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Entropy representation of optimal logarithmic utility for insiders in Lévy markets

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1 Introduction

The problem of asymmetric markets in continuous time mathematical finance has been considered since Karazas and Pikovsky [3]. They regarded the insider's information as an enlargement of filtration. Corcuera et al. [2] considered insiders whose knowledge of asset price at maturity period is perturbed by an additional noise. Without such a noise, insider's optimal logarithmic utility up to maturity period is infinite. They showed that under some condition on the strength of the noise, the optimal logarithmic utility up to maturity period become finite. The markets considered above are driven by Brownian motion. Kohatsu and Yamazato [4] considered similar problem for markets driven by Lévy processes.

In this paper, we deal with the same setting as [4]. We show that optimal logarithmic utility is represented as a minimum of relative entropies of base probability measure w.r.t. equivalent martingale measures in a certain class. Kunita [5] considered such a problem for normal investor. Our result is its extension to insider model. In our case, some difficulties arize. Compensators are not deterministic and are not adapted to insider's filtration. By these reasons, integrability of the optimal portfolio is not clear and the class of equivalent martingale measures is not determined. This means that the martingale representation theorem is not known. Contrary, the martingale representation is known for noninsider (see Kunita [6]).

Amendinger et al. [1] showed that the insider's additional logarithmic utility at maturity period coincides with the entropy of the insider's extra knowledge G ($\mathcal{F}_T$-measurable random variable) for markets driven by Brownian motion. Here, $\mathcal{F}_T$ is a σ-algebra generated by an information up to maturity period T. We remark that this does not hold in our case (markets driven by Lévy processes).

2 Optimal portfolios for insiders in Lévy markets

In this section, we summarize a part of rsults in [4]. Let W_t be a 1-dimensional Brownian motion and let N be a Poisson random measure on $\mathbb{R}\times[0,T]$ with compensator $\overline{N}(dx,ds) = F(dx)ds$. We assume that $\int |x|\wedge|x|^2F(dx) < \infty$. Let $\widetilde{N} = N-\overline{N}$ be a maringale part of N. Let

$$Z_t = cW_t + \int_0^t\int_{|x|\le 1} x\widetilde{N}(dx,ds) + \int_0^t\int_{|x|>1} xN(dx,ds)$$

be a Lévy process and define a stock price S by

$$S_t = S_0\exp\left(\left(b-\frac{c^2}{2}\right)t + Z_t\right).$$

$\widehat{S}_t = e^{-rt}S_t$ be the discounted stock price. Let

$$Z'_t = cW'_t + \int_0^t\int_{|x|\le 1} x\widetilde{N}'(dx,ds) + \int_0^t\int_{|x|>1} xN'(dx,ds)$$

be another Lévy process independent of Z_t, where W' is a Brownian motion and N' is a Poisson random measure and $\widetilde{N}'$ is it's martingale part. We assume that the compensator is $F(dx)ds$, same as for N. The process Z' is considered as an additional noise added to the information of an insider. Let $\mathcal{F}_t = \sigma\{Z_s : s\le t\}$ and let

$$\mathcal{G}_t = \mathcal{F}_t \vee \sigma\{Z_T + Z'(g(T-s)) : s\le t\}.$$

Let π_s be an insider's portfolio, i.e. $(\mathcal{G}_s)$-predictable process. Discounted wealth process $\widehat{V}_t$ satisfies

$$\widehat{V}_t = V_0 + \int_0^t \frac{\pi_{s-}\widehat{V}_{s-}}{\widehat{S}_{s-}}d\widehat{S}_s.$$

Let

$$F_s(dx) = \frac{\int_s^T N(dx,du) + \int_0^{g(T-s)} N'(dx,du)}{T-s+g(T-s)},$$

$$\beta(s) = \frac{W(T)-W(s)+W'(g(T-s))}{T-s+g(T-s)}.$$

By [4], we have that

$$Z_t - \int_0^t \frac{Z_T - Z_s + Z'(g(T-s))}{T-s+g(T-s)}ds$$

is a $\mathcal{G}$-martingale. However,

$$M(dx,dt) = N(dx,dt) - F_t(dx)$$

is not $\mathcal{G}$-adapted. We consider a bigger filtration. Let

$$\mathcal{H}_t = \mathcal{F}_t \vee \sigma(W(T) + W'(g(T-s)); s \le t) \vee \sigma(F_s(dx)du; u \le t).$$

