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APPROXIMATION OF A FIXED POINT OF A ϕ-STRONGLY PSEUDOCONTRACTIVE MAPPING IN A BANACH SPACE

HIROKO MANAKA TAMURA

ABSTRACT. In the present paper, we show the stability result of the following unified iterative scheme

$$x_{n+1} = t_n T v_n + (1 - t_n) x_n + u_n$$

for a given sequence $\{v_n\}$ in a Banach space X and a ϕ-strongly pseudocontractive mapping T with a coefficient sequence $\{t_n\}$ in $[0, 1]$ and an error term sequence $\{u_n\}$ in X. This stability result implies a convergence theorem of Mann, Ishikawa and Stević's iterations. We also try to show convergence rateestimates to a fixed point of T.

1. INTRODUCTION

Let X be an arbitrary Banach space and let $T : X \to X$ be a nonlinear mapping such that the set $F(T)$ of fixed points of T is nonempty. In the last four decades, numerous papers have been published on the iterative approximation of fixed points of nonlinear mappings T in Banach spaces. Let J denote the normalized duality mapping from X into 2^{X^*} given by

$$J(x) = \{f \in X^* : \langle x, f\rangle = \|x\|^2 = \|f\|^2\},$$

where X^* denotes the dual space of X and $\langle \cdot, \cdot \rangle$ denotes the duality pairing. A mapping $T : D(T) \subset X \to X$ is called a strong pseudocontraction if there exists $t > 1$ such that for all $x, y \in D(T)$, there is $j(x - y) \in J(x - y)$ satisfying

$$\langle Tx - Ty, j(x - y)\rangle \leq \frac{1}{t} \|x - y\|^2 .$$

Let $\phi : [0, \infty) \to [0, \infty)$ be a strictly increasing function with $\phi(0) = 0$. A mapping $T : D(T) \subset X \to X$ is called ϕ-strongly pseudocontractive if for all $x, y \in D(T)$ there exists $j(x - y) \in J(x - y)$ such that

$$\langle Tx - Ty, j(x - y)\rangle \leq \|x - y\|^2 - \phi(\|x - y\|) \|x - y\| .$$

We call a ϕ-strongly pseudocontractive mapping a ϕ-strong pseudocontraction. The class of strong pseudocontractions and the class of ϕ-strong pseudocontractions have been studied extensively by several authors (see [1], [6], [8], [14]). It was shown in [14] that the class of strong pseudocontractions is a proper subset of the class of ϕ-strong pseudocontractions. It is well-known that if $T : X \to X$ is continuous and strongly pseudocontractive, then T has a unique fixed point (see [18]). Let

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C be a closed convex subset of X. If $T : C \to C$ is continuous and ϕ-strongly pseudocontractive, then T has a unique fixed point (see [6], [7]).

A mapping $f(T,\cdot) : X \to X$ is said to be an iterative scheme when $f(T,\cdot)$ is considered as a procedure, involving T, which yields a sequence of points $\{x_n\} \subset X$ defined by $x_{n+1} = f(T, x_n)$ for $n \geq 0$, where $x_0 \in X$ is given. Suppose that the sequence $\{x_n\}$ converges strongly to a $p \in F(T)$, and then it is denoted by $x_n \to p \in F(T)$. Let $\{y_n\}$ be an arbitrary sequence in X. If $\lim_{n\to\infty} \|y_{n+1} - f(T, y_n)\| = 0$ implies $y_n \to p \in F(T)$, then an iterative scheme $f(T,\cdot)$ is said to be T-stable or stable with respect to T (see [3]). We say that an iterative scheme $f(T,\cdot)$ is almost T-stable or almost stable with respect to T if $\sum_{n=1}^{\infty} \|y_{n+1} - f(T, y_n)\| < \infty$ implies $y_n \to p \in F(T)$. (cf. [17], [22]). Clearly, an iterative scheme $f(T,\cdot)$ which is T-stable is almost T-stable. In [17] Osilike gave an example showing that an iterative scheme which is almost T-stable may fail to be T-stable. In [3], Harder and Hicks pointed out the importance of stability of iteration schemes from the view point of practical use of iterations, and gave some results for the stability of iteration schemes. Recently, the stability of iterative scheme for nonlinear mappings was investigated by several authors (cf. [15], [16], [17], [19], [20], [21], [22], [23], [24]).

