数理解析研究所講究録 1474 巻 2006 年 127-143

A geometrical similarity between diffusion of biological particles in mathematical biology and migration of human population in mathematical sociology

生物学的拡散現象と社会学的人口移動現象の幾何学的相似性について

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Introduction

Since 1990's, many free trade areas have been established and planned such as CEFTA ('93), AFTA ('93), NAFTA ('94), SADC ('94), ANCOM ('95), and FTAA (2005). Within such free trade areas, goods and services are traded freely, but geographic population mobility is strictly restricted. However, there is a new move to entirely abolish the restriction. For example, the EU has been moving toward an economic integration of 15 countries, and it plans to remove the restriction in the near future. If the restriction is abolished, then population will move to more desirable locations. Such a socioeconomic phenomenon is an example of *interregional migration*, and has been studied fully in various articles (see, e.g., Åndersson & Philipov (1982), Haag & Weidlich (1986), and Zhang (1989)). In particular it is known in quantitative sociodynamics that the interregional migration is described by a system of nonlinear differential equations, which is called *the master equation* (see, e.g., Helbing (1995), Weidlich (2000), and Weidlich & Haag (1983, 1988)).

In the present paper, we take a master-equation approach to the interregional migration. We consider that each agent of the model represents an individual. We assume that each agent relocates in a discrete bounded domain in order to obtain higher *utility*, where the utility denotes a quantity representing socioeconomic desirability. In a real world we often observe that the socioeconomic desirability depends on the population density. Hence, in the present paper we assume that the utility is a function of the density of agents.

In the real world each individual evaluates the socioeconomic desirability according to his/her own preference. However, preferences vary a great deal, and it is almost impossible to explicitly know the preference of each individual. Hence, the desirability needs to be evaluated statistically. Moreover, in a real world various unpredictable socioeconomic events occur stochastically, and have a large influence on each individual in evaluating the socioeconomic desirability. Hence, we need to assume that the utility changes stochastically, i.e., that the utility contains a *random variable.*

We assume that each agent relocates in a discrete domain, i.e., that the model has a discrete space variable. Moreover we assume that each agent moves discretely in time, i.e., that the model has a discrete time variable. We reasonably assume that *the least unit of discrete time variable* and *the least unit of discrete space variable* are sufficiently small in order that the agent-based model can describe the phenomenon accurately in space and time. Furthermore, the total number of agents needs to be sufficiently large, since a large number of individuals relocate in a real world. Therefore we will impose the following assumption on the stochastic agent-based model:

Assumption 1. (i) The total number of agents is sufficiently large.

(ii) The least unit of discrete time variable is sufficiently small.

(iii) The least unit of discrete space variable is sufficiently small.

In the present paper we will prove that if we describe the interregional migration in terms of the stochastic agent-based model, then the description is almost the same as that done by the discrete master equation in quantitative sociodynamics, i.e., that the agent-based model can accurately describe the interregional migration in almost the same way as the discrete master equation. Hence the agent-based approach to the interregional migration is very effective in almost the same way as the discrete master equation.

Remark 1. (i) The theory of interregional migration described by the discrete master equation is based upon macroscopic observations about the mobility of population, and has few microscopic foundation. However, our agent-based model is based on microscopic assumptions imposed on each agent. Hence the agent-based model is more suitable than the discrete master equation when we investigate the interregional migration with emphasis laid on a microscopic aspect of the interregional migration. In the present paper, we use the technical terms "microscopic" and "macroscopic" in the sense employed in *statistical mechanics* in the same way as Tabata et al. (2002b, Remark 1.1). See, e.g., Cercignani (1990) for these technical terms.

(ii) In Helbing (1995, (1.5a), (1.5b)) the discrete master equation is simply called *the master equation*. However, in the present paper we call the equation the discrete master equation, because we need to distinguish the discrete master equation from *the continuous master equation*, which is introduced in Remark 4.

(iii) In the real world, there exist relocation, growth, new entry, and exit of population. Indeed it is natural to consider these kinds of behavior, but it makes our model extremely complicated to take all of them into account. Hence, in the present paper, as a first step we consider only relocation of agents, and we take neither growth, new entry, nor exit into consideration.

(iv) The method of agent-based approach has one of its origins in computer science (see, e.g., Russell&Norvig (1995)), and now plays a very important role also in *computational social sciences* (see, e.g., Epstein&Axtell (1996) and Tesfatsion (2002)).