Then $M(dx,dt)$ is a $\mathcal{H}_t$-martingale random measure and $B(t) = W(t) - \int_0^t \beta(s)ds$ is a $\mathcal{H}$-Brownian motion. Discounted stock price satisfies the following equation :

$$\begin{aligned}
\widehat{S}_t &= S_0 + \int_0^t (b - r + c\beta(s))\widehat{S}_{s-}ds + c\int_0^t \widehat{S}_{s-}dB(s) \\
&\quad + \int_0^t \int_{|x|\le 1} (e^x - 1)\,\widehat{S}_{s-}M(dx,ds) + \int_0^t \int_{|x|>1} (e^x - 1)\,\widehat{S}_{s-}N(dx,ds) \\
&\quad + \int_0^t \int_{|x|\le 1} (e^x - 1 - x)\,\widehat{S}_{s-}F_s(dx)ds \\
&\quad + \int_0^t \int_{|x|\le 1} x\widehat{S}_{s-}\,(F_s(dx) - F(dx))\,ds.
\end{aligned}$$

Hence the wealth equation is

$$\begin{aligned}
\widehat{V}_t &= V_0 + \int_0^t (b - r + c\beta(s))\pi_{s-}\widehat{V}_{s-}ds + c\int_0^t \pi_{s-}\widehat{V}_{s-}dB(s) \\
&\quad + \int_0^t \int_{|x|\le 1} (e^x - 1)\,\pi_{s-}\widehat{V}_{s-}M(dx,ds) \\
&\quad + \int_0^t \int_{|x|>1} (e^x - 1)\,\pi_{s-}\widehat{V}_{s-}N(dx,ds) \\
&\quad + \int_0^t \int_{|x|\le 1} (e^x - 1 - x)\,\pi_{s-}\widehat{V}_{s-}F_s(dx)ds \\
&\quad + \int_0^t \int_{|x|\le 1} x\pi_{s-}\widehat{V}_{s-}\,(F_s(dx) - F(dx))\,ds = V_0 + \int_0^t \widehat{V}_{s-}d\widehat{R}_t
\end{aligned}$$

where

$$\begin{aligned}
&\widehat{R}_t \\
&= \int_0^t (b - r + c\beta(s))\pi_{s-}ds + c\int_0^t \pi_{s-}dB(s) \\
&\quad + \int_0^t \int_{|x|\le 1} (e^x - 1)\,\pi_{s-}M(dx,ds) + \int_0^t \int_{|x|>1} (e^x - 1)\,\pi_{s-}N(dx,ds) \\
&\quad + \int_0^t \int_{|x|\le 1} (e^x - 1 - x)\,\pi_{s-}F_s(dx)ds + \int_0^t \int_{|x|\le 1} x\pi_{s-}\,(F_s(dx) - F(dx))\,ds.
\end{aligned}$$

The unique solution is the Doleans-Dade exponential of $\widehat{R}_t$. i.e.,

$$\begin{aligned}\widehat{V}_t &= V_0\mathcal{E}(\widehat{R})_t \\ &= \exp(\widehat{R}_t - \widehat{R}_0 - \frac{1}{2}\langle \widehat{R}^c\rangle_t)\prod_{s\leq t}(1+\Delta\widehat{R}_s)e^{-\Delta\widehat{R}_s},\end{aligned}$$

We say that a portfolio π is admissible ($\pi \in \mathcal{A}$) if π is self financed, $\mathcal{G}$-predictable, $V_t^\pi > 0$,

$$\begin{aligned}&E(\int_0^t |\pi_s|^2 ds) < \infty, \\ &E(\int_0^t \int_{|x|\leq 1} |\pi_s x|^2 F_s(dx)ds) < \infty, \\ &E\left(\int_0^t \int \{\log(1+(e^x-1)\pi_s)\}^2 \mathbf{1}(-1 < (e^x-1)\pi_s < -\frac{1}{2})F_s(dx)ds\right) < \infty, \\ &E\left(\int_0^t \int_{|x|>1} |\log(1+(e^x-1)\pi_s)| F_s(dx)ds\right) < \infty\end{aligned}$$

for all $t < T$.