In this paper, first we show a stability result of the following iteration scheme $f(T, v_n, \cdot)$ involving a ϕ-strong pseudocontraction T and a sequence $\{v_n\}$ in X :

$$\text{(1.1)} \qquad \begin{cases} x_0 \in X, \\ x_{n+1} = f(T, v_n, x_n) \\ \qquad = t_n T v_n + (1 - t_n) x_n + u_n, \end{cases}$$

where $\{t_n\}$ and $\{u_n\}$ are a coefficient sequence in $[0,1]$ and an error term sequence in X, respectively. This iteration scheme gives the Mann iteration as a special case when $v_n = x_n, (n \geq 0)$ ([13]). Moreover we show that this stability result implies the stability of an iterative scheme for a family of k selfmappings $T_1, T_2, ..., T_k$ with k coefficient sequences $\{t_n^{(j)}\}$ and k error term sequences $\{u_n^{(j)}\}$, $(j = 1, ..., k)$ such as

$$\text{(1.2)} \qquad \begin{cases} x_0 \in X, \\ x_{n+1} = f(T_1, T_2, ..., T_k, x_n) \\ \qquad = t_n^{(1)} T_1 x_n^{(1)} + (1 - t_n^{(1)}) x_n + u_n^{(1)}, \end{cases}$$

where

$$x_n^{(1)} = t_n^{(2)} T_2 x_n^{(2)} + (1 - t_n^{(2)}) x_n + u_n^{(2)},$$

$$\cdots$$

$$x_n^{(k-1)} = t_n^{(k)} T_k x_n + (1 - t_n^{(k)}) x_n + u_n^{(k)},$$

and

$$x_n^{(k)} = x_n.$$

This iteration generalizes the Mann, Ishikawa, Das & Debata and Stević iterations, and we obtain them as special cases when we put $T_1 = T, T_1 = T_2 = T, T_1 \neq T_2$,and $T_1 = T_2 = ... = T_k = T$, respectively. (See [2], [4], [13], [22], [25], etc.) Our iteration defined by (1.1) is a unified approach for these iterations, when we put $v_n = x_n^{(1)}$. With respect to the stability results of Mann and Ishikawa iterations, there are many results by several authors (see [11], [15], [16], [17], [21], [24]). Osilike gave the result of Mann and Ishikawa iterations for a ϕ-strong pseudocontraction in [17],

and Stević generalized the stability result of Osilike in [24] . The unified approach of (1.1) gives a natural generalization of Stević's result and a strong convergence theorem for iterations defined by (1.2). Secondary we try to show how fast the iterative sequence converges to a fixed point. We give the convergence rate estimates for Mann iteration involving a Lipschitz continuous ϕ-strong pseudocontraction T on a closed convex subset C in an arbitrary Banach space X.

2. Preliminaries

We shall give some lemmata which are needed to present our statements and to prove the main results.

Lemma 1. [9] *Let $\{a_n\}, \{b_n\}$ and $\{c_n\}$ be three non-negative real number sequences satisfying the difference inequality*

$$a_{n+1} \leq (1 - t_n)a_n + b_n + c_n. \tag{2.1}$$

Suppose

$$\{t_n\} \subseteq [0,1], \quad \sum_{n=1}^{\infty} t_n = \infty, \quad b_n = t_n o(1), \text{ and } \sum_{n=1}^{\infty} c_n < \infty .$$

Then

$$\lim_{n\to\infty} a_n = 0.$$

Lemma 2. [24] *Let $\{a_n\}, \{b_n\}$ and $\{c_n\}$ be three non-negative real number sequences satisfying the difference inequality (2.1). Suppose $\{t_n\} \subseteq [0,1]$, $\sum_{n=1}^{\infty} t_n = \infty$, $b_n = O(t_n)$, and $\sum_{n=1}^{\infty} c_n < \infty$. Then $\{a_n\}$ is bounded.*

The following lemma plays an important role in the proof of the main theorem.

Lemma 3. [24] *Let $\{a_n\}, \{b_n\}, \{t_n\}, \{\delta_n\}$ and $\{\rho_n\}$ be non-negative real number sequences satisfying the following conditions $(a) - (g)$:*

(a) $$a_{n+1} \leq \left(1 - t_n \frac{f_1(b_n)}{f_2(b_n)}\right) a_n + t_n\delta_n + \rho_n,$$

where f_1 and f_2 are non-negative increasing functions on $[0,\infty)$, and $f_2(0) > 0$,