(v) Assumption 1 has its origin in statistical mechanics. For example, such an assumption is employed in obtaining the so-called *Boltzmann equation* (see, e.g., Bogoljubov (1962) and Lanford (1975)).

(vi) The discrete master equation has its origin in statistical mechanics. By making use of the method developed in the theory of nonlinear differential equations, we can obtain mathematical results on the discrete master equation (see Tabata et al. (1998, 1999, 2000, 2001a, 2001b, 2002a, 2002b, 2002c, 2002d)). In the present paper, we apply those mathematical results to the agent-based model. Hence the subject of the paper is an application of the methods in statistical mechanics, the theory of nonlinear differential equations, and computer science to quantitative sociodynamics.

The discrete variables

By D we denote a bounded domain in which the agents relocate. For simplicity, we assume that the domain D is a rectangle contained in the 2-dimensional Euclidean space, i.e., that

$$D \equiv [0,a)\times[0,b), \tag{1}$$

where a and b are sufficiently large positive constants. In a real world, it is impossible to *always* observe each individual. Moreover, each measurement of the density of individuals is performed statistically. Hence, we reasonably assume that we can discretely (in time) measure only the mean value (in space) of the density of agents in each small subset of D. This is the reason for making our model discrete in space and time.

In order to make our model discrete in space, we divide the domain D into small disjoint rectangles as follows:

$$D = \cup_{i,j=1,\dots,N}[(i-1)a/N, ia/N)\times[(j-1)b/N, jb/N), \tag{2}$$

where N is a sufficiently large natural number, and we regard the pair $(a/N,b/N)$ as the least unit of discrete space variable. If N is sufficiently large, then the agent-based model satisfies Assumption 1, (iii). We call these small rectangles *sections*. We number the sections from 1 to N^2, and we denote them by d_j, $j = 1,\dots,N^2$. We have

$$D = \cup_{j=1,\dots,N^2} d_j. \tag{3}$$

We introduce a quantity that represents the influence of one agent on the whole model, and we call the quantity *the size* of agents. The product of the total number of agents and the size of agents can be regarded as the total amount of influence of all the agents on the whole model. We assume that if the total number of agents becomes larger and larger, then the size of agents becomes smaller and smaller with the condition that the product is identically equal to a positive constant, because if the size of agents is identically equal to a positive constant independent of the total number of agents, then the total amount of influence of all the agents tends to infinity as the total number of agents tends to infinity. It follows from Assumption 1, (i), that the size of agents is sufficiently small. We denote by R the total number of agents contained in D. If R is sufficiently large, then the agent-based model satisfies Assumption 1, (i). We see that the size of agents has the form, γ/R, were γ is a positive constant. If $R \rightarrow +\infty$, then the total number of agents tends to infinity and the size of agents converges to 0 with the condition that the product of the total number of agents and the size of agents is identically equal to the positive constant γ (we can regard γ as the total amount of influence of all the agents on the whole model).

We assume that we can measure only the mean value of the density of agents in each section. Hence, by assuming that agents are distributed uniformly in each section, we define *the density of agents* $\mathbf{f} = \mathbf{f}(t,x)$, $(t,x) \in [0,+\infty)\times D$, as follows:

$$\mathbf{f} = \mathbf{f}(t,x) \equiv (\gamma/R)R_j(t)/(ab/N^2) \text{ if } x \in d_j,\ j = 1,\ldots,N^2, \tag{4}$$

where $R_j = R_j(t)$ denotes the number of agents located in d_j at time $t \geq 0$, $j = 1,\ldots,N^2$. Note that the factor ab/N^2 is equal to the area of each section.

As mentioned in the preceding section, we assume that each agent relocates stochastically. Hence, we regard $R_j = R_j(t)$, $j = 1,\ldots,N^2$, as random variables depending on the time variable $t \geq 0$. Hence $\mathbf{f} = \mathbf{f}(t,x)$ is a random variable depending on $(t,x) \in [0,+\infty)\times D$. We reasonably assume that our model contains the random variable $\mathbf{f} = \mathbf{f}(t,x)$ as an *endogenous variable*, since the purpose of the model is to describe the interregional migration.

