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Instability of vortex filaments and stabilization effect of finite core

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Richard Pelz made a trip to Kyushu, based at Fukuoka, on June 5–8, 2002. He was invited to Ropponmatsu Functional Equations Seminar held on Friday 7th; mathematicians working on partial differential equations get together, from all over the Kyushu area, to Ropponmatsu Campus of Kyushu University every Friday afternoon. His talk "The search for finite–time blowup solutions to the equations of hydrodynamics" blew a stimulating wind into people's minds. S. L. had been staying at Graduate School of Mathematics, Kyushu University, under the Invitation Fellowships Program for Research in Japan offered by the Japan Society for Promotion of Science since April 22. We three met in Hakozaki Campus of Kyushu University, traveled together to Ropponmatsu Campus and enjoyed discussions at the seminar and sushi, sashimi and sake after the seminar. Rich's suggestions and comments pointed to new and essential viewpoints and created our future problems, and he has greatly influenced our lives. We express the deepest thanks to Rich.

1 Rich's trip to Kyushu, a western island

In September 2001, Y. F. was very glad to see Rich again in Zakopane, a scenic mountain resort in Poland, and to hear that he will come to Japan in 2002. They attended an IUTAM Symposium on "Tubes, Sheets and Singularities in Fluid Dynamics" organized by Keith Moffatt (Cambridge) and Konrad Bajer (Warsaw). Both of the authors had known him only by his great papers on numerical simulations of turbulence for long years but Y. F. had not met Rich before September 2000 when the Program "Geometry and Topology of Fluid Flows" was held at the Isaac Newton Institute in Cambridge, U. K. S. L. further waited till June 2002 to get acquainted with Rich. Rich was one of the program organizers. He friendly entertained and guided Y. F. and his aid made Y. F.'s two and half-month stay in Cambridge fruitful. Above all, Y. F. was shocked by his 'pedagogical' lecture on his own numerical identification of spontaneous appearance of a singularity in a periodic Euler flow with high symmetry or the Kida flow, at the first

Program Workshop organized by Renzo Ricca [1], and quickly realized that he is the leading runner of the singularity business which attracts many active mathematicians and physicists. Therefore Y. F. gave Rich the warmest words of invitation to Kyushu University when he knew Rich's plan of visiting Japan, in Zakopane, and it came true. At that time, Rich could already manipulate a few of the Japanese language and his name cards in Japanese businessmen's way.

The authors knew each other by the courtesy of Hidenori Hasimoto (Tokyo). Under the Invitation Fellowships Program of the JSPS, S. L. joined Y. F. at Kyushu University, just after Y. F. returned from his two-weeks visit to Los Alamos, U. S. A., hosted by Darryl Holm. S. L. stayed at Mathematics Department for two months from April 22 till June 19. We were excited about wiggly strings produced by a laser-matter interaction, and made an attempt to understand the structure from the viewpoint of vortex-flow and vortex-vortex interactions. We managed to get a consistent picture relying upon the localized induction approximation for motion of a vortex filament. The result was published in *Physics Letters A, Vol. 308 (2003) pp.375–380* [2] and constitute the main body of this article. Rich's comments and suggestions contributed to an improvement of the paper, and created new research projects, as will be described.

S. L. was absent from Fukuoka when Rich arrived at Hakata Station in Fukuoka City, the entrance to Kyushu Island, by the super express Nozomi at 18:49 on the evening of Wednesday June 5th. Kyushu Island is located in the westernmost part of Japan; actually Fukuoka is closer to Seoul, South Korea, than to Kyoto, perhaps. On the same day, S. L. gave a seminar organized by Katsuya Honda at Shinshu University in Matsumoto City, located at the foot of Japanese Alps in the mid-Japan, and, on that evening, further extended his travel to Tokyo to meet Miki Wadati and Masaki Sano and to give a seminar at Department of Physics, Tokyo University, on the next day (6th).