Using Ito's formula wealth process can be written as : $\widehat{V}_t = V_0\exp(R_t)$, where

$$\begin{aligned}R_t &= \int_0^t \left((b-r+c\beta(s))\pi_{s-} - \frac{c^2}{2}\pi_{s-}^2\right)ds + c\int_0^t \pi_{s-}dB(s) \\ &+ \int_0^t \int_{|x|\leq 1} x\pi_{s-}\left(F_s(dx) - F(dx)\right)ds \\ &+ \int_0^t \int_{|x|\leq 1} \log\left(1+(e^x-1)\pi_{s-}\right)M(dx,ds) \\ &+ \int_0^t \int_{|x|>1} \log\left(1+(e^x-1)\pi_{s-}\right)N(dx,ds) \\ &+ \int_0^t \int_{|x|\leq 1} \left(\log(1+(e^x-1)\pi_{s-}) - x\pi_{s-}\right)F_s(dx)ds.\end{aligned}$$

We set $V_0 = 1$. Under the assumption $\pi_s \in \mathcal{A}$, logarithmic utility $u(t,\pi) = E(\log(V_t)) = E(R_t)$ is finite for $t < T$. We want to maximize the logarithmic utility

$$\begin{aligned}u(t,\pi) &= \int_0^t \Big[E\left((b-r+c\beta(s))\pi_{s-} - \frac{c^2}{2}\pi_{s-}^2\right) \\ &+E\left(\int_{|x|\leq 1} x\pi_{s-}(F_s(dx) - F(dx))\right) \\ &+E\Big(\int_{\mathbb{R}} \{\log\left(1+(e^x-1)\pi_{s-}\right) - x1_{|x|\leq 1}(x)\pi_{s-}\}E(F_s(dx))\Big)\Big]ds.\end{aligned}$$

Since π_s is $\mathcal{G}$-predictable, we consider

$$\begin{aligned} f(y) &= (b-r+c\beta(s))y - \frac{c^2}{2}y^2 \\ &+y\int_{|x|\leq 1} xE(F_s(\omega,dx)-F(dx)|\mathcal{G}_s) \\ &+\int_{\mathbb{R}}\{\log\left(1+(e^x-1)\,y\right) - yx1_{|x|\leq 1}(x)\}E(F_s(dx)|\mathcal{G}_s). \end{aligned}$$

Then

$$f''(y) = -c^2 - \int \frac{(e^x-1)^2}{(1+(e^x-1)y)^2}E(F_s(dx)|\mathcal{G}_s) \leq 0.$$

Hence $f(y)$ is concave. The maximal point of $f(y)$ satisfies

$$\begin{aligned} (b-r+c\beta(s)) - c^2y + \int_{\mathbb{R}}\{\frac{e^x-1}{1+(e^x-1)y} - x1_{|x|\leq 1}\}E(F_s(dx)|\mathcal{G}_s) \\ +\int_{|x|\leq 1} xE(F_s(dx)-F(dx)|\mathcal{G}_s) = 0 \end{aligned} \tag{1}$$

This equation for noninsider is

$$(b-r) - c^2y + \int_{\mathbb{R}}\{\frac{e^x-1}{1+(e^x-1)\,y} - x1_{\{|x|\leq 1\}}(x)\}F(dx) = 0.$$

If $c \neq 0$ or, supp $\mathrm{F} \cap (-\infty,0) \neq \emptyset$ and supp $\mathrm{F} \cap (0,\infty) \neq \emptyset$, then the solution is unique. Obviously, we have

$$\max_{\pi\in\mathcal{G}} E(R(t)) \geq \max_{\pi\in\mathcal{F}} E(R(t)).$$

3 Entropy representation of optimal logarithmic utility for insider

We consider a characterization of the optimal utility as the minimum entropy of base probability measure w.r.t. equivalent martingale measures in a certain class.

