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The Reachability and Related Decision Problems for Semi-Constructor TRSs

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Abstract
This paper shows that reachability is undecidable for confluent monadic and semi-constructor TRSs, and joinability and confluence are undecidable for monadic and semi-constructor TRSs. Here, a TRS is monadic if the height of the right-hand side of each rewrite rule is at most 1, and semi-constructor if all defined symbols appearing in the right-hand side of each rewrite rule occur only in its ground subterms.

1 Introduction
In this paper, we consider the reachability problem for confluent monadic and semi-constructor TRSs posed by our previous paper [4]. Here, a TRS is monadic if the height of the right-hand side of each rewrite rule is at most 1, and semi-constructor if all defined symbols appearing in the right-hand side of each rewrite rule occur only in its ground subterms. We give a negative answer to this problem. This undecidability result is compared with the decidability results of joinability and unification for the same class [4, 3]. Moreover, we show that joinability and confluence are undecidable for monadic and semi-constructor TRSs.

2 Preliminaries
We assume that the reader is familiar with standard definitions of rewrite systems [1] and we just recall here the main notations used in this paper.

Let $F$ be a finite set of operation symbols graded by an arity function $\text{ar} : F \rightarrow \mathbb{N} = \{0, 1, 2, \ldots\}$, $F_n = \{f \in F | \text{ar}(f) = n\}$. We use $x, y$ as variables, $f$ as an operation symbol, $r, s, t$ as terms. Let $V(s)$ be the set of variables occurring in $s$. The height of a term is defined as follows: $\text{height}(a) = 0$ if $a$ is a variable or a constant and $\text{height}(f(t_1, \ldots, t_n)) = 1 + \max\{\text{height}(t_1), \ldots, \text{height}(t_n)\}$ if $n > 0$. The root symbol of a term is defined as $\text{root}(a) = a$ if $a$ is a variable and $\text{root}(f(t_1, \ldots, t_n)) = f$.

A position in a term is expressed by a sequence of positive integers, and positions are partially ordered by the prefix ordering $\leq$. Let $O(s)$ be the set of positions of $s$. For a set of positions $\mathcal{W}$, let $\text{Min}(\mathcal{W})$ be the set of its minimal positions (w.r.t. $\leq$).

Let $s_{i,p}$ be the subterm of $s$ at position $p$. For a sequence $(p_1, \ldots, p_n)$ of pairwise parallel positions and terms $t_1, \ldots, t_n$, we use $s[t_1, \ldots, t_n|p_1, \ldots, p_n]$ to denote the term obtained from $s$ by replacing each subterm $s_{i,p_i}$ by $t_i(1 \leq i \leq n)$. For a set of function symbols $F$, let $O_F(s) = \{p \in O(s) | \text{root}(s|p) \in F\}$. For a string of unary function symbols $u = a_1 a_2 \cdots a_k$ and a term $t$, let $u(t)$ be an abbreviation for $a_1(a_2(\cdots a_k(t)))$.

A rewrite rule $\alpha \rightarrow \beta$ is a directed equation over terms. A TRS $R$ is a set of rewrite rules. Let $\rightarrow$ be the inverse of $\leftrightarrow = \rightarrow \cup \leftarrow$, and $\bot = \rightarrow^* \leftarrow^*$. $t$ is reachable from $s$ if $s \rightarrow^* t$. $R$ is confluent on TRS $R$ if for every $s \rightarrow_R^* t \rightarrow_R^* s$, $s \bot t$. A TRS $R$ is confluent if every $\rightarrow$ is confluent on $R$. Let $\gamma : s_1 \leftarrow a_2 \cdots \leftarrow a_k \rightarrow s_n$ be a rewrite sequence. This sequence is abbreviated to $\gamma : s_1 \leftarrow a_2 \cdots \leftarrow a_k \rightarrow s_n$. Let $|\gamma|$ be the number of steps of $\gamma$. $\gamma$ is called $p$-invariant if $q > p$ for any redex position $q$ of $\gamma$, and we write $\gamma \leftarrow P = \gamma : s_1 \leftarrow a_2 \cdots \leftarrow a_k \rightarrow s_n$.