Y. F. came to Hakata Station to say welcome to Rich and took him to Green Hotel 1, located within three-minutes walking distance from the Station, where he stayed for three nights. We dined at an "Izakaya" (Restaurant & Sake Bar) in a building adjacent to the Station. Fukuoka faces the Japan Sea. Fishes swim against violent streams and waves, their muscles are highly developed, and as a consequence the taste of "sashimi" (raw fishes) is excellent. The price is not high, compared with Tokyo and Kyoto area. Fukuoka people have an advantage in pleasing travelers with a variety of delicious raw fish dishes. They wandered from topics to topics and confirmed their friendship. Y. F. was silent about S. L.'s stay till the noon of Friday 7th as he intended to surprise Rich. Y. F. was very happy when he received from Rich, as souvenirs, postcards and a name card holder designed by Michael Graves. In Fukuoka, there are a hotel, Hyatt-Regency Fukuoka Hotel, in the area called Canal City and a building near Momoch Beach designed by this worldwide famous architect. Every time Y. F. passes by these building, the memory of those days shared with Rich revives within his eyes.

On Thursday 6th, Rich and Y. F. went on an excursion to Mt. Aso by Y. F.'s car. Mt. Aso is an active volcano located in the middle of Kyushu Island, 120–130 Km south of Fukuoka. It is famous for the magnificent Caldera structure; five peaks, among which the "Middle Peak" constantly ejects steams, are surrounded by a ridge of radius 30 Km. People can reach the very tip of active crater directly by car. Y. F. picked Rich up at the Green Hotel at about a quarter past ten am, and entered the Kyushu Motorway down to Kumamoto City, 100 Km from Fukuoka. At this stage, they became rather intimate and began to talk frankly about the life and culture of U. S. and Japan, several issues of academic systems of U. S. and Japan, their favorite subjects in fluid mechanics and so on. Rich was wondering how to delight his wife and daughter when they joined him in Kyoto after a month or two. Y. F. suggested him to go to some hot spring resort. There are many good hot springs scattered from Hokkaido to Kyushu. We exited the Motorway at Kumamoto and drove on a route to Mt. Aso. When the slope of road started gradually increasing, one of them found the sign of *raw horse meet.* Hisashi Okamoto and Koji Ohkitani (Kyoto) had informed Rich of tasty raw horse meet dishes available near Mt. Aso. Rich showed the greatest interest in this eccentric dish. It was past noon. After a short-time break at the foot of a beautiful green hill called "Rice Storehouse" as it so looks, they droved up to the car park near the Cable Car Station only a 2 Km-drive remaining to the crater of the Middle Peak. There they entered one of the Restaurants and took lunch. Among the dishes we chose is raw horse meet, of course. Rich showed a slight hesitation. He slowly put a piece into his mouth, and found it acceptable. He ate a few pieces, and, Y. F. believe, could appreciate their taste.

Fig. 1. Richard Pelz (right) and Y. F. (left) with the crater of Mt. Aso for the background.

After lunch, they had a short ride up to the crater. The weather was perfect. They were lucky! The volcanic activity was mild. The wind blew in the right directions.

If the direction were wrong, people suffer from volcanic gas including some poisonous ingredients, and approaching the crater is banned. This was not the case for the afternoon of 6th. A spectacular scene of the crater jumped into their eyes. Rich was highly excited, like a child, by the crater and boiling magma, of beautiful emerald green, in it. It was the first experience for him to view the active crater in such a close distance, and was, to our great sadness, also the last. They walked around along the ridge of crater, enjoying this unusual space, and took photos. One of them is shown above (Fig. 1). They then found a convenient place to sit and stay for a while. The steam released from the boiling water was rolled up into a series of vertical columns, the columns wandering about erratically above the water surface, sometimes fast and sometimes slow, interacting with each other and the crater boundary. This scene fascinated them. They discussed the underlying turbulence phenomena as the steam served as means of visualizing the flow field. Rich was impressed also by the stripe pattern of the crater skin telling its long long history.

They drove back on a different route along a winding country road, and it was night when they returned to Fukuoka. At the same time, S. L. was on the way flying back from Tokyo after a long journey and reached Kyushu University accommodation at very late at night.