Lemma 1 *$M_{\mathcal{G}}(dx,dt) = N(dx,dt) - E(F_t(dx)|\mathcal{G}_t)dt$ is a $\mathcal{G}$-martingale.*

Proof. For $A \in \mathcal{B}(\mathbb{R})$,

$$\begin{aligned}
& E(N(A,(0,t]) - \int_0^t E(F_u(A)|\mathcal{G}_u)du|\mathcal{G}_s) \\
= \; & N(A,(0,s]) - \int_0^s E(F_u(A)|\mathcal{G}_u)du \\
& + E\Big(N(A,(s,t]) - \int_s^t E(F_u(A)|\mathcal{G}_u)du\Big|\mathcal{G}_s\Big) \\
= \; & N(A,(0,s]) - \int_0^s E(F_u(A)|\mathcal{G}_u)du \\
& + E\Big[E\Big(N(A,(s,t]) - \int_s^t E(F_u(A)|\mathcal{G}_s)du\Big|\mathcal{H}_s\Big)\Big|\mathcal{G}_s\Big] \\
= \; & N(A,(0,s]) - \int_0^s E(F_u(A)|\mathcal{G}_u)du \\
& + E\Big[E\Big(N(A,(s,t]) - \int_s^t F_u(A)du\Big|\mathcal{H}_s\Big)\Big|\mathcal{G}_s\Big] \\
= \; & N(A,[0,s]) - \int_0^s E(F_u(A)|\mathcal{G}_u)du.
\end{aligned}$$

Hence, $M_{\mathcal{G}}(dx,dt) = N(dx,dt) - E(F_t(dx)|\mathcal{G}_t)dt$ is a $\mathcal{G}$-martingale. ∎

We denote by $\mathcal{P}$ the $\mathcal{G}$ -predictable σ-field. Let $\mathcal{L}$ be a set of pairs (f,g) of $\mathcal{G}$-predictable $f = f(s,\omega)$ and $\mathcal{B}(R) \times \mathcal{P}$-measurable $g = g(x,s,\omega)$ such that

$$\begin{aligned}
& \alpha_t(f,g) \\
= \; & \exp\Big[\int_0^t f(s)dB(s) - \frac{1}{2}\int_0^t f(s)^2 d\langle B\rangle_s + \int_0^t \int_{|x|>1} g(x,s)N(dx,ds) \\
& - \int_0^t \int_{|x|>1} (e^{g(x,s)} - 1)E(F_s(dx)|\mathcal{G}_s)ds + \int_0^t \int_{|x|\le 1} g(x,s)M_{\mathcal{G}}(dxds) \\
& - \int_0^t \int_{|x|\le 1} (e^{g(x,s)} - 1 - g(x,s))E(F_s(dx)|\mathcal{G}_s)ds\Big]
\end{aligned}$$

is well defined, $E(\alpha_T(f,g)) = 1$ and satisfies

$$\begin{aligned}
& b - r + c\beta(t) + cf(t) + \int_{|x|>1} e^{g(x,t)}(e^x - 1)E(F_t(dx)|\mathcal{G}_t) \\
& + \int_{|x|\le 1} xE(\{F_t(dx) - F(dx)\}|\mathcal{G}_t) \\
& + \int_{|x|\le 1} \{e^{g(x,s)}(e^x - 1) - x\}E(F_t(dx)|\mathcal{G}_t) = 0. \qquad (2)
\end{aligned}$$

Nonnegative process $\{\alpha_t\}$ satisfies

$$d\alpha_t(f,g) = \alpha_t(f,g)\left[f(t)dB(t) + \int (e^{g(x,t)} - 1)M_{\mathcal{G}}(dx,dt)\right].$$

This means that α is the stochastic exponential w.r.t.

$$\int_0^t f(s)dB(s) + \int_0^t \int (e^{g(x,s)} - 1)M_{\mathcal{G}}(dx, ds).$$

We denote $\mathcal{M} = \{Q : dQ = \alpha_T(f,g)dP,\ (f,g) \in \mathcal{L}\}$. By Theorem 3.7 in Lepingle and Mémin [7], we have that if

$$E\Big[\exp\Big(\int_0^T \int (e^{-g(x,s)} - 1 + g(x,s))N(dx,ds) + \frac{1}{2}\int_0^T f(s)^2 ds\Big)\Big] < \infty,$$

then $\alpha_t(f,g)$ is a $\mathcal{G}$-martingale.