The set $D_R$ of defined symbols for a TRS $R$ is defined as $D_R = \{\text{root}(\alpha) | \alpha \rightarrow \beta \in R\}$. A term $s$ is semi-constructor if for every subterm $t$ of $s$, $t$ has no variable or $\text{root}(t)$ is not a defined symbol.

Definition 1 A rule $\alpha \rightarrow \beta$ is monadic if $\text{height}(\beta) \leq 1$, semi-constructor if $\beta$ is semi-constructor. A TRS $R$ is monadic if every rule in $R$ is monadic, semi-constructor if every rule in $R$ is semi-constructor.
3 Undecidability of joinability for monadic and semi-constructor TRSs

We have shown that joinability is undecidable for linear semi-constructor TRSs [4]. In this section, we show that joinability for monadic and semi-constructor TRSs is undecidable by a reduction from the Post's Correspondence Problem (PCP). Let $P = \{(u_i, v_i) \in \Sigma^* \times \Sigma^* | 1 \leq i \leq n\}$ be an instance of the PCP. The corresponding TRS $R_P$ is constructed as follows. Let $F_0 = \{0, c, d, \}$, $F_1 = \{e_i | 1 \leq i \leq n\}(= E) \cup \Sigma$, $F_2 = \{f, g\}$.

$$R_P = \{0 \rightarrow e_i(0) | 1 \leq i \leq n\} \cup \{0 \rightarrow f(c, d)\}$$

$$\cup \{b \rightarrow a(b), b \rightarrow a(\$) | b \in \{c, d\}, a \in \Sigma\}$$

$$\cup \{f(x, x) \rightarrow g(x, x)\}$$

$$\cup \{e_i(g(u_i(x), v_i(y))) \rightarrow g(x, y) | 1 \leq i \leq n\}$$

$R_P$ is monadic. Here, $D_{R_P} = \{0, c, d, f\} \cup E$, so $R_P$ is semi-constructor.

Lemma 2 $0 \rightarrow^*_R g(\$,\$) iff PCP $P$ has a solution.

Proof. $0 \rightarrow^*_R g(\$,\$) iff there exists $i_1: \cdots : i_m \in \{1, \cdots, n\}^*$ such that $0 \rightarrow^m e_m \cdots e_1(f(c, d)) \rightarrow^* e_m \cdots e_1(f(u_1 \cdots u_m(\$,\$)), u_1 \cdots u_m(\$,\$))) \rightarrow^* e_m \cdots e_1(g(u_1 \cdots u_m(\$,\$), u_1 \cdots u_m(\$,\$))) \rightarrow^m g(\$,\$) iff u_1 \cdots u_m = u_1 \cdots u_m.$

Since $g(\$,\$)$ is a normal form, the following theorem holds.

Theorem 3 Both joinability and reachability for monadic and semi-constructor TRSs are undecidable.

4 Undecidability of reachability for confluent monadic and semi-constructor TRSs

We give a stronger result for reachability, that is, reachability for confluent monadic and semi-constructor TRSs is undecidable. Note that joinability is decidable for the same class [4, 3]. Let $P = \{F \cup \{1\}\}.

$$\hat{R}_P = R_P \cup \{(\$,\$ \rightarrow 1) \cup \{a(1) \rightarrow 1 | a \in \Sigma\} \cup \{e_i(g(1, v_i(y))) \rightarrow g(1, y), e_i(g(u_i(x), 1)) \rightarrow g(x, 1), e_i(g(1, 1)) \rightarrow g(1, 1) | 1 \leq i \leq n\}$$

$\hat{R}_P$ is monadic. Here, $D_{\hat{R}_P} = D_{R_P} \cup \{\$,\$\} \cup \Sigma$, so $\hat{R}_P$ is semi-constructor. First, we show the confluence of $\hat{R}_P$.