On the morning of Friday 7th, Rich came to Hakozaki Campus by subway by himself.On the previous night, Y. F. gave him a simple guidance of how to get to Kyushu University Station from Hakata Station, and asked him to call Y. F. at the office when he arrived at University Station. After the call, Y. F. walked to the Station but could not find Rich around the exit. Several seconds passed, and then Rich came up stairs. Rich told that he called Y. F. from the wrong Station. Y. F. understood what was wrong and felt sorry for his forgetting the fact that there are two University Stations close to each other, "Kyushu University Hospital Station" and "Hakozaki Kyushu University Station". Rich got off at the former and called from there.

On this third day, they could at last find time to discuss their hot topics in front of a black board. Both had an interest in vortex-filament interactions. Y. F. introduced his recent asymptotic analysis incorporating the effect of finite core thickness, a collaboration with Keith Moffatt. Rich manifested his great brightness. He was very quick to understand the geometry of flow structure, and applied this analysis to an interaction of anti-parallel vortex tubes in three dimensions. They were driven to a speculation that the finite thickness of core hinders catastrophic collapse of two vortex tubes, and thus formation of a singularity in the Euler flows. They had several other topics to be discussed, but the discussion was interrupted by lunch time. Y. F. took action of surprising Rich, and called S. L. at his office for asking him to join lunch. The office is a bit distant from Y. F.'s office. S. L. felt tired but had come to his office on the late morning. When the telephone bell rung, he was just outside his office. He heard the bell, ran back to his office, but was too late to lift the receiver. Y. F. guessed that S. L. took a rest at his accommodation. Y. F. knocked the door of his colleague Tatsuyuki Nakaki, and the three went to the Staff Restaurant of University. Tatsuyuki Nakaki was also an attendant of Zakopane IUTAM Symposium, thus knew each other, and they spent enjoyable and relaxed lunch time.

The Friday was a busy day. Soon after lunch, 1:30 pm say, Rich must leave Hakozaki Campus for Ropponmatsu Campus of Kyushu University where his seminar, the main event of the day, was scheduled from 3:00 pm. It was a one-hour bus ride between the two Campuses. Y. F. could narrowly catch S. L. at his office and introduced him to Rich. S. L. was very happy to meet this distinguished researcher in the Far East. It was optional for S. L., depending on his physical condition, whether he joined the seminar or not. Moved by this happy encounter and Y. F.'s suggestion, S. L. decided to attend the seminar, quickly wrapped up his luggage, and followed Rich and Y. F. This decision turned out to be over rewarding. Rich kindly glued S. L. to one of his colleague. We three enjoyed chatting in the bus. S. L. could explain to Rich the outline of his experimental work. The conversation on the bus and in the rest of the day tied S. L. closely to Rich. Later after Rich came back to U. S., he kindly informed Norman Zabusky, his colleague of Rutgers University, of S. L.'s work. In 2003, S. L. has received invitation for a short-term visit to Rutgers University from Zabusky.

The seminar was successful. The title was "The search for finite–time blowup solutions to the equations of hydrodynamics". He lively presented some of his original results, the numerical simulation of the Kida flow by the pseudo spectral method, the numerical solution by Taylor expansions in time and its Padé approximation, the Biot-Savart simulation of a dodecahedral configuration of six pairs of anti-parallel vortex-filament couples. All of his results are very important and are well known. The authors feel no need to add about the detail of his talk. His talk strongly impressed the people working on partial differential equations, specifically the Navier-Stokes equations. Among the audience were Naoyasu Kita, Takayuki Kobayashi, Mitsuhiro Nakao, Takayoshi Ogawa and Naoki Yamada. Hiroshi Shibata, a physicist on nonequlibrium statistical mechanics, came to the seminar from Kumamoto. After the seminar, we went to an "Izakaya" (sake bar) in Akasaka, the area adjacent to the city center Tenjin, and entertained Rich with sushi, sashimi, tempura and a combination of beer and local sake. This was not the end of our way of entertaining a guest. After two hours of enjoying meals and talks, we took Rich to a "Yatai" (stand of Hakata special). We continued to drink wine and sake and to eat grilled chickens. The last menu of one-day course was "Hakata soup noodle". Rich looked enjoying the last night in Fukuoka. We thanked Rich and S. L. took Rich to Hakata Station by subway. On the morning of Saturday 8th, Rich left Hakata for Kyoto by the super express Nozomi.

