical applications in many fields ; the Grassmann manifold is a rather new subject treated as a sample space.

We usually consider, as population distributions, exponential-type distributions on the manifolds; e.g., the matrix Langevin and the matrix Bingham distributions including the uniform distributions. These distributions are relatively tractable like the normal distributions on the Euclidean space. However, the distribution forms and inference problems based on these exponential distributions involve hypergeometric functions with matrix arguments and the exact solutions are expressed in integral or differential forms involving these hypergeometric functions. Thus we need to evaluate these problems asymptotically, that is, for large sample size (and small concentration), for large concentration, and for high dimension.

We concentrate our discussion on the Stiefel manifold (and the Grassmann manifold as well if time is permitted). The Stiefel manifold $V_{k,m}$ is the space a point of which is a set of k orthonormal vectors in $R^m (k \leq m)$, so that $V_{k,m} = \{X(m \times k); X'X = I_k\}$, where I_k is the $k \times k$ identity matrix. Special cases are the unit hypersphere $V_{1,m}$ of directed vectors and the orthogonal group $O(m) = V_{m,m}$ of $m \times m$ orthonormal matrices. The analysis of data on $V_{k,m}$ is required especially for $k \leq m \leq 3$ in practical applications in Geological Sciences, Astrophysics, Biology, Meteorology, Medicine and other fields.

The matrix Langevin $L(m,k;F)$ distribution is of exponential type whose density function is given by

$$\exp(tr\ F'X)/{}_0F_1(\frac{1}{2}m; \frac{1}{4}F'F),$$

for an $m \times k$ matrix F of rank $p(\leq k)$ with the singular value decomposition of F

$$F = \Gamma\Lambda\Theta', \text{ where } \Gamma \in V_{p,m}, \Theta \in V_{p,k}, \text{ and } \Lambda = diag(\lambda_1, \ldots, \lambda_p),\ \lambda_1 \geq \cdots \geq \lambda_p > 0,$$

(see Downs (1972)). Some more distributions of the $L(m,k;F)$ distribution may be found in e.g., Jupp and Mardia (1979), Watson (1983), Prentice (1986), and Chikuse (1998, 1999, 2002b).

2 Asymptotic Results

Large sample asymptotic theory is concerned in connection with tests for uniformity ($\Lambda = 0$) of the $L(m,k;F)$ distribution (see Chikuse (1991)). Asymptotic properties, near the uniformity, are discussed of the estimation of the orientation parameters Γ and Θ and the concentration parameter Λ, and of some optimal tests for uniformity (the likelihoo ratio, the locally best invariant, the Rao score and the Rayleigh-style tests).

Asymptotic theory is developed for the concentrated $L(m,k;F)$ distributions (i.e., for large Λ), discussing the estimation of the large concentration parameter Λ, tests for hypotheses of the orientation parameters Γ and Θ, classifications of matrix Langevin distributions, measures of orthogonal association and orientational regressions (see e.g., Chikuse (2000a)).

This papar investigates, in particular, the high dimensional asymptotic behavior of some matrix statistics and related functions constructed from some main distributions, including the matrix Langevin, the matrix Bingham and the uniform distributions. We derive asymptotic expantions for the distributions of the *standardized* sample mean matrices, using the results on the invariant polynomials and the generalized Hermite and Laguerre polynomials with matrix arguments.

We generalize the *Stam's (first and second) limit theorems*, for high dimensional samples taken from the hypersphere, to some non-uniform distributions on the Stiefel manifold. We discuss high dimensional asymptotic properties of the parameter estimation and the tests of hypotheses for the $L(m,k;F)$ distributions. We are also concerned with large sample asymptotic properties of the inference based on the (profile) score functions when the dimension is large.

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