(b) $$\{t_n\} \subset [0,1] \quad \text{and} \quad \lim_{n\to\infty} t_n = 0,$$

(c) $$\lim_{n\to\infty} \delta_n = 0,$$

(d) $$\sum_{n=1}^{\infty} \rho_n < \infty,$$

(e) $$\{a_n\} \text{ is bounded} \quad \text{and} \quad \liminf_{n\to\infty} a_n = 0,$$

(f) $$\lim_{n\to\infty} (a_{n+1} - a_n) = 0,$$

(g) $$\lim_{n\to\infty} (a_n - b_n) = 0.$$

Then

$$\lim_{n\to\infty} a_n = 0.$$

Lemma 4. (*cf.* [5]) *Let X be a Banach space, and let T be ϕ-strongly pseudocontractive. For any $x, y \in X$ and $r > 0$,*

$$||x - y|| \leq ||x - y + r\{(I - T - \gamma(x,y))x - (I - T - \gamma(x,y))y\}||,$$

where

$$\gamma(x,y) = \frac{\phi(||x-y||)}{1 + \phi(||x-y||) + ||x-y||}.$$

The following lemma can be proved using Lemma 4, similar to the method used for a pseudocontraction (cf. [10], [24]).

Lemma 5. *Let T be ϕ-strongly pseudocontractive on X, and let y^* be defined by*

$$y^* = tTv + (1-t)y + u,$$

for $y, u, v \in X$, and $t \in [0,1]$. Suppose $p \in F(T) \neq \emptyset$. Then

$$||y^* - p|| \leq \frac{1 + t(1-\gamma^*)}{1+t}||y - p|| + \frac{t^2(2-\gamma^*)}{1+t}||y - Tv|| + \frac{t}{1+t}||Ty^* - Tv|| + \frac{1 + t(2-\gamma^*)}{1+t}||u||,$$

where

$$\gamma^* = \gamma(y^*, p) = \frac{\phi(||y^* - p||)}{1 + \phi(||y^* - p||) + ||y^* - p||}.$$

3. Stability and Strong convergence theorem

We shall show a stability result and a strong convergence theorem.

Theorem 1. [26] *Let X be an arbitrary Banach space and let T be a uniformly continuous and ϕ-strongly pseudocontractive selfmapping on X with a bounded range. For a sequence $\{v_n\}$ in X, let an iterative scheme $f(T, v_n, \cdot\ )$ with a coefficient sequence $\{t_n\}$ in $[0,1]$ and an error term sequence $\{u_n\}$ in X be defined by (1.1). Suppose that $\{t_n\}$ and $\{u_n\}$ satisfy the following conditions :*

$$(h)\ \sum_{n=1}^{\infty} ||u_n|| < \infty, \quad (i)\ \sum_{n=1}^{\infty} t_n = \infty, \quad (j)\ \lim_{n\to\infty} t_n = 0.$$

Then the following statements hold.

(I) If $\{y_n\}$ in X satisfies

$$||y_n - v_n|| \to 0\ (n \to \infty) \quad \text{and} \quad \sum_{n=1}^{\infty} ||y_{n+1} - f(T, v_n, y_n)|| < \infty,$$

then $y_n \to p \in F(T)$.

(II) If $w_n \to p \in F(T)$, then

$$||w_{n+1} - f(T, v_n, w_n)|| \to 0\ (n \to \infty).$$

Proof. (I) Suppose $\{y_n\}$ in X satisfies

$$||y_n - v_n|| \to 0 \quad (n \to \infty) \quad \text{and} \quad \sum_{n=1}^{\infty} ||y_{n+1} - f(T, v_n, y_n)|| < \infty.$$

Let ε_n be $||y_{n+1} - f(T, v_n, y_n)||$ for $n \geqq 1$, then

$$\begin{aligned} ||y_{n+1} - p|| &\leq \varepsilon_n + ||t_n T v_n + (1 - t_n) y_n + u_n - p|| \\ &\leq \varepsilon_n + t_n ||T v_n - p|| + (1 - t_n) ||y_n - p|| + ||u_n||. \end{aligned}$$

Since T has a bounded range, there exists M such that $\sup_n ||T v_n - p|| = M < \infty$. Thus the above inequality implies

$$||y_{n+1} - p|| \leq (1 - t_n) ||y_n - p|| + t_n M + (\varepsilon_n + ||u_n||).$$