We reasonably assume that there exists no error in counting the total number of agents contained in D, because even if each measurement of population is performed statistically, then the measurement of the total population can be regarded as sufficiently accurate in a real world. Hence, we treat the total number of agents R not as a random variable but as a positive constant. From Remark 1, (iii), we obtain *the conservation law of total number of agents*. Hence the following condition is imposed on the random variables, $R_j = R_j(t)$, $j = 1,\ldots,N^2$:

$$R = \Sigma_{j=1,\ldots,N^2} R_j(t), \text{ for each } t \geq 0. \tag{5}$$

In order to make our model discrete in time, we assume that each agent relocates discretely in time, i.e., we make the following assumption:

Assumption 2. (i) No agent relocates at the initial time $t = 0$. At each time t of the form,

$$t = n\Delta t,\ n \in \mathbb{N}, \tag{6}$$

agents can relocate, where we denote the set of all natural numbers by $\mathbb{N}$, and Δt is a sufficiently small positive constant representing the least unit of discrete time variable.

(ii) For each time interval of the form,

$$(n\Delta t,(n+1)\Delta t),\ n \in \mathbb{N} \cup \{0\}, \tag{7}$$

no agent relocates.

It does not follow from Assumption 2, (i), that each agent relocates at a time t of the form (6). There exists the possibility that some agents do not relocate at a time t of the form (6). Whether agents can move or not at the initial time $t = 0$, we can construct almost the same model. Hence, for simplicity we assume that no agent relocates at $t = 0$. If $\Delta t > 0$ is sufficiently small, then the model satisfies Assumption 1, (ii). From Assumption 2 and (4) we see that $\mathbf{f} = \mathbf{f}(t,x)$ is constant in each subset of the form,

$$(n\Delta t,(n+1)\Delta t) \times d_j, \tag{8}$$

where $j = 1,\ldots,N^2$, and $n \in \mathbb{N} \cup \{0\}$.

The stochastic agent-based model and the continuous model

Let us construct the agent-based model. The following socioeconomic and economic variables (u_i), $i = 1,\ldots,5$, are introduced in Weidlich&Haag (1988, (4.15-19), pp. 82-83) in order to consider the desirability: (u_1) *unemployment rate*, (u_2) *export structure index* (industrial minus agricultural export divided by the total export), (u_3) *overnight stays per capita*, (u_4) *percentage of total employment in the tertiary sector*, and (u_5) *population density*. In Weidlich&Haag (1988) and Helbing (1998), the variable (u_5) is treated as an endogenous variable. However, the relations between the other variables (u_1-u_4) are extremely complicated, and it is almost impossible to investigate the time evolution of the variables (u_1-u_4) within a simple framework. Hence the variables (u_1-u_4) are treated as *exogenous variables* in Weidlich&Haag (1988). In our agent-based model we introduce four exogenous variables representing (u_i), $i = 1,...,4$, respectively. Noting that the variable (u_i), $i = 1,...,4$, depend on the time variable and the space variable, we assume that the four exogenous variables depend on $(t,x) \in [0,+\infty)\times D$. We denote them by $g_i = g_i(t,x)$, $i = 1,...,4$, respectively. We impose the following assumption on $g_i = g_i(t,x)$, $i = 1,...,4$:

Assumption 3. (i) $g_i = g_i(t,x)$, $i = 1,...,4$, are given step functions which are constant in each subset of the form $[n\Delta t,(n+1)\Delta t)\times d_j$, $n \in \mathbb{N}\cup\{0\}$, $j = 1,...,N^2$.

(ii) $g_i = g_i(t,x)$, $i = 1,...,4$, are *uniformly bounded*, i.e., $\sup_{(t,x)\in[0,+\infty)\times D}|g_i(t,x)| < +\infty$, $i = 1,...,4$.

In Weidlich&Haag (1988, (4.19)) and Helbing (1998), a quantity is introduced in order to describe the desirability. The quantity is called *utility*. Also in our model we will employ a similar quantity in order to describe the desirability. We call the quantity *utility* also in our model; no confusion will arise. In our model we assume that the utility depends on $\mathbf{f} = \mathbf{f}(t,x)$ and the exogenous variable $g = g(t,x)$, where we define

$$g = g(t,x) \equiv (g_1(t,x),...,g_4(t,x)). \tag{9}$$

Recalling that the utility needs to contain a random variable (see Introduction), we make the following assumption:

Assumption 4. The utility has the following form:

$$\mathbf{U} = \mathbf{U}(t,j) \equiv U(\mathbf{f}(t,X_j),g(t,X_j)) + S(t,j), j = 1,...,N^2, \tag{10}$$

where we denote the utility of a section d_j at time t of the form $t = n\Delta t$, $n \in \mathbb{N}\cup\{0\}$, by $\mathbf{U} = \mathbf{U}(t,j)$, $j = 1,...,N^2$, $U = U(\mathbf{f},g)$ is a sufficiently smooth given function of $(\mathbf{f},g) \in [0,+\infty)\times\mathbb{R}^4$, and $S = S(t,j)$ is a nonnegative-valued random variable depending on $(t,j) \in \{t = n\Delta t;\ n \in \mathbb{N}\cup\{0\}\}\times\{1,...,N^2\}$ (see

Assumption 2, (i), for $\mathbb{N}$). We denote the center of d_j by X_j, $j = 1,\ldots,N^2$.

Remark 2. (i) The utility is employed not only in quantitative sociodynamics but also in economics. However, the definition of utility in the former is different from that in the latter. In the present paper we employ the definition given in quantitative sociodynamics.

(ii) The utility employed in Weidlich&Haag (1988, (4.19)) is a function of the exogenous variables (u_1-u_4), the endogenous variable (u_5), and *the remaining residuals of regression analysis.* The utility employed in our model is a given function of $g = g(t,x)$, $\mathbf{f} = \mathbf{f}(t,x)$, and the random variables $S = S(t,j)$, $j = 1,\ldots,N^2$. However, we do not afford a microscopic foundation to the given function.

(iii) We have no need to assume that $U = U(\mathbf{f},g)$ is uniformly bounded in $[0,+\infty)\times\mathbb{R}^4$.

(iv) The utility is defined at each time t of the form $t = n\Delta t$, $n\in\mathbb{N}\cup\{0\}$. However, we can extend the domain of the utility from the discrete set $\{t = n\Delta t;\ n \in \mathbb{N}\cup\{0\}\}\times\{1,\ldots,N^2\}$ to $[0,+\infty)\times\{1,\ldots,N^2\}$ by defining $\mathbf{U}(t,j) \equiv \mathbf{U}(n\Delta t,j)$ if $n\Delta t \leq t < (n+1)\Delta t$, $n \in \mathbb{N}\cup\{0\}$.

(v) Note that $g = g(t,x)$ and $\mathbf{f} = \mathbf{f}(t,x)$ are constant with respect to the space variable x in each section. By replacing $x\in d_i$ by X_i, $i = 1,\ldots,N^2$, we can regard the functions as those defined in $[0,+\infty)\times\mathbf{D}$, where we define

$$\mathbf{D} \equiv \{X_i;\ i = 1,\ldots,N^2\}. \tag{11}$$

In a real world, there is the possibility that the random variable $S = S(s,i)$ depends on the random variable $S = S(t,j)$ for some s, t, i, and j such that $(s,i) \neq (t,j)$. Furthermore, there is the possibility that the density function of $S = S(s,i)$ is not equal to that of $S = S(t,j)$ for some s, t, i, and j such that $(s,i) \neq (t,j)$. However, in the present paper, for simplicity we assume that the random variables $S = S(t,j)$, $t = n\Delta t$, $n \in \mathbb{N}\cup\{0\}$, $j = 1,\ldots,N^2$, are independent of each other, and that the density functions of $S = S(t,j)$, $t = n\Delta t$, $n \in \mathbb{N}\cup\{0\}$, $j = 1,\ldots,N^2$, are the same. We denote the density function by $\rho = \rho(S)$. We easily see that the agent-based model changes drastically depending on $\rho = \rho(S)$, i.e., that the density function $\rho = \rho(S)$ determines the model. We reasonably assume that

$$\rho(S) \to 0 \text{ as } S \to +\infty. \tag{12}$$

Noting that the random variables are nonnegative-valued, we will make the following simple assumption in order that $\rho = \rho(S)$ can satisfy (12):

Assumption 5. $\rho(S) \equiv \exp(-S)$ for each $S \geq 0$.

If an individual moves from one section to another in a real world, then he/she needs to bear *the cost of moving*, which increases with the distance between the sections. Hence we reasonably make the following assumption:

Assumption 6. If an agent moves from one section to another, then he/she needs to bear the cost of moving. The cost incurred in moving from a section d_j to a section d_i is equal to $C = C(|X_i - X_j|)$, for each $i,j = 1,\ldots,N^2$, where $C = C(r)$ is a sufficiently smooth increasing nonnegative-valued given function of $r \geq 0$ (see Assumption 4 for X_j, $j = 1,\ldots,N^2$).