4.1 Confluence of $\hat{R}_P$

To show the confluence of $\hat{R}_P$, we need some definitions and lemmata.

Definition 4 The set of $\Sigma$-strings is defined as follows.

- $1, c, d$ and $\$ are $\Sigma$-strings.
- $a(t)$ is a $\Sigma$-string if $t$ is a $\Sigma$-string and $a \in \Sigma$.

Lemma 5 For any $\Sigma$-string $s$, the following properties hold.

1. For any $\gamma : s \leftrightarrow^* t$, $t$ is a $\Sigma$-string.
2. $s \rightarrow^* 1$.

Proof. (1) By induction on $|\gamma|$. (2) By induction on the structure of $s$.

Corollary 6 Every $\Sigma$-string is confluent.

Lemma 7 Let $\gamma : u(s) \rightarrow^* t$ where $u \in \Sigma^+$. Then, if root$(s) \notin \{1, c, d, \}$ $\cup \Sigma$ and $u(s)_{p1} = s$ then $\gamma$ is $\rho$-invariant.

Proof. By induction on $|\gamma|$.

Definition 8 The set of $E$-strings is defined as follows.

- $0, f(t_1, t_2)$ and $g(t_1, t_2)$ are $E$-strings if $t_1, t_2$ are $\Sigma$-strings.
- $e_i(t)$ is an $E$-string if $t$ is an $E$-string and $i \in \{1, \cdots, n\}$.

Lemma 9 For any $E$-string $s$, the following properties hold.

1. For any $\gamma : s \leftrightarrow^* t$, $t$ is an $E$-string.
2. $s \rightarrow^* g(1, 1)$.

Proof. (1) By induction on $|\gamma|$. (2) By induction on the structure of $s$. Basis: For any $\Sigma$-strings $s_1, s_2$, $f(s_1, s_2) \rightarrow f(1, 1) \rightarrow g(1, 1)$ and $g(s_1, s_2) \rightarrow g(1, 1)$ by Lemma 5(2), and $0 \rightarrow f(c, d) \rightarrow g(1, 1)$. Thus, $s \rightarrow^* g(1, 1)$ if $s = f(s_1, s_2), g(s_1, s_2)$ or 0. Induction step: Let $s = e_i(s')$ for some $i \in \{1, \cdots, n\}$. By the induction hypothesis, $s' \rightarrow^* g(1, 1)$. Thus, $e_i(s') \rightarrow^* g(1, 1)$.

Corollary 10 Every $E$-string is confluent.
The following lemma is used as a component of the proof of Lemma 12.

Lemma 11 For any $i \in \{1, \ldots, n\}$ and terms $r_1, r_2$, the following properties hold.

1. If $t \not\in e_i(g(r_1, r_2)) \rightarrow^* t$ then there exist terms $t_1, t_2$ such that $t \rightarrow^* g(t_1, t_2)$.

2. If $g(s_1, s_2) \leftarrow e_i(g(r_1, r_2)) \rightarrow^* g(t_1, t_2)$ and $g(r_1, r_2)$ is confluent then $g(s_1, s_2) \rightarrow^* g(t_1, t_2)$.

Proof.

1. Let $t = e_i(g(t_1, t_2))$. If $r_1$ is a $\Sigma$-string then $t_1 \rightarrow^* 1$ by Lemma 5. Otherwise, $r_1 \neq 1$. Thus, $r_1 = u_i(r_1')$ for some term $r_1'$ by $e_i(g(r_1, r_2)) \rightarrow s$. By Lemma 7, $t_1' = u_i(t_1')$, where $r_1' \rightarrow^* t_1'$. Similarly, $t_2 \rightarrow^* 1$ or $t_2' = u_i(t_2')$ for some term $t_2'$. Thus, $t \rightarrow^* g(t_1, t_2)$, where $t_1 \in \{1, t_1'\}$ and $t_2 \in \{1, t_2'\}$.