Lemma 2 *If $(f,g) \in \mathcal{L}$, then $\widehat{S}_t$ is an $\alpha_t(f,g)P$- local martingale.*

Proof. First, recall that

$$\begin{aligned}
\widehat{S}_t &= S_0 \exp\Big((b - r - \frac{c^2}{2})t + cB_t + c\int_0^t \beta(s)ds \\
&\quad + \int_0^t \int_{|x|\le 1} x(N-F)(dx,ds) + \int_0^t \int_{|x|>1} xN(dx,ds)\Big). \\
&= S_0 \exp\Big((b - r - \frac{c^2}{2})t + cB_t + c\int_0^t \beta(s)ds + \int_0^t \int_{|x|\le 1} xM_{\mathcal{G}}(dx,ds) \\
&\quad + \int_0^t \int_{|x|\le 1} xE[(F_s - F)(dx)|\mathcal{G}_s]ds + \int_0^t \int_{|x|>1} xN(dx,ds)\Big).
\end{aligned}$$

Then

$$\begin{aligned}
\alpha_t\widehat{S}_t = \exp\Big[&\int_0^t (f(t) + c)dB(s) + \int_0^t (b - r - \frac{c^2}{2} + c\beta(s) - \frac{1}{2}f(s)^2)ds \\
&+ \int_0^t \int_{|x|\le 1} (x + g(x,s))M_{\mathcal{G}}(dx,ds) + \int_{|x|>1} (x + g(x,s))N(dx,ds) \\
&- \int_0^t \int_{|x\le 1} \{e^{g(x,s)} - 1 - g(x,s))\}E(F_s(dx)|\mathcal{G}_s)ds \\
&- \int_0^t \int_{|x|>1} (e^{g(x,s)} - 1)E(F_s(dx)|\mathcal{G}_s)ds + \int_0^t \int_{|x|\le 1} xE(\{F_s - F\}(dx)|\mathcal{G}_s)ds\Big]
\end{aligned}$$

satisfies

$$\begin{aligned}
& d(\alpha_t \widehat{S}_t) \\
= \; & \alpha_t \widehat{S}_t \Big\{ (f(t)+c)dB(t) + (b - r + c\beta(t) + cf(t))dt \\
& + \int_{|x|\le 1} (e^x e^{g(x,t)} - 1) M_{\mathcal{G}}(dx,dt) + \int_{|x|>1} (e^x e^{g(x,t)} - 1) N(dx,dt) \\
& + \int_{|x|\le 1} \{ e^{g(x,t)}(e^x - 1) - x \} E(F_t(dx)|\mathcal{G}_t) dt \\
& + \int_{|x|\le 1} x E(\{F_t - F\}(dx)|\mathcal{G}_t) dt + \int_{|x|>1} e^{g(x,t)}(e^x - 1) E(F_t(dx)|\mathcal{G}_t) dt \Big\}.
\end{aligned}$$

Since f and g satisfies the equation (2), $\widehat{S}_t$ is an $\alpha_t P$-local martingale. ∎

The relative entropy of P with respect to Q is defined by

$$H_t(P|Q) = \begin{cases} E\left[\log \frac{dP}{dQ}|_{\mathcal{G}_t}\right] & \text{if } P << G, \\ \infty & \text{otherwise} \end{cases}$$

Theorem 1 *Assume that* $\pi^* \in \mathcal{A}$. *If the maximal utility of the insider is finite and* $\{\alpha_t(f,g)\}$ *with* $f(t) = -c\pi_t^*$ *and* $g(x,t) = -\log(1 + \pi_t^*(e^x - 1))$ *is a* $\mathcal{G}$*-martingale, then*

$$u(t,\pi^*) = \inf\{H_t(P|Q) : Q \in \mathcal{M}\}.$$

Proof. Let $(f,g) \in \mathcal{L}$. Then $\widehat{S}_t$ is an $\alpha_t P$- local martingale. Hence a measure Q defined by $dQ = \alpha_T dP$ is an equivalent martingale measure. The relative entropy of P with respect to Q is given by

$$H_t(P|Q) = E[\log E(\frac{1}{\alpha_T}|\mathcal{G}_t)] = E[\log \frac{1}{\alpha_t}] = -E[\log \alpha_t].$$