To his happiness, S. L. could soon see Rich again in Kyoto on Monday June 17th. S. L. visited the Research Institute of Mathematical Sciences, Kyoto University to give a talk at the seminar organized by Shigeo Kida (Toki) and Koji Ohkitani (Kyoto). His seminar was scheduled from 3:00 pm. He had arrived at Kyoto on the previous evening. On Monday morning, S. L. went on a walking trip to "Ginkakuji Temple" (Silver Pavilion) and to nearby temples guided by Koji Ohkitani. Rich joined the lunch at a Japanese Restaurant near University and they had a good time remembering that day in Fukuoka. Figure 2 is a photo of Rich and S. L. in front of the restaurant. Rich listened to S. L.'s talk "Complex Structures of Vortex Filaments Generated in Laser-Matter Interactions". He gave S. L. several valuable comments and questions to the talk and to the authors' collaboration. One question that stuck S. L. was the significance of baroclinic effect. S. L. and Y. F. had paid little attention to this effect, but Rich claimed that he could not believe the generation of such an abundance of vortex filaments without baroclinic effect. Rich was familiar with Norman Zabusky's numerical simulation of compressible inhomogeneous flows, and pointed S. L. to this aspect. The term of S. L.'s fellowships almost came to an end. S. L. came back to Fukuoka on the Monday night. On Tuesday 18th, S. L. and Y. F. discussed the issue raised by Rich, and recognized the importance of baroclinic effect by looking at Zabusky's paper [3] which was being submitted to the Proceedings of Zakopane IUTAM Symposium. Currently, "vortex projectiles" put forward by Zabusky is of our primary concern.

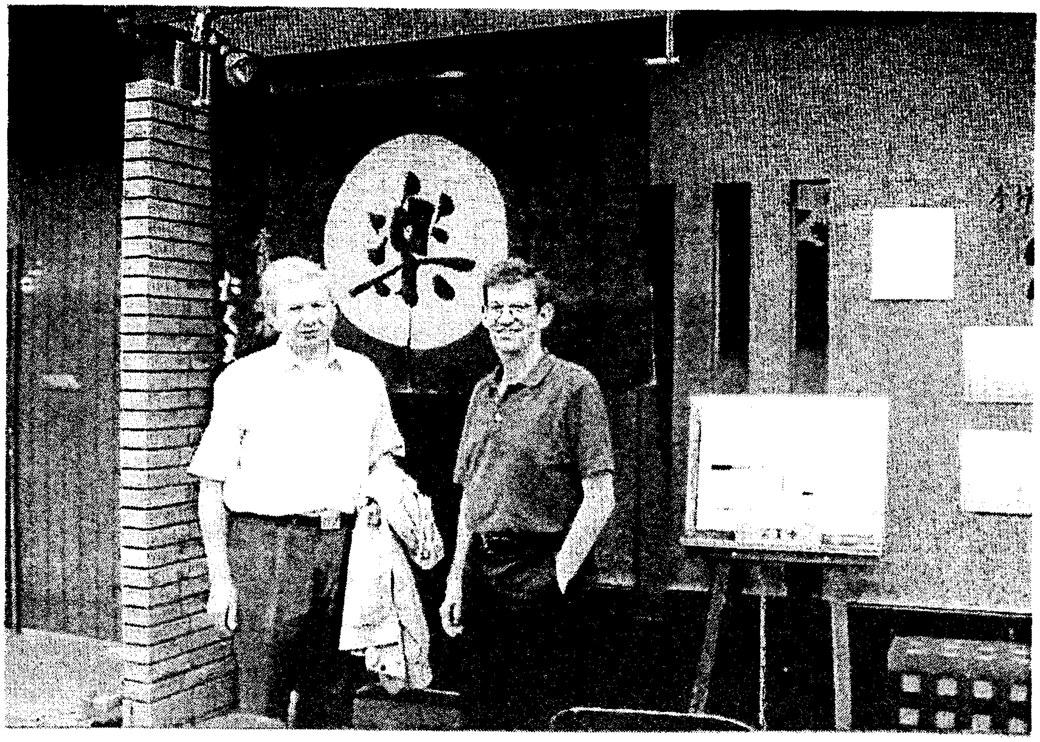

Fig. 2. Richard Pelz (right) and S. L. (left) in front of a restaurant near Kyoto University.