2. By the definition of $\mathcal{R}_\gamma$, $e_i(g(r_1, r_2)) \rightarrow^* e_i(g(s_1', s_2')) \rightarrow g(s_1', s_2') \rightarrow g(s_1, s_2)$ and $e_i(g(s_1', s_2')) \rightarrow g(t_1', t_2') \rightarrow g(t_1, t_2)$. Thus, $s_1 \leftarrow r_1 \rightarrow^* t_1', s_1' \rightarrow^* s_1$ and $t_1' \rightarrow^* t_1$. First, we show that $s_1 = t_1$.

Case of $s_1' = t_1' = 1$: Obviously, $s_1' = s_1 = t_1' = 1$.

Case of $s_1' = 1$ and $t_1' = u_i(t_1')$: Obviously, $s_1' = s_1 = 1$. By Lemma 5, $t_1$ is a $\Sigma$-string and $t_1 \rightarrow^* 1$.

Case of $s_1' = u_i(s_1'')$ and $t_1' = 1$: Similar to the previous one.

Case of $s_1' = u_i(s_1'')$ and $t_1' = u_i(t_1'')$: By confluency of $g(r_1, r_2)$, $r_1$ is confluent. Thus, $u_i(s_1) \downarrow u_i(t_1)$. If $s_1$ is a $\Sigma$-string then $s_1 \downarrow t_1$ by Corollary 6. Otherwise, $s_1 \downarrow t_1$ by Lemma 7.

Similarly, $s_2 \downarrow t_2$. Thus, $g(s_1, s_2) \rightarrow^* g(t_1, t_2)$.

Now, we show the confluence of $\mathcal{R}_\gamma$.

Lemma 12 $\mathcal{R}_\gamma$ is confluent.

Proof. We show that for any $\gamma: s \not\in r \rightarrow^* t$, $s \downarrow t$ by induction on height($r$).

Basis: If $r \in \{a, d\}$ then $s \downarrow t$ by Corollary 6, else if $r = 0$ then $s \downarrow t$ by Corollary 10. Otherwise, $s = r$ since $r$ is a normal form.

Induction step: If $\gamma$ is $\epsilon$-invariant then $s \downarrow t$ by the induction hypothesis. So, we consider that $\gamma$ has an $\epsilon$-reduction. Let $\gamma : r \rightarrow^* s$ and $\gamma : r \rightarrow^* t$.

Without loss of generality, we assume that $\gamma$ has an $\epsilon$-reduction and root($r$) $\in \Sigma \cup \{f\} \cup E$.

Case of root($r$) $\in \Sigma$: $\gamma : r = a(r_1) \rightarrow^* a(1) \rightarrow 1 = s$ holds for some $a \in \Sigma$ and $r_1$. By Lemma 5, $t \rightarrow^* 1$.

Case of root($r$) $\in \Sigma$: $\gamma : r = f(r_1, r_2) \rightarrow^* f(r_2') \rightarrow g(r_1, r_2) = s$ holds for some terms $r_1, r_2, r', s_1, s_2$. If $\gamma$ is $\epsilon$-invariant then $t = f(t_1, t_2)$ where $r_1 \rightarrow^* t_1$ and $r_2 \rightarrow^* t_2$. In this case, $s \not\in g(r_0, r_0) \rightarrow^* t$ for some $r_0$ by Figure 1(i). If $\gamma$ has an $\epsilon$-reduction then $\gamma : r = f(r_1, r_2) \rightarrow^* f(r_2', r_3') \rightarrow g(r_2', r_3') \rightarrow^* g(t_1, t_2) = t$ holds for some terms $r', t_1, t_2$. In this case, $s \not\in g(r_0, r_0) \rightarrow^* t$ for some $r_0$ by Figure 1(ii).

