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Pointwise and Sequential Continuity in Constructive Analysis

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We discuss various continuity properties, especially pointwise and sequential continuity, in Bishop's constructive mathematics; see [1, 2, 11] for Bishop's constructive mathematics and [3, 4, 5, 9] for various continuity properties. We say that a mapping $f$ between metric spaces $X$ and $Y$ is sequentially continuous if $x_n \rightarrow x$ implies that $f(x_n) \rightarrow f(x)$; pointwise continuous if for each $x \in X$ and $\epsilon > 0$ there exists $\delta > 0$ such that $d(x, y) < \delta$ implies $d(f(x), f(y)) < \epsilon$ for all $y \in X$. We first show the following theorem.

**Theorem 1** The following are equivalent.

1. Every sequentially continuous mapping of a separable metric space into a metric space is pointwise continuous.

2. Every sequentially continuous mapping of a complete separable metric space into a metric space is pointwise continuous.

3. BD-N. Every countable pseudo-bounded subset of $\mathbb{N}$ is bounded.

Here a subset $A$ of $\mathbb{N}$ is said to be pseudo-bounded if for each sequence $\{a_n\}$ in $A$, $a_n < n$ for all sufficiently large $n$. Note that although BD-N holds in classical mathematics, intuitionistic mathematics and constructive recursive mathematics of Markov’s school, a natural recursivisation of BD-N is independent of Heyting arithmetic [3, 5, 8, 10].

We also show that very important theorems in functional analysis – Banach's inverse mapping theorem, the open mapping theorem, the closed graph theorem, the Banach-Steinhaus theorem and the Hellinger-Toeplitz theorem – can be proved in Bishop's constructive mathematics for sequentially continuous linear mappings [6, 7]. However it has emerged that the theorems for pointwise continuous linear mappings are equivalent to BD-N.
参考文献