We have

$$\begin{aligned}
& E[\log \alpha_t] \\
= \; & E\Big[\int_0^t f(s)dB(s) - \frac{1}{2}\int_0^t |f(s)|^2 d\langle B\rangle_s \\
& + \int_0^t \int_{|x|>1} g(x,s) N(dx,ds) - \int_0^t \int_{|x|>1} (e^{g(x,s)} - 1) E(F_s(dx)|\mathcal{G}_s) ds \\
& + \int_0^t \int_{|x|\le 1} g(x,s) M_{\mathcal{G}}(dxds) \\
& - \int_0^t \int_{|x|\le 1} (e^{g(x,s)} - 1 - g(x,s)) E(F_s(dx)|\mathcal{G}_s) ds \Big] \\
= \; & E\Big[-\frac{1}{2}\int_0^t |f(s)|^2 ds - \int_0^t \int (e^{g(x,s)} - 1 - g(x,s)) E(F_s(dx)|\mathcal{G}_s) ds \Big].
\end{aligned}$$

Hence

$$H_t(P|Q) = E[\frac{1}{2}\int_0^t |f(s)|^2 ds + \int_0^t \int (e^{g(x,s)} - 1 - g(x,s))E(F_s(dx)|\mathcal{G}_s)ds].$$

is a convex functional of f and g. We find

$$\inf_{Q\in\mathcal{M}}\{\frac{1}{2}\int_0^t |f(s)|^2 ds + \int_0^t \int (e^{g(x,s)} - 1 - g(x,s))E[F_s(dx)|\mathcal{G}_s]ds\}$$

under the constraint (2), using Lagrange multiplier. We show that the infimum is attained by $dQ^* = \alpha_T(f^*, g^*)dP$. i.e., $\inf_{Q\in\mathcal{M}} H_t(P|Q) = H(P|Q^*)$ where $f^* = -c\pi_t^*$, $g^* = -\log(\pi_t^*(e^x - 1) + 1)$. We minimize

$$\begin{aligned}
&\frac{1}{2}f(t)^2 + \int (e^{g(x,t)} - 1 - g(x,t))E[F_t(dx)|\mathcal{G}_t] \\
&+\lambda\Big[cf(t) + b - r + c\beta(t) + \int_{|x|>1} e^{g(x,t)}(e^x - 1)E[F_t(dx)|\mathcal{G}_t] \\
&+ \int_{|x|\le 1} x(E[F_t(dx)|\mathcal{G}_t] - F(dx)) \\
&+ \int_{|x|\le 1} \{e^{g(x,t)}(e^x - 1) - x\}E[F_t(dx)|\mathcal{G}_t]\Big]. \qquad (3)
\end{aligned}$$

We have

$$\begin{aligned}
\lim_{\epsilon\to 0}\frac{1}{\epsilon}\Big[&\frac{1}{2}(f(t) + \epsilon\psi(t))^2 + \lambda(cf(t) + \epsilon\psi(t)) \\
&-\frac{1}{2}(f(t) + \psi(t))^2 + \lambda(cf(t) + \psi(t))\Big] = 0
\end{aligned}$$

and

$$\begin{aligned}
&\lim_{\epsilon\to 0}\frac{1}{\epsilon}\Big[\int (\exp\{g(x,t) + \epsilon\phi(t,x)\} - 1 - g(x,t) - \epsilon\phi(t,x))E[F_t(dx)|\mathcal{G}_t] \\
&+\lambda\Big(\int_{|x|>1} \exp\{g(x,t) + \epsilon\phi(t,x)\}(e^x - 1)E[F_t(dx)|\mathcal{G}_t] \\
&\int_{|x|\le 1} \{\exp\{g(x,t) + \epsilon\phi(s,x)\}(e^x - 1) - x\}E[F_t(dx)|\mathcal{G}_t]\Big) \\
&- \int (\exp\{g(x,t) + \phi(t,x)\} - 1 - g(x,t) - \phi(s,x))E[F_t(dx)|\mathcal{G}_t] \\
&-\lambda\Big(\int_{|x|>1} \exp\{g(x,t) + \phi(t,x)\}(e^x - 1)E[F_t(dx)|\mathcal{G}_t] \\
&\int_{|x|\le 1} \{\exp\{g(x,t) + \phi(t,x)\}(e^x - 1) - x\}E[F_t(dx)|\mathcal{G}_t]\Big)\Big] = 0.
\end{aligned}$$