Case of root($r$) $\in E$: $\gamma : r = e_i(r_1) \rightarrow^* e_i(g(r_1, r_2)) \rightarrow g(s_1', s_2') \rightarrow^* g(s_1, s_2) = s$ holds for some terms $r_1, s_1', s_2', s_1, s_2$ and $i \in \{1, \ldots, n\}$. If $\gamma$ is $\epsilon$-invariant then $t = e_i(t_1)$ where $r_1 \rightarrow^* t_1$. By the induction hypothesis, there exists a term $t'$ such that $e_i(g(s_1', s_2')) \rightarrow^* t' \rightarrow^* t$. By Lemma 11(i), $t' \rightarrow^* g(t_1', t_2)$ for some $t_1', t_2$. Here, $g(s_1', s_2')$ is confluent by the induction hypothesis and $r_1 \rightarrow^* g(s_1', s_2')$. Thus, $s \not\in g(t_1', t_2)$ by Lemma 11(ii). (See Figure 1(iii).) If $\gamma$ has an $\epsilon$-reduction then $\gamma : r = e_i(r_1) \rightarrow^* e_i(g(t_1', t_2)) \rightarrow g(t_1, t_2) \rightarrow^* g(t_1', t_2') \rightarrow g(t_1', t_2') = t$ holds for some terms $t_1', t_2, t_1', t_2, t_1', t_2'$, and $t_1, t_2$. There exists a term $t'$ such that $s \not\in t' \rightarrow^* e_i(g(t_1', t_2))$ as shown in Figure 1(iii). Here, root($s'$) = $g$ by root($s$) = $g$. By the induction hypothesis and $r_1 \rightarrow^* g(t_1', t_2)$, $g(t_1', t_2)$ is confluent. Thus, $s' \not\in t$ by Lemma 11(ii). (See Figure 1(iv).)

4.2 Reachability for confluent monadic and semi-constructor TRSs

Lemma 13 For any $\gamma: s \not\in \mathcal{R}_\gamma t$, if $s$ has 1 as its subterm then so does $t$.

Proof. Since for any $\alpha \rightarrow \beta \in \mathcal{R}_\gamma$, $\mathcal{V}(\alpha) = \mathcal{V}(\beta)$ and $\alpha$ has 1 as its subterm then so does $\beta$.

Lemma 14 $0 \not\in \mathcal{R}_\gamma g(s, \delta)$ if $0 \not\in \mathcal{R}_\gamma g(\delta, \delta)$.

Proof. Only if part: Let $\gamma: 0 \not\in \mathcal{R}_\gamma g(\delta, \delta)$. We assume to the contrary that $\gamma$ must have $\mathcal{R}_\gamma \setminus \mathcal{R}_\gamma$ reduction, i.e., $\gamma: 0 \not\in \mathcal{R}_\gamma g(s, \delta)$ for some $s, t$. By the definition of $\mathcal{R}_\gamma$, $t$ has 1 as its subterm. By Lemma 13, $g(\delta, \delta)$ has 1 as its subterm, a contradiction. If part: By $\mathcal{R}_\gamma \subseteq \mathcal{R}_\gamma$.

By Lemmata 2, 12, and 14, the following theorem holds.

Theorem 15 Reachability for confluent monadic and semi-constructor TRSs is undecidable.
5 Undecidability of confluence of monadic and semi-constructor TRSs

We show that confluence of monadic and semi-constructor TRSs is undecidable.

Let $F' = F_0' \cup F_1'$ where $F_0' = \{2\}$, $F_1' = \{h\}$.

\[ R = \{ h(x) \rightarrow h(0), h(g(s, s)) \rightarrow 2 \} \]

$\bar{R}_P \cup R$ is monadic. Here, $D_R = \{h\}$, so $\bar{R}_P \cup R$ is semi-constructor.