Setting $a_n = ||y_n - p||$, $b_n = t_n M$, and $c_n = \varepsilon_n + ||u_n||$, Lemma 2 gives the boundedness of $\{y_n\}$. Let M_0 denote $\max\{M, \sup_n ||y_n - p||\}$. Let $y_{n+1}^* = f(T, v_n, y_n)$. From Lemma 5, putting $\gamma_n = \gamma(y_{n+1}^*, p)$, we have an estimate of $\left\|y_{n+1}^* - p\right\|$ as follows:

$$\begin{aligned} \left\|y_{n+1}^* - p\right\| \leq{}& \frac{1 + t_n(1 - \gamma_n)}{(1 + t_n)} ||y_n - p|| + \frac{t_n^2(2 - \gamma_n)}{(1 + t_n)} ||y_n - T v_n|| \\ &+ \frac{t_n}{(1 + t_n)} \left\|T y_{n+1}^* - T v_n\right\| + \frac{1 + t_n(2 - \gamma_n)}{(1 + t_n)} ||u_n||. \end{aligned}$$

Using the inequality

$$\frac{1 + t_n(1 - \gamma_n)}{(1 + t_n)} \leq 1 - t_n \gamma_n + (t_n)^2,$$

we have

$$\begin{aligned} ||y_{n+1} - p|| &\leq \left\|y_{n+1}^* - p\right\| + \varepsilon_n \\ &\leq (1 - t_n \gamma_n) ||y_n - p|| + t_n \left\|T y_{n+1}^* - T v_n\right\| + (\varepsilon_n + 3 ||u_n||) \\ &\quad + t_n \left(t_n ||y_n - p|| + 2 t_n (||y_n - p|| + ||T v_n - p||)\right) \qquad (3.1) \\ &\leq (1 - t_n \gamma_n) ||y_n - p|| + (\varepsilon_n + 3 ||u_n||) \\ &\quad + t_n \left(\left\|T y_{n+1}^* - T v_n\right\| + 5 t_n M_0\right). \end{aligned}$$

Since

$$\left\|y_{n+1}^* - v_n\right\| \leq t_n ||T v_n - y_n|| + ||y_n - v_n|| + ||u_n||,$$

from our assumptions we have

$$\left\|y_{n+1}^* - v_n\right\| \to 0 \quad (n \to \infty).$$

The uniform continuity of T implies

$$\left\|T y_{n+1}^* - T v_n\right\| \to 0 \quad (n \to \infty).$$

Suppose $\liminf_{n \to \infty} \gamma_n = \gamma > 0$. For any $\varepsilon \in (0, \frac{\gamma}{2})$ and sufficiently large n, $\gamma - \varepsilon < \gamma_n$. Then the inequality (3.1) implies

$$\begin{aligned} ||y_{n+1} - p|| \leq{}& (1 - t_n(\gamma - \varepsilon)) ||y_n - p|| + (\varepsilon_n + 3 ||u_n||) \\ &+ t_n \left(\left\|T y_{n+1}^* - T v_n\right\| + 5 t_n M_0\right). \end{aligned}$$

By Lemma 1 we obtain $\lim_{n\to\infty} \|y_n - p\| = 0$.

On the other hand, if $\liminf \gamma_n = \gamma = 0$, then $\liminf \left\|y_{n+1}^* - p\right\| = 0$. Setting $b_n = \left\|y_{n+1}^* - p\right\|$, $f_1(t) = \phi(t)$, $f_2(t) = 1 + \phi(t) + t$, we can apply Lemma 3 to (3.1) and we have $\lim_{n\to\infty} \|y_n - p\| = 0$ (cf. [24]). This means that (I) hold.

(II) Let $w_n \to p$, then

$$\|w_{n+1} - f(T, v_n, w_n)\| = \|w_{n+1} - t_n T v_n - (1 - t_n) w_n - u_n\|$$

$$\leq \|w_{n+1} - p\| + t_n \|Tv_n - p\| + (1 - t_n) \|w_n - p\| + \|u_n\|$$

$$\to 0 \quad (n \to \infty),$$

by the conditions of the theorem. From this (II) follows. □

Let $T_1, T_2, ..., T_k$ be k selfmappings of an arbitrary Banach space X. We consider an iterative scheme $f(T_1, ..., T_k, \cdot) = f(\tau, \cdot)$ defined by (1.2) for a family of selfmappings $\tau = \{T_1, ..., T_k\}$ with k coefficient sequences $\{t_n^{(j)}\}$ in $[0, 1]$ and k error term sequences $\{u_n^{(j)}\}$ in X for $j = 1, 2, ..., k$. Suppose that a sequence $\{x_n\}$ is defined by $x_{n+1} = f(\tau, x_n)$ for $x_1 \in X$. Letting $F(\tau)$ denote a set of common fixed points of τ, we suppose that $x_n \to p \in F(\tau) \neq \emptyset$. Let $\{y_n\}$ be an arbitrary sequence in X. If