We make the following assumption, which gives a microscopic foundation for the relocation of agents:

Assumption 7. (i) At each time t of the form (6) each agent decides whether or not to attempt to relocate, where the decision is made stochastically under the following condition:

$$\Pr\{\text{an agent determines to attempt to relocate}\} = k\Delta t, \tag{13}$$

where we denote the probability of an event E by $\Pr\{E\}$, and k is a positive constant such that $k\Delta t < 1$.

(ii) If an agent decides not to attempt to relocate at a time $t = n\Delta t$, $n \in \mathbb{N}$, then he/she stays in the same section in the time interval $[n\Delta t,(n+1)\Delta t)$, but if an agent determines to attempt to relocate at a time $t = n\Delta t$, $n \in \mathbb{N}$, then he/she chooses one section at random at the time t.

(iii) Assume that an agent contained in a section d_j determines to attempt to relocate and chooses a section d_i at a time $t = n\Delta t$, $n \in \mathbb{N}$, where i and j are such that $i,j \in \{1,\ldots,N^2\}$. The agent contained in d_j decides whether or not to move to the chosen section d_i by comparing the utility of d_j with that of d_i. If

$$\mathbf{U}(t,i) - \{\mathbf{U}(t,j) + C(|X_i - X_j|)\} > A, \tag{14}$$

where A is a certain positive-valued function, then the agent moves from d_j to d_i at the time t. If not, i.e., if

$$\mathbf{U}(t,i) - \{\mathbf{U}(t,j) + C(|X_i - X_j|)\} \leq A, \tag{15}$$

then the agent does not relocate to d_i at the time t, and stays in d_j in the time interval $[n\Delta t,(n+1)\Delta t)$.

By Assumption 7, (i), we see that there exists the possibility that an agent decides not to attempt to move at a time of the form (6). Moreover, it follows from Assumption 7, (ii), that even if an agent decides to attempt to move, then the agent needs to choose one section and to compare the utility of the chosen section and that of the section containing himself/herself. Hence, even if an agent attempts to move, then there is the possibility that the agent fails to move. In Assumption 7, (iii), we accept the case where $i = j$, i.e., we allow each agent to choose the section containing himself/herself; no contradiction will arise.

Let us discuss the condition (13). The following lemma will be proved in the last section:

Lemma 1. If we replace (13) by the following condition in Assumption 7, (i):

$$\Pr\{\text{an agent determines to attempt to relocate}\} = \kappa, \tag{16}$$

where κ is a positive constant such that $\kappa < 1$, then in each bounded time interval an infinite number of times each agent decides to attempt to move when $\Delta t \to 0+0$.

The phenomenon described in the lemma above is unnatural, since such an event does not occur in a real world. Hence we need to assume that the probability in the left-hand side of (16) becomes smaller and smaller as Δt becomes smaller and smaller. Hence we make not (16) but (13) in Assumption

7, (i).

Let us discuss the function A. Considering (14-15), we find that the left-hand sides of (14-15),

$$B = B(i,j) \equiv \mathbf{U}(t,i) - \{\mathbf{U}(t,j) + C(|X_i - X_j|)\}, \tag{17}$$

can be regarded as a *benefit* gained in moving from d_j to d_i, $i,j = 1,\dots,N^2$. It follows from Assumption 7, (iii), that if an agent contained in d_j chooses d_i, and if the benefit $B = B(i,j)$ is larger than (smaller than or equal to, respectively) A, then the agent moves (does not move, respectively) from d_j to d_i. Hence, we see that A is the *threshold* of benefit. If the threshold A is small (large, respectively), then the inequality (14) ((15), respectively) holds easily. Hence, the activity of agents depends on the threshold A in such a way that the activity becomes greater and greater (smaller and smaller, respectively) as the threshold A becomes lower and lower (higher and higher, respectively).