Lemma 16 For any $s$ with root($s$) $\in F'$, the following properties hold.

(1) If $s \rightarrow^{\ast}_{\bar{R}_P \cup R} t$ then root($t$) $\in F'$.

(2) If $0 \rightarrow^{\ast}_{\bar{R}_P} g(s, s)$ then $s \rightarrow^{\ast}_{\bar{R}_P \cup R} 2$.

The proof is straightforward, so omitted.

Lemma 17 Let $s \rightarrow^{\ast}_{\bar{R}_P \cup R} t$, $\text{Min}(O_{F'}(s)) = \{p_1, \ldots, p_m\}$, and $\text{Min}(O_{F'}(t)) = \{q_1, \ldots, q_n\}$.

Then, $s[2, \ldots, 2][p_1, \ldots, p_m] \rightarrow_{\bar{R}_P} t[2, \ldots, 2][q_1, \ldots, q_n]$ or $s[2, \ldots, 2][p_1, \ldots, p_m] = t[2, \ldots, 2][q_1, \ldots, q_n]$.

Proof. Let $s \rightarrow^{\ast}_{\bar{R}_P \cup R} t$. If there exists $i \in \{1, \ldots, m\}$ such that $p_i \leq p$ then $\{p_1, \ldots, p_m\} = \{q_1, \ldots, q_n\}$ by Lemma 16(1).

Thus, $s[2, \ldots, 2][p_1, \ldots, p_m] = t[2, \ldots, 2][q_1, \ldots, q_n]$. Otherwise, obviously $s \rightarrow_{\bar{R}_P} t$. Since every function symbol in $F'$ does not occur in $\bar{R}_P$, $s[2, \ldots, 2][p_1, \ldots, p_m] \rightarrow_{\bar{R}_P} t[2, \ldots, 2][q_1, \ldots, q_n]$. \[\square\]

Lemma 18 $\bar{R}_P \cup R$ is confluent iff $0 \rightarrow_{\bar{R}_P} g(s, s)$.

Proof. Only if part: By $h(0) \rightarrow_{\bar{R}_P} h(g(s, s)) \rightarrow_{\bar{R}_P} 2$, confluence ensures that $h(0) \rightarrow_{\bar{R}_P \cup R} 2$. Since 2 is a normal form, $h(0) \rightarrow_{\bar{R}_P \cup R} 2$. Thus, there exists a shortest sequence $\gamma$ that satisfies $\gamma : h(0) \rightarrow_{\bar{R}_P \cup R} h(g(s, s)) \rightarrow_{\bar{R}_P \cup R} 2$. Since $\gamma$ is shortest, $h(0) \rightarrow_{\bar{R}_P \cup R} h(g(s, s))$. Thus, there exists $\gamma' : 0 \rightarrow_{\bar{R}_P \cup R} g(s, s)$. Obviously, every function symbol occurring in $\gamma'$ belongs to $\bar{F}$. Thus, $0 \rightarrow_{\bar{R}_P} g(s, s)$. By Lemma 14, $0 \rightarrow_{\bar{R}_P} g(s, s)$.

If part: Let $s \rightarrow_{\bar{R}_P \cup R} r \rightarrow_{\bar{R}_P \cup R} t$. By Lemma 17, $s[2, \ldots, 2][p_1, \ldots, p_m] \rightarrow_{\bar{R}_P} r[2, \ldots, 2][q_1, \ldots, q_n]$, where $\text{Min}(O_{F'}(r)) = \{q_1, \ldots, q_n\}$, $\text{Min}(O_{F'}(s)) = \{p_1, \ldots, p_m\}$, and $\text{Min}(O_{F'}(t)) = \{q_1, \ldots, q_n\}$. Since $\bar{R}_P$ is confluent by Lemma 12, $s[2, \ldots, 2][p_1, \ldots, p_m] \rightarrow_{\bar{R}_P} t[2, \ldots, 2][q_1, \ldots, q_n]$. By $0 \rightarrow_{\bar{R}_P} g(s, s)$ and Lemma 16(2), $s \rightarrow_{\bar{R}_P \cup R} s[2, \ldots, 2][p_1, \ldots, p_m]$ and $t \rightarrow_{\bar{R}_P \cup R} t[2, \ldots, 2][q_1, \ldots, q_n]$. Thus, $s \rightarrow_{\bar{R}_P \cup R} t$. \[\square\]