S. L. departed from Fukuoka Airport on the evening of Wednesday 19th and from Tokyo Narita Airport on the morning of 20th. After that, the authors communicated with each other using e-mail and fax. We wrote up the first manuscript of our paper shown below around the middle of September, taking Rich's suggestions into account. It was Rich whom we wanted to read our manuscript. S. L. sent Rich the manuscript by e-mail, and looked forward to hearing from him. No reply came from him, but the sad news toward the end of September.

Y. F. is reminded of Rich's words of wisdom that were given on the Kyushu Motorway to go to Mt. Aso; many years ago when Rich was young, a famous professor told him.

"Rich, look around me! I had many students, but all have gone, because they didn't listen to my advice. Don't write so many papers! Don't write so many grant proposals!"

The following is the article of our work made under the support of the JSPS program and has been recently published in *Phys. Lett. A*, Vol. **308** (2003) pp.375–380 [2].

An attempt is made at gaining, from a frozen picture of nano-second laser-matter interactions, information on the vortex-filament dynamics by estimating the hydrodynamic parameters of the shear layer of molten metal surface. The parameter values are consistent with the filament instability, and are in a range that admits breakup of waves, formation of loop solitons and reconnections of filaments.

2 Introduction to vortex filaments in laser-matter interactions

Understanding of dynamics and organization of vortex filaments is one of major challenges, because they play a crucial role in many physical systems extending from the atomic to the astrophysical scales [4]. Although many studies have been made from geometric and topologoical as well as hydrodynamic aspects [1], experiments are demanded that would generate vortex filaments with a diversity of organizational structures.

Recently, it was demonstrated that a nanosecond laser-matter interaction (*ns-LMI*) with metal surface, in the regime of pulsed vaporization, generates such string structures akin to vortex filaments [5, 6, 7]. The turbulent field of the vaporizing metal surface generated in ns-LMI may be conveniently looked upon as a "two fluid" system [8]. Such a system consists of two interacting "fluids"; one is a highly random background sea, while the other is composed of isolated coherent vortex structures of microscale, embedded in the first. The random fluid obeys the traditional view of the energy cascade. In contrast, coherent structures - vortex filaments - experience interactions both with the background sea and between themselves. The latter brings a coherence and an intermittent large fluctuation that affect the energy cascade. Vortex filaments are supposed to appear as a consequence of the Kelvin–Helmholtz instability in the shear layer of liquid metal. Owing to ultrafast cooling after termination of the short laser pulse (τ = 16–20 ns), they stay frozen permanently, enabling an *a posteriori* analysis [7]. Various types of filament organization, from simple to highly complex, are observed. Among them are parallel filaments, double helices, loop solitons, braided and tangled structures, and their inhomogeneous combinations [7, 8].

A fundamental question is: how do these patterns form? At present, they pertain only to nanosecond phenomena. Creation of the non-trivial patterns seems to become realizable by virtue of an accidental adjustment of physical parameters. Moreover it is not fully understood that the observed strings are vortex filaments in fluids. As a first step toward an understanding of these very thin entities, we make a simple analysis to reconstruct a crude picture of dynamics, by anticipating that the strings are vortex filaments in a shear flow of a virtually incompressible fluid. We give, in this letter, an estimation of material and dynamical parameters of the shear layer and the filaments, relying on the linear stability theory of a vortex filament [17]. A speculation is given to nonlinear development of waves on a filament leading to a catastrophic instability (breakup), and to formation of a series of loop solitons, phenomena conceivably connected with the coherence and the intermittent energy cascade as mentioned above.

