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<th>Title</th>
<th>Algorithms for Table Transformation (New Aspects of Theoretical Computer Science)</th>
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Abstract:
Tables with heterogeneous cells are commonly used in computer human interface and documentation. We proposed an attribute multi edge graph representation for tables that considers editing and drawing in [7]. In this paper, we give algorithms for basic operations in table editing.

We provide a cell unification algorithm that runs in $O(1)$ time. We also provide a column insertion algorithm that runs in $O(n + m)$, for $n \times m$ heterogenous tables. It is noted that the column insertion algorithm runs in $O(\sqrt{N})$ time for $N = n \times n$ cell square tables while known methods require $O(N)$ time.

Keywords: Data structures and algorithms, graphs, table interface

1 Introduction

In this paper, we deal with the representation of tables while considering editing and drawing. Several representation methods have been proposed for tables: rectangular duals of planar graphs [1], and quadtrees [3]. Although, we do not know which representation method is used in present table processing systems.

In this paper, we considers another graph representation [7] for tables, in which the table editing is executed efficiently. For drawing and editing problems, see [4, 5].

*This paper partly appeared in our previous study Tomoe Motohashi, Kensei Tsuchida and T. Yaku, Algorithm on attribute graphs for table editing, The 3rd Hungarian-Japanese Sumpos. Discrete Math. & Its Appl. and [6].
In Section 2, we propose a representation of tables by an attribute multi-edge graph. Several properties of the graphs are shown. In Section 3, we show several algorithms that execute table editing based on the representation. We provide algorithms for unifying cells, changing column width, and insertion column. Section 4 provides conclusions.

2 ATTRIBUTE GRAPHS FOR TABLES

We described several definitions concerning tables in [7]. We represent a table by a certain type of tabular diagram satisfying Condition 2.1 in [7]. That is, the tabular diagrams have perimeter cells.

Example 2.1 Figure 1A illustrates a tabular diagram $D_1 = (T_1, P_1, g_1)$, where $P_1 = \{(1,1),(2,1)\}, \{(1,2)\}, \{(1,3)\}, \{(2,2),(2,3)\}$ is a partition over a $(2,3)$-table $T_1$. A grid $g_1$ is defined by $g_{1\text{row}}(0) = 0, g_{1\text{row}}(1) = 1, g_{1\text{row}}(2) = 2, \text{ and } g_{1\text{column}}(0) = 0, g_{1\text{column}}(1) = 2, g_{1\text{column}}(2) = 4, \text{ and } g_{1\text{column}}(3) = 6$. For a cell $c = \{(2,2),(2,3)\}$, we define the north wall $nw(c) = g_{1\text{row}}(1) = 1$, south wall $sw(c) = g_{1\text{row}}(2) = 2$, east wall $ew(c) = g_{1\text{column}}(3) = 6$, and west wall $ww(c) = g_{1\text{column}}(1) = 2$. Figure 1B illustrates a tabular diagram $D_{1p}$ with perimeter cells. For the table drawing, it corresponds to $D_1$.

Now, we introduce an attribute graph. Then, we show how to represent a tabular diagram as an attribute graph.

Definition 2.1 An attribute graph is a 6-tuple $G = (V, E, L, \lambda, A, \alpha)$, where $(V, E)$ is a multi-edge undirected graph, $L$ is the set of labels for edges, $\lambda : E \rightarrow L$ is the label function, $A$ is the set of attributes, and $\alpha : V \rightarrow A$ is the attribute map.

A tabular diagram $D = (T, P, g)$ is represented as an attribute graph $G_D = (V_D, E_D, L, \lambda_D, A, \alpha_D)$, where $V_D$ is identified by a partition $P$ (we denote a node corresponding to a cell $c$ in $P$ by $v_c$, we call $v_c$ a perimeter node (resp. inner node) if $c$ is a perimeter cell (resp. inner cell)), $E_D$ is defined by Rules 1-4, $L = \{nw, sw, ew, enw\}$, $\lambda_D : E_D \rightarrow L$ is defined by Rules 1-4, $A = \mathbb{R}^4$, and $\alpha_D : V_D \rightarrow \mathbb{R}^4$ are defined by $\alpha_D(v_c) = (nw(c), sw(c), ew(c), enw(c))$.

