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<th>Title</th>
<th>ON JOINT SPECTRA OF NON-COMMUTING HYPONORMAL OPERATORS (Structure of operators and related current topics)</th>
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ON JOINT SPECTRA OF NON-COMMUTING HYPONORMAL OPERATORS

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Let $H$ be a complex Hilbert space and let $B(H)$ denote the Banach algebra of all (bounded linear) operators on $H$.

For $n$-tuple $T = (T_1, \ldots, T_n)$ of operators on $H$ a spectral set $\gamma(T)$ is defined as follows:

$$\gamma(T) = \{ (\lambda_1, \ldots, \lambda_n) \in \mathbb{R}^n : \sum_{j=1}^{n} (T_j - \lambda_j)^2 \text{ is not invertible in } B(H) \}.$$  

(Here we write as usual $T_j - \lambda_j$ instead of $T_j - \lambda_j \text{id}_H$.) This set was introduced by McIntosh and Pryde ([1, 2]) and has proved useful not only in the spectral theory of self-adjoint operators but also in comparing various types of joint spectra of commuting families of operators (see [3]). One advantage of the set $\gamma(T)$ over other joint spectra is that it can be easily computed. In [4] it was shown that this set is also useful in the multiparameter spectral theory of normal operators.

We recall some necessary definitions. An operator $T \in B(H)$ is hyponormal (cohyponormal) if $\|T^*x\| \leq \|Tx\|$ ($\|Tx\| \leq \|T^*x\|$ respectively) for all $x \in H$. Clearly if an operator $T$ is hyponormal, then $T^*$ is cohyponormal. Moreover an operator $T$ is normal if it is both hyponormal and cohyponormal.

Let $T = (T_1, \ldots, T_n)$ be an $n$-tuple of operators. A point $\lambda = (\lambda_1, \ldots, \lambda_n) \in \mathbb{C}^n$ is not in the left (joint) spectrum of $T$ if there exist operators $U_1, \ldots, U_n \in B(H)$ such that $\sum_{j=1}^{n} U_j (T_j - \lambda_j) = \text{id}_H$. The left spectrum of $T$ will be denoted by $\sigma_l(T)$. The right spectrum, $\sigma_r(T)$, is defined analogously. The Harte spectrum of $T$ (in $B(H)$), denoted by $\sigma_H(T)$, is the union of the left and right joint spectra, i.e.

$$\sigma_H(T) = \sigma_l(T) \cup \sigma_r(T).$$

All these spectra are compact (possibly empty) subsets of $\mathbb{C}^n$. Notice that for a single operator $T$ the Harte spectrum $\sigma_H(T)$ coincides with the usual spectrum $\sigma(T)$. It is well-known that

$$\sigma_l(T) = \{ \lambda \in \mathbb{C}^n : \inf_{\|x\|=1} \sum_{j=1}^{n} \| (T_j - \lambda_j)x \| = 0 \}$$

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(the approximate point spectrum) and
\[ \sigma_r(T) = \{ \lambda \in \mathbb{C}^n: \sum_{j=1}^{n} ((T_j - \lambda_j)(H)) \neq H \} \]
(the defect spectrum). Let us introduce the following notation. For a single operator \( T \) symbols \( \text{Re} T \) and \( \text{Im} T \) will denote as usual its real and imaginary part. Hence \( T = \text{Re} T + i \text{Im} T \). If \( T = (T_1, \ldots, T_n) \) is an \( n \)-tuple of operators, then \( \text{Re} T = (\text{Re} T_1, \ldots, \text{Re} T_n) \), \( \text{Im} T = (\text{Im} T_1, \ldots, \text{Im} T_n) \), and \( II(T) = (\text{Re} T, \text{Im} T) \).

Letter \( p \) will denote the polynomial map \( p(z_1, \ldots, z_{2n}) = (z_1 + iz_{n+1}, \ldots, z_n + iz_{2n}) \).

We present a generalisation of one of the results proved in [4] to \( n \)-tuples of (not necessarily commuting) hyponormal operators. The result is as follows:

**Theorem.** If \( T = (T_1, \ldots, T_n) \) is an arbitrary \( n \)-tuple of hyponormal (cohyponormal) operators, then
\[ \sigma_l(T) = p(\gamma(II(T))) \]
(and respectively
\[ \sigma_r(T) = p(\gamma(II(T))) \].

It is easy to see that one cannot replace in the theorem the left spectrum (or the right spectrum) by the Harte spectrum if the operators \( T_j \) are not normal.

**References**