3 Outline of experiment

Small samples of Co-coated steel of $1 \times 1 \times 0.5$ mm have been exposed to a focused beam of XeCl excimer laser of energy $E \sim 250$ mJ and of the pulse duration time $\tau_p \sim 16$–20 ns. As described in ref [7, 8], the sample surface is scratched in micron scale parallel lines, extending in a direction. A local coordinate system is laid down on the scratched surface which is oriented in such a way that the x-axis is directed along the scratched lines, while that the y-axis is transversal to them. The scratched surface is the origin of perturbation of the shear layer which leads to the formation of vortex filaments possibly by the Kelvin–Helmholtz instability; when the sample surface is irradiated with the laser pulse, an interface, or a shear layer, is formed with the velocity in the radial direction (along the y-axis locally) and with velocity variation along the z-direction. An oscillatory shock propagates in the radial direction. The parallel scratch-lines cause a transversal perturbation of form $\psi = A\cos(k_p y)$ when shock waves pass, which is then accompanied by wave generation and roll-up process. Here A is the perturbation amplitude of the same order as the height of the scratch wall, and k_p is the perturbation wavenumber; $k_p = 2\pi/l$, where l is the typical scratch–scratch distance. Thus vortex filaments are generated, along the x-direction and parallel between themselves, in the liquid shear layer on molten metal surface with different fluid velocities at the vapour/liquid interface (top/bottom layer).

Fig. 3. Pattern of parallel vortex filaments comprising a series of the loop solitons, generated by the XeCl excimer laser ($E = 250$ J; $t = 16$ ns), on Co-coated steel surface. The length L of the filaments is about 500 μm, and the core size is $d \sim 5 - 7$ μm. The horizontal and vertical dimensions of the picture are 1,100 μm and 300 μm.

4 Results and discussion

Parallel, though wavy, vortex filaments on Co-coated steel surface with parallel scratches, appear as a consequence of the shear layer instability. Every vortex filament undergoes deformation with a set of loop solitons on it that are considered to grow up from the instability waves on the filament (Fig. 3). The size of the loops is mostly in a range of 20–25 μm. Some are however much larger and reach the size greater than 30 μm.

Estimate of parameters of shear layer and vortex filaments

To shed light on the dynamical process behind the formation of a series of vortex filaments in ns-LMI, we pursue hydrodynamic parameters of the shear layer and of the filaments themselves. Comparing with the literature of classical (non-laser) hydrodynamic experiments and of numerical simulations, we estimate that the *Reynolds number* Re relevant to the observed pattern ranges from 2000 to 5000 [10]. We choose $Re = 3000$, the mean value in this range. For the typical fluid velocity U in ns-LMI, we assume that $U \sim$ 6000 m/sec, being close to the mean value obtained in a set of experimental measurements associated with the surface jetting phenomena [11]. Measurement of dimensions of the frozen vortex-filament in micro-graphs has shown that the typical length $L \sim 500$ μm and the typical core size $d \sim 5$–7 μm, while the filament curvature radius $R \sim 50$ μm $(d < R < L)$. From the definition of the Reynolds number

$$Re = UR/\nu \sim 3000\,, \tag{4.1}$$

we obtain an estimate of the kinematic viscosity ν to be $\nu \sim 10^{-4}$ m^2/sec (= 1 cm^2/sec). Care should be exercised that the viscosity is sensitive to local temperature, and therefore varies drastically in space and time. This value is taken to be a gross average in the whole dynamical process. Using $d \sim 5$ μm, the *circulation* Γ is estimated as

$$\Gamma = 2\pi dU \sim 0.2\mathrm{m}^2/\mathrm{sec}\,. \tag{4.2}$$

The *circulation Reynolds number* Re_Γ is then found to be

$$Re_\Gamma = \Gamma/\nu \sim 2000\,, \tag{4.3}$$

in agreement with (4.1). The *strength of the shear field* F_s is

$$F_s = U/d \sim 10^9\mathrm{sec}^{-1}\,, \tag{4.4}$$

the inverse of which provides the *growth time* τ_g of the wiggles on the filament:

$$\tau_g = d/U \sim 0.8\,\mathrm{ns}\,. \tag{4.5}$$

An elaboration of the growth rate $1/\tau_g$ can be made using the argument described later in this section.