$$\lim_{n\to\infty} \|y_{n+1} - f(\tau, y_n)\| = 0 \quad \textit{implies} \quad y_n \to p,$$

then an iterative scheme $f(\tau, \cdot)$ is said to be τ-stable or stable with respect to τ. We say that an iterative scheme $f(\tau, \cdot)$ is almost τ-stable or almost stable with respect to τ if

$$\sum_{n=1}^{\infty} \|y_{n+1} - f(\tau, y_n)\| < \infty \quad \textit{implies} \quad y_n \to p.$$

By applying Theorem 1 under assumption that T_1 is ϕ-strongly pseudocontractive, we obtain Theorem 2 which proves strong convergence and almost stability of the iterative scheme $f(\tau, \cdot)$ defined by (1.2). This theorem generalizes Stević's result in [24]. Its proof is obtained from Theorem 1 by putting

$$v_n = t_n^{(2)} T_2 x_n^{(2)} + (1 - t_n^{(2)}) x_n + u_n^{(2)},$$

because Lemma 2 gives the boundedness of $\{x_n\}$ and we have $\lim_{n\to\infty} \|v_n - x_n\| = 0$.

Theorem 2. [26] *Let X be an arbitrary Banach space and let $T_1, ..., T_k$ be selfmappings of X ($k \geq 2$). Let $f(\tau, \cdot)$ be the iterative scheme defined by (1.2) for $\tau = \{T_1, T_2, ..., T_k\}$. Suppose T_1 is a uniformly continuous ϕ -strong pseudocontraction with a bounded range, and that T_2 has a bounded range. Suppose $F(\tau) \neq \emptyset$ and that the following conditions hold :*

$$\begin{aligned}
&(l) \quad \textstyle\sum_{n=1}^{\infty} \left\|u_n^{(1)}\right\| < \infty, \\
&(m) \quad \lim_{n\to\infty} \left\|u_n^{(2)}\right\| = 0, \\
&(n) \quad \textstyle\sum_{n=1}^{\infty} t_n^{(1)} = \infty, \\
&(q) \quad \lim_{n\to\infty} t_n^{(j)} = 0, \quad \textit{for } j = 1, 2.
\end{aligned}$$

Then the following statements hold.
(I) The sequence $\{x_n\}$ defined by (1.2) converges strongly to $p \in F(\tau)$.
(II) The iterative scheme $f(\tau, \cdot)$ is almost τ-stable.
(III) For any sequence $\{y_n\}$ satisfying that $y_n \to p \in F(\tau)$,

$$\lim_{n\to\infty} ||y_{n+1} - f(\tau, y_n)|| = 0.$$

4. Convergence rate estimates

Let Φ be the set of all strictly increasing functions $f : [0,\infty) \to [0,\infty)$ with $f(0) = 0$. Let C be a bounded closed and convex subset of an arbitrary Banach space X and set $\delta(C) = M$, where $\delta(C)$ is the diameter of C. Consider $\phi \in \Phi$ such that the function ψ defined by

$$\psi(t) = \begin{cases} \frac{\phi(t)}{t} & if \quad t \in (0, M], \\ 0 & if \quad t = 0 \end{cases}$$

is increasing and $0 \le \psi(t) \le 1$ for all $t \in [0, M]$. Such a ϕ is said to be a P-function on $[0, M]$. For example,

$$\phi(t) = \left(\frac{t}{M}\right)^2 \quad and \quad \phi(t) = \frac{t}{1 + \log M - \log t}$$

are P-functions on $[0, M]$. We can prove the following lemma by using Kato's lemma (see [5]).

Lemma 6. [12] *Let C be a convex subset of an arbitrary Banach space X, and let $T : C \to C$ be a ϕ-strong pseudocontraction with $\phi \in \Phi$. Then for any $x, y \in C$ with $x \neq y$, the following inequality holds:*

$$||x - y + t\{(I - T - \gamma_{xy} I)x - (I - T - \gamma_{xy} I)y\}|| \ge ||x - y|| \quad (t > 0),$$

where $\gamma_{xy} = \frac{\phi(||x-y||)}{||x-y||} = \psi(||x - y||)$.

Using Lemma 6, we obtain the following important lemma.