Rule 1 If $nw(c) = nw(d)$ and there is no cell between $c$ and $d$ having an equal north wall, then $[v_c, v_d]$ is in $E_D$ and $\lambda_D[v_c, v_d] = enw$. In this case, $[v_c, v_d]$ is called a north wall edge. Similarly, Rules 2, 3 and 4 define the south wall, east wall, and west wall edges, respectively.

An attribute graph $G_D$ is called a tessellation graph. Note that the degree $\#v$ of each node $v$ in $G_D$ is at most 8. The location values of the inner cells are evaluated from the location values of perimeter cells and linked edges. So, we assume in the latter part of this paper, that the $\alpha_D$ values of the inner cells are null.

Note that we consider tabular diagrams with perimeter cells. Then,
Proposition 2.1 Let $G_D$ be a tessellation graph for a tabular diagram $D$ of an $(n, m)$-table. Let $k$ be the number of inner cells in $G_D$. For the number $\# E_D$ of edges in $G_D$, we have $2\# E_D = 6(2n - 4) + 6(2m - 4) + 8k + 16$.

3 ALGORITHMS

This section provides algorithms for tessellation graphs. The following algorithm execute unification of two adjacent inner cells in the tabular diagram.

ALGORITHM UnifyCells($G_D, v_c, v_d, G_E$)

INPUT
$G_D = (V_D, E_D, L, \lambda_D, A, \alpha_D)$: a tessellation graph for a tabular diagram $D$, $v_c$: a node in $G_D$ corresponding to an inner cell $c$, $v_d$: a node in $G_D$ corresponding to an inner cell $d$ which is adjacent to the south wall of $c$ that is $ww(c) = ww(d), ew(c) = ew(d), sw(c) = nw(d)$.

OUTPUT
$G_E = (V_E, E_E, L, \lambda_E, A, \alpha_E)$: a tessellation graph for a tabular diagram $E$, where $E$ is obtained from $D$ by the unification of cells $c$ and $d$ to $c$.

METHOD
begin
Initially let $G_E \leftarrow G_D$ ;
/* change of vertical edges concerning to $d$ */
delete two vertical edges between $v_d$ and $a(\neq v_c)$ from $E_E$ ;
add edges between $v_c$ and $a$ to $E_E$ ;
put $\lambda_E[v_c, a] \leftarrow \lambda_D[v_d, a]$ ;
delete two vertical edges between $v_c$ and $v_d$ from $E_E$ ; (See Fig.3)
/* change of south wall edges concerning to $c$ */
choose two nodes $f$ and $f'(f \neq f')$ such that $\lambda_D[f, v_c] = \lambda_D[v_c, f'] = esw$ ;
delete south wall edges $[f, v_c]$ and $[v_c, f']$ ;
add an edge $[f, f']$ to $E_E$ and put $\lambda_E[f, f'] \leftarrow esw$ ;
/* change of south wall edge concerning to $d$ */
choose two nodes $h$ and $h'(h \neq h')$ such that $\lambda_D[h, v_d] = \lambda_D[v_d, h'] = esw$;
delete south wall edges $[h, v_2]$ and $[v_d, h']$;
add $[h, v_3]$ and $[v_d, h']$ to $E_F$ and put $\lambda_E[h, v_3] \leftarrow esw, \lambda[v_2, h'] \leftarrow esw$;
/* change of north wall edges concerning to $d$ */
choose two nodes $g$ and $g'(g \neq g')$ such that $\lambda_D[g, v_d] = \lambda_D[v_d, g'] = enw$;
delete north wall edges $[g, v_d]$ and $[v_d, g']$ from $E_F$;
add an edge $[g, g']$ to $E_F$ and put $\lambda_E[g, g'] \leftarrow enw$;
delete a node $d$
end.

**Theorem 3.1** Let $D$ be a tabular diagram, and $c$ be a cell in $D$. Suppose that there is an adjacent cell $d$ at south side in $D$ such that $ew(c) = ew(d)$, $ww(c) = ww(d)$ and $sw(c) = nw(d)$. Let $E$ be a tabular diagram obtained from $D$ by the unification of cells $c$ and $d$ to $c$. Let $G_D$ and $G_E$ be the tessellation graphs for $D$ and $E$, respectively. Then $G_E$ is obtained from $G_D$ in constant time.