The *reconnection time* τ_r of vortex filaments can also be estimated, by assuming the Gaussian distribution of vorticity $\omega(r,t)$

$$\omega(r,t) = \frac{\Gamma}{4\pi\nu}\exp\left(-r^2/4\nu t\right)\,, \tag{4.6}$$

or equivalently from the local azimuthal velocity $v(r,t)$

$$v(r,t) = \frac{\Gamma}{2\pi r}\left[1 - \exp(-r^2/4\nu t)\right] , \tag{4.7}$$

of a diffusing vortex filament [12]. Here r is the distance from the core center and t is the time. The vortical core spreads as $r \sim \sqrt{4\nu t}$. and imposition of $r = d$ supplies a desired estimate of time as

$$\tau_r = d^2/4\nu \sim 60\,\mathrm{ns}\,. \tag{4.8}$$

This result suggests that filament-filament reconnections are possible during the laser pulse exposition (16–20 ns), consistently with the observation [8].

It is remembered that the speed of a vortex ring is about $(\Gamma/4\pi R)\log(R/d)$ [12] (see eq.(4.10) below). By use of the value $R \sim 50$ μm, the time τ_{dyn} for the *vortex filament to traverse by itself a distance of a diameter* $2R$ is gained as

$$\tau_{dyn} = \frac{8\pi R^2}{\Gamma \log(R/d)} \sim 140\,\mathrm{ns}\,, \tag{4.9}$$

being longer than the pulse duration. This implies that the self-induced motion of filaments is of minor influence on the pattern formation, a conclusion acceptable from Fig. 3.

Translative instability

The above argument is elaborated and thus comparison with the experiment becomes feasible by appealing to the *localized induction approximation (LIA)* [13, 14, 15] in which a point $\boldsymbol{x}$ on a vortex filament, embedded in an external flow U_{ext}, evolves according to

$$\frac{\partial \boldsymbol{x}}{\partial t} = C\kappa \boldsymbol{b} + U_{ext}\boldsymbol{e}_x\,; \quad C = \frac{\Gamma}{4\pi}\log\left(\frac{R}{d}\right) , \tag{4.10}$$

where κ and $\boldsymbol{b}$ are the curvature and the binormal vector of the filament curve, and $\boldsymbol{e}_x$ is the unit vector in the x-direction.

Inspired by numerical simulations [14, 16], Pierrehumbert [17] addressed the linear stability of a vortex filament subjected to a simple shear flow based on the model (4.10) and provided an interpretation in terms of the "translative instability". This instability may serve as the seeds of the loop solitons. We give a brief summary of the result of [17]. A sinusoidal disturbances is superposed on a straight filament in the form:

$$\boldsymbol{x} = (0,0,z) + (\tilde{x},\tilde{y},0)\exp[i(\alpha z - nt)]\,. \tag{4.11}$$

which results in the dispersion relation:

$$n = \pm C\left[\alpha^2(\alpha^2 + U'/C)\right]^{1/2} . \tag{4.12}$$

The second term with U' arises from the advection by the external shear field U_{ext} whose strength is

$$U' = \partial U_{ext}/\partial y|_{y=0} \sim U/d\,. \tag{4.13}$$

The instability occurs when $-U'/C > 0$. The most dangerous instability occurs at the wavenumber

$$|\alpha| = (-U'/2C)^{1/2} . \tag{4.14}$$

By substitution from the above parameter values, we get the wavelength of the most unstable mode as

$$2\pi/|\alpha| \sim 20\,\mu\mathrm{m} . \tag{4.15}$$

This value is acceptable from the measurements; for choice of a filament, measurement of the breakup wavelength λ gives $\lambda \sim 30$ μm, and for the second filament chosen from a separated domain, a larger wavelength have been obtained as $\lambda \sim 48$ μm.