By the above equalities, we have

$$
\begin{aligned}
&f(t) + \lambda c = 0,\\
&\int (e^{g(x,t)} - 1)\phi(t,x)E[F_t(dx)|\mathcal{G}_t]\\
&+\lambda\Big[\int_{|x|>1} e^{g(x,t)}(e^x - 1)\phi(t,x)E[F_t(dx)|\mathcal{G}_t]\\
&+\int_{|x|\le 1}(e^{g(x,t)} - 1)\phi(t,x)E[F_t(dx)|\mathcal{G}_t]\Big] = 0
\end{aligned}
$$

for any ϕ satisfying some regularity properties. Hence

$$
\begin{cases} f(t) &= -c\lambda,\\ g(x,t) &= -\log(1+\lambda(e^x - 1)). \end{cases}
$$

Substituting above f and g into (2), we have

$$
\begin{aligned}
&b - r + c\beta(t) - c^2\lambda + \int_{|x|>1}\frac{e^x - 1}{1+\lambda(e^x-1)}E(F_t(dx)|\mathcal{G}_t)\\
&+\int_{|x|\le 1} x(E(F_t(dx)|\mathcal{G}_t) - F(dx))\\
&+\int_{|x|\le 1}\{\frac{e^x - 1}{1+\lambda(e^x-1)} - x\}E(F_t(dx)|\mathcal{G}_t) = 0.
\end{aligned}
$$

This is equation (1) for λ. The unique solution of the above equation is $\lambda = \pi_t^*$. Hence we have

$$
\begin{cases} f(t) &= -c\pi_t^*,\\ g(x,t) &= -\log(1+\pi_t^*(e^x - 1)). \end{cases}
$$

Since π^* is $\mathcal{G}$-predictable and the above f and g generate a martingale measure $Q^* \in \mathcal{M}$, we have

$$
\begin{aligned}
&\inf\{H_t(P|Q) : Q \in \mathcal{M}\}\\
=\;& E\Big(\inf_{Q\in\mathcal{M}}\Big\{\frac{1}{2}\int_0^t |f(s)|^2 ds + \int_0^t\int (e^{g(x,s)} - 1 - g(x,s))E[F_s(dx)|\mathcal{G}_s]ds)]\Big\}\Big)\\
=\;& E\Big[\frac{1}{2}c^2\int_0^t (\pi_s^*)^2 ds\\
&+\int_0^t\int\Big\{\frac{1}{1+\pi_s^*(e^x-1)} - 1 + \log(1+\pi_s^*(e^x-1))\Big\}E(F_s(dx)|\mathcal{G}_s)ds\Big]\\
=\;& \int_0^t E\Big[-\frac{1}{2}c^2(\pi_s^*)^2 + (b - r + c\beta(s))\pi_s^* + \int_{|x|\le 1} x\pi_s^*(E(F_s(dx)|\mathcal{G}_s) - F(dx))\\
&+\int\{\log(1+\pi_s^*(e^x-1)) - x\pi_s^* 1_{|x|\le 1}\}E(F_s(dx|\mathcal{G}_s)\Big]ds\\
=\;& \max_{\pi\in\mathcal{A}} E(R_t).
\end{aligned}
$$

■

Remark 1 *By [7], we have that if*

$$E\Big[\exp\Big\{\int_0^T\int\Big(\pi_t^*(e^x-1)+\log(1+\pi_t^*(e^x-1))\Big)N(dx,ds)+\int_0^T(c\pi_t^*)^2dt\Big\}\Big]<\infty,$$

then $\alpha_t(f^*,g^*)$ is a $\mathcal{G}$-martingale.

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