**Theorem 3.2** Let $D$ be a tabular diagram, and $c$ be an inner cell in $D$. Let $\delta$ be a movement value. Suppose $\Delta + \delta > 0$, where $\Delta > 0$ is the width of a perimeter cell in the column which has equal east wall to $c$. Let $E$ be a tabular diagram obtained from $D$ by the changing width using $\delta$ of cells that have the equal east wall for $c$. Let $G_D$ and $G_E$ be the tessellation graphs for $D$ and $E$, respectively. Then $G_E$ is obtained from $G_D$ in $O(n + m)$ time by the algorithm CHANGECOLUMNWIDTH [$\delta$], where $n$ is the number of rows in $D$.

The following algorithm executes insertion of a column at the west side of a focused cell into the tabular diagram.

**ALGORITHM** INSERTCOLUMN($G_D, v_c, G_E$)

**INPUT**

$G_D$ : a tessellation graph for a tabular diagram $D = (T, P, g)$, $v_c$ : a node in $G_D$ corresponding to a cell $c$, where the cell, that is adjacently located at the west-side of $c$, exists.

**OUTPUT**

$G_E$ : a tessellation graph for $E$, where $E$ is a tabular diagram obtained from $D$ by insertion of a column with width $\delta$ at the west side of $c$, where $\delta$ is the width of a perimeter cell in the column including $c$.

**METHOD**

begin
Initially, put $G_E \leftarrow G_D$
traverse upward through the west wall edges from $v_c$ until a perimeter node $v_0$ (see Fig.5)
let $\delta$ be the width of the cell corresponding to $v_0$
add a node $v_0$
put $i \leftarrow 0$
/* insert a column */
while a node $v_i$ is not the lowermost node do begin
let $w_i$ be an adjacently west-side node linked to $v_i$ by a north wall edge
delete the north wall edge between $w_i$ and $v_i$
add a north wall edge between $w_i$ and $u_i$
deform $G_E$ similarly for a south wall edge
Figure 4: Insertion of the Column at the West-Side of $c$

add a north wall edge and south wall edge between $u_i$ and $v_i$; let $v_{i+1}$ be a lower node linked to $v_i$ by a west wall edge; add a node $u_{i+1}$; add a west wall edge and east wall edge between $u_i$ and $u_{i+1}$; $i \leftarrow i + 1$
end; (see Fig.6)
deform $G_E$ for the lowermost node $v_i$, similarly for the north and south wall edge; /* the existing column shifts to the east */
let $u_0$ be the uppermost node in $u_i$'s;
let $v_x$ be an adjacently west-side node linked to the node in the northeast corner in $G_E$;
put $G_{E_0} \leftarrow G_E$;
CHANGECOLUMNWIDTH($G_{E_0}, v_x, \delta, G_E$);
put $G_{E_0} \leftarrow G_E$;
let $x_0$ be a node adjacent west-side of $v_x$;
put $i \leftarrow 0$;
while $ww(u_0) \leq ww(x_i)$ do begin
  MOVEEASTWALL($G_{E_i}, x_i, \delta, G_{E_i+1}$);
  let $x_{i+1}$ be an adjacent west-side node linked to $x_i$ by a north wall edge;
  put $i \leftarrow i + 1$;
end;
put $G_E \leftarrow G_{E_i}$
end.

Theorem 3.3 Let $D$ be a tabular diagram, and $c$ be a cell in $D$. Suppose that $E$ is the tabular diagram obtained from $D$ by insertion of a column with width $\delta$ at the west side of the column including $c$, where $\delta$ is the width of a perimeter cell of the column including $c$. Let $G_D$ and $G_E$ be the tessellation graphs for $D$ and $E$, respectively. Then $G_E$ is obtained from $G_D$ in $O(n + m)$ time, where $n$ and $m$ are the number of rows and columns in $D$, respectively.

It is noted that the column insertion algorithm runs in $O(\sqrt{N})$ time for $N = n \times n$ cell square diagrams, while known methods require $O(N)$ time. The following table illustrates the features of representation methods for $N$ cell square tables with respect to the column insertion.
4 CONCLUSION

We introduced attribute graphs and algorithms for table drawing and editing. We note that, we have determined the necessary and sufficient condition where an attribute graph represents a tabular diagram by a graph grammar. [9]. We are designing a processing system for table editing based on our model in [8].

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References