Lemma 7. [12] *Let C be a bounded closed and convex subset of an arbitrary Banach space X with $\delta(C) = M$, and let ϕ be a P-function on $[0, M]$. Let $T : C \to C$ be a $\phi-$strongly pseudocontractive and Lipschitz continuous mapping with a Lipschitz constant L. Then for Mann iteration $\{x_n\}$ defined by (1.1), we obtain the following estimate: for a unique fixed point $p \in F(T)$,*

$$||x_{n+1} - p|| \le (1 - \gamma_n t_n + \tilde{L} t_n^2)\, ||x_n - p|| \qquad (n \ge 0),$$

where $\gamma_n = \psi(||x_{n+1} - p||)$ and $\tilde{L} = 3 + 3L + L^2$.

Let N be the set of all natural numbers, and let N_0 be $N \cup \{0\}$. For $\beta \in (0, 1)$ and $K \in N_0$, let $C(\beta^K, \tilde{L}) = (1 - \frac{1}{4\tilde{L}} \psi(M\beta^K)^2)$. It is trivial that $0 < C(\beta^K, \tilde{L}) < 1$. Since $\psi(t)$ is increasing, we also have that $C(\beta^K, \tilde{L}) \le C(\beta^{K+1}, \tilde{L})$. Define

$$m_K = \min\{m \in N : \ C(\beta^K, \tilde{L})^m \le \beta \ \},$$

and then define $n : N \to N$ by $n(0) = 0$ and $n(K) = n(K-1) + m_K$, i.e.,

$$n(K) = n(0) + \sum_{j=1}^{K} m_j = \sum_{j=1}^{K} m_j.$$

We have $0 = n(0) < n(1) < \cdots < n(K) < \cdots$. So, for each $n \in N_0$ we can find $K \in N_0$ with $n(K-1) \leq n < n(K)$. Then, define $t_n \in (0,1)$ by

$$t_n = \frac{1}{2\tilde{L}}\psi(M\beta^K).$$

Now, we can consider the Mann iteration $\{x_n\} \subset C$ with this coefficient sequence $\{t_n\}$ as follows :

$$\begin{cases} x_0 \in C, \\ x_{n+1} = t_n T x_n + (1 - t_n)x_n. \end{cases}$$

For such a sequence $\{n(K)\}_{K \in N_0}$ and the Mann iteration $\{x_n\} \subset C$, we obtain the following result.

Theorem 3. [12] *Let $\beta \in (0,1)$ be fixed. Then for the $\{n(K)\}_{K \geq 0}$ and $\{x_n\}$ defined above, we obtain the following estimate:*

$$\|x_n - p\| \leq M\beta^K, \qquad (\ n \geq n(K)\ ).$$

Next for a given β and each P-function ϕ, we give the convergence rate estimates for the Mann iteration involving $\phi-$strong pseudocontraction T.

Theorem 4. [12] *Under the assumption of the previous theorem, we obtain the following estimate of $n(K)$:*

$$n(K) \leq K(1 + \log\beta) - 8\tilde{L}(\log\beta)\sum_{j=1}^{K} \frac{1}{\psi(M\beta^j)^2}, \qquad (K \geq 1).$$

Moreover, for $\beta = \frac{1}{2}$, we have the following estimates: (1) if $\phi(t) = \left(\frac{t}{M}\right)^2$, i.e., $\psi(t) = \frac{t}{M^2}$, then we have

$$n(K) \leq K(1 - \log 2) + \frac{32\tilde{L}}{3}(\log 2)M^2 4^K, \qquad (K \geq 1).$$

(2) if $\phi(t) = \frac{t}{1+\log M - \log t}$, i.e., $\psi(t) = \frac{1}{1+\log M - \log t}$, then we have

$$\begin{aligned} n(K) \leq\ & (1 + (8\tilde{L} - 1)(\log 2))K + 8\tilde{L}(\log 2)^2 K(K+1) \\ & + \frac{4}{3}\tilde{L}(\log 2)^3 K(K+1)(2K+1), \qquad (K \geq 1). \end{aligned}$$

One can see that the Mann iteration which involves a ϕ-strongly pseudocontraction T with $\phi(t) = \frac{t}{1+\log M - \log t}$ converges strongly to $p \in F(T)$ faster than one with $\phi(t) = \left(\frac{t}{M}\right)^2$.

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DEPARTMENT OF MATHEMATICAL AND COMPUTING SCIENCES, TOKYO INSTITUTE OF TECHNOLOGY,, OHOKAYAMA,MEGURO-KU,TOKYO 152-8552, JAPAN

E-mail address: Hiroko.Tamura@is.itech.ac.jp