A numerical simulation for the development of initially localized disturbances that disperse into strong planar undulations of the filament was conducted by Aref and Flinchem [16]. A similar picture was recovered, though the wavelength being longer, by use only of the linearized equations [17]. These studies suggest a scenario for genesis of developed waves and filament breakup from a localized disturbance. A catastrophic filament instability may immediately follow the matured stage of the translative instability in a strong background shear flow.

Generation of loop solitons

The loop solitons would result from the breakup of fully developed waves on filaments. Their dynamics can be understood on the basis of the LIA (4.10). Ignoring the background flow, the nonlinear perturbations are described by a couple system of intrinsic equations, known as the Betchov–Da Rios equations [13]. These equations, written in the language of the local curvature κ and torsion τ, are transformed, by introduction of a complex function Ψ (Hasimoto transformation)

$$\Psi = \kappa \exp(i \int \tau ds) \tag{4.16}$$

into the nonlinear Schrödinger equation (NLS):

$$\frac{1}{i}\frac{\partial \Psi}{\partial t} = C\left\{\frac{\partial^2 \Psi}{\partial s^2} + \frac{1}{2}\left(|\Psi|^2 + A\right)\Psi\right\} , \tag{4.17}$$

where s is the arclength along the filament and A is an arbitrary function of time t [18]. For a single loop-soliton formation, the curvature takes the form

$$\kappa = 2\eta\,\mathrm{sech}[\eta(s - 2C\tau t)] , \tag{4.18}$$

where η is a constant, and the torsion τ and thus the velocity of the curvature on the filament are constant. On the micrograph, instead, a series of loops appear. Multisoliton dynamics can be also described by the LIA, using Hirota's and other methods [19, 20] (see also [21] for formation of loops in a shear flow).

Summarizing the results of this study, it is plausible that a set of parallel vortex filaments is formed in ns-LMI on Co-coated steel surface, as a result of the Kelvin–Helmholtz instability. However, during preparation of this paper, a suggestion was given by Pelz [22] that vortex filaments could also be created by the action of the "baroclinic term" due to the large density gradient across the melted layer which originates from the large temperature gradient. The baroclinic term originates from the pressure pulse, with a delay from the laser pulse. In this case also, a rich pattern of filaments could be obtained. Also suggested is that the scratch walls could cause the formation of elongated vortex dipole structures.

Estimation of the hydrodynamic parameters of the shear layer and of the vortex filaments showed that our system mimics the classical (non-laser) experiments as well as the numerical simulations [10]. A series of loop solitons (multisolitons) on filaments appear as a consequence of nonlinear dynamics, similarly to other physical systems. However, they appear in a way richer than other systems and offer a good opportunity for studying their characteristics and organization. Since the loop–loop distance on some parts of a filament is almost equidistant, they may represent a one-dimensional (regular) lattice, or the "Toda lattice" The other parts show more or less irregular loop-loop organization. To our knowledge, the treatment of a series of loop solitons on a filament, in the context of a regular or an irregular Toda lattice, has not been done yet.

For completeness, we mention that vortex filaments with a series of loop solitons are tied with the Euler elastica [21]. In the latter context, transition from regular or irregular to chaotic loop soliton organization was considered by El Naschle and Elneschale [23]. However, the loop solitons in Fig. 3 are mostly regularly or irregularly organized along the filament, rather than chaotically (overlapping each other). A detailed analysis of Fig. 3 reveals that some of the loops are very close to and even touch each other. In other words, they show some pretransitional behavior to chaotic organization, that may be reached asymptotically. Intuitively, the chaotic loop-loop organization would happen for a longer laser pulse or for a larger number of laser pulses, which turns out to be the case [24]. The catastrophic instability (breakup) of filaments are considered to follow directly from the translative instability. Although the breakup wavelength in our case is in agreement with the wavenumber predicted for the linear process, the nonlinear nature due to "soliton breakup" predicted by Aref and Flinchem [16] can not be excluded.

The ns-LMI with metal target - as a very specific method of generation of vortex filaments - has raised several theoretical questions. To have a clear picture of all processes occurring in an extremely short time, further sophisticated experiments are needed, and, at the same time, new models for nonlinear filament dynamics have to be developed.

Acknowledgments

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