By Lemma 2 and 18, the following theorem holds.

Theorem 19 Confluence of monadic and semi-constructor TRSs is undecidable.

6 Confluence of flat TRSs

In [2], the undecidability of confluence of flat TRSs has been claimed, but we found that the proof is incorrect. In this section, we explain its flaw.

Definition 20 [2] A rule $\alpha \rightarrow \beta$ is flat if $\text{height}(\alpha) \leq 1$ and $\text{height}(\beta) \leq 1$.

In [2], first the undecidability of reachability has been obtained by showing that $0 \rightarrow_{\bar{R}_S} 1$ iff there
exists a solution for PCP for the following TRS $R_1$.

$$R_1 = R_2 \cup \{0 \rightarrow f(q_A^{(3)}, q_A^{(4)}, q_B^{(13)}, q_B^{(14)}, q_A^{(15)}, q_B^{(16)}),
   f(x_1, x_2, x_3, y_{11}, y_{12}, y_{11}, y_{12}) \rightarrow
   g(x_1, x_2, x_3, y_{11}, y_{12}, y_{11}, y_{12}),
   g(x_0, x_0, y_{17}, y_{18}, y_{18}, y_{10}, y_{10}) \rightarrow 1\}$$

Here, $R_2$ has many rules, so omitted (see [2], p.267).

Next, the undecidability of confluence has been obtained by showing the claim that $R_1 \cup R_2$ is confluent iff $0 \rightarrow^*_{R_1} 1$ for the following TRS $R_2$.

$$R_2 = \{2 \rightarrow 0, 2 \rightarrow 1 \} \cup \{c \rightarrow 0 \mid c \in \Xi \}$$

$$\cup \{d(x) \rightarrow 0, d(1) \rightarrow 1 \mid d \in \Xi_1\}$$

$$\cup \{f(x_1, \cdots, x_8) \rightarrow 1, g(x_1, \cdots, x_8) \rightarrow 1 \mid$$

$\text{one of the } x_i \text{ is } 1,$

the others are distinct variables$\}$

Here, $\Xi = \Xi_0 \cup \Xi_1 \cup \{f, g\}$, which is a set of function symbols occurring in $R_1$. $\Xi_0, \Xi_1$ have many symbols, so omitted (see [2], p.267). Note that $\Xi_0$ has

$q_A^{(3)}, q_A^{(4)}, q_A^{(5)}, q_B^{(13)}, q_B^{(14)}, q_B^{(15)}, q_B^{(16)}$.

However, the proof of the only-if part of the claim is incorrect. The proof claims that if $0 \rightarrow^*_{R_1} 1$ does not hold then $R_1 \cup R_2$ is not confluent because of the peak $0 \leftarrow_{R_2} 2 \rightarrow_{R_2} 1$.

But, the claim overlooks that $0 \rightarrow_{R_1} f(q_A^{(3)}, q_A^{(4)}, q_B^{(13)}, q_B^{(14)}, q_A^{(15)}, q_B^{(16)}) \rightarrow_{R_2} f(0, 0, 0, 0, 0, 0, 0, 0) \rightarrow_{R_1} g(0, 0, 0, 0, 0, 0) \rightarrow_{R_1} 1$. Thus, the undecidability of confluence of flat TRSs has not been shown. Now, Jacquemard claims that the proof can be corrected.

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