The following socioeconomic and economic variables (v_i), $i = 1,\dots,4$, are introduced in Weidlich&Haag (1988, (4.13-14), pp. 82-83) in order to describe the activity of population: (v_1) *total job vacancies*, (v_2) *total income*, (v_3) *investment structure index* (expansionary minus rationalizing investment divided by total investment), and (v_4) *total population*. However, the relations between the variables $(v_1$-$v_3)$ are extremely complicated, and it is almost impossible to investigate the time evolution of the variables $(v_1$-$v_3)$ within a simple framework. Therefore, the variables $(v_1$-$v_3)$ are regarded as exogenous variables in Weidlich&Haag (1988). Moreover, from Remark 1, (iii), we see that the total population is constant. Hence, in our agent-based model we introduce three exogenous variables which represent (v_i), $i = 1,2,3$. Noting that the variables $(v_1$-$v_3)$ depend only on the time variable t, we assume that the three exogenous variables depend only on $t \geq 0$. We denote them by $h_i = h_i(t)$, $i = 1,\dots,3$. We make the following assumption:

Assumption 8. (i) $h_j = h_j(t)$, $j = 1,\dots,3$, are known step functions which are constant in each subset of the form $[n\Delta t,(n+1)\Delta t)$, $n \in \mathbb{N} \cup \{0\}$.

(ii) $h_j = h_j(t)$, $j = 1,\dots,3$, are uniformly bounded, i.e., $\sup_{t\geq 0}|h_j(t)| < +\infty$, $j = 1,\dots,3$.

We reasonably make the activity of agents depend on $h_i = h_i(t)$, $i = 1,...,3$. In order to do so, we assume that the threshold A is a function of $h = h(t)$, where

$$h = h(t) \equiv (h_1(t),...,h_3(t)). \tag{18}$$

Hence we write

$$A = A(h(t)). \tag{19}$$

We impose the following assumption on $A = A(\cdot)$:

Assumption 9. (i) $A = A(z)$ is a sufficiently smooth function of $z \in \mathbb{R}^3$.

(ii) $A_- > \delta_+$, where we define

$$A_- \equiv \inf_{t\geq 0} A(h(t)), \tag{20}$$

$$\delta_+ \equiv \sup_{0\leq y\leq \gamma/(ab/N^2),(t,x)\in[0,+\infty]\times D} U(y,g(t,x))$$

$$-\inf_{0\leq y\leq \gamma/(ab/N^2),(t,x)\in[0,+\infty)\times D} U(y,g(t,x)). \tag{21}$$

Making use of Assumptions 3-4, Assumption 8, and Assumption 9, (i), we see that

$$0 \leq \delta_+, A_- < +\infty. \tag{22}$$

Broadly speaking, we can say that if the threshold A is sufficiently large in comparison with the range of the utility $U = U(y,g(t,x))$ when

$$0 \leq y \leq \gamma/(ab/N^2) \text{ and } (t,x)\in[0,+\infty)\times D, \tag{23}$$

then the inequality of Assumption 9, (ii), holds. In Proof of Lemma 4 we will explain the reason for restricting y not to $0 \leq y < +\infty$ but to $0 \leq y \leq \gamma/(ab/N^2)$ in (21). We have no need to assume that $A = A(z)$ is a uniformly bounded function of z. We will employ Assumption 9 in the last section.

On the basis of Assumptions 1-9, we can construct a stochastic agent-based model. We denote the model by **M**. We can consider that the behavior of each agent is sufficiently rational, and that the model **M** has a microscopic foundation for the relocation of agents. We see that the model **M** can describe the interregional migration.

Remark 3. (i) We assume that each agent can evaluate the variables $g = g(t,x)$ and $h = h(t)$. As mentioned above, we treat these variables as exogenous variables. In the present paper, we do not afford a microscopic foundation to these exogenous variables (see Remark 2, (ii)).

(ii) If we impose an assumption different from Assumption 5 on $\rho = \rho(S)$, e.g., if we assume that $\rho = \rho(S)$ is equal to a *normal distribution*, then we can construct a stochastic agent-based model which is very different from **M**. This subject will be fully discussed in another paper.

Let us construct a continuous model described by the discrete master equation, which is the following N^2-dimensional system of nonlinear ordinary differential equations:

$$df(t,X_i)/dt = -w(f(t,\cdot);t,X_i)f(t,X_i) + \Sigma_{j=1,\dots,N^2} W(f(t,\cdot);t;X_i|X_j)f(t,X_j)(ab/N^2),\ i = 1,\dots,N^2, \quad (24)$$

where $f = f(t,x)$ denotes the unknown function of $(t,x) \in [0,+\infty)\times\mathbf{D}$ (see (11) for **D**). The kernel $W = W(f(t,\cdot);t;x|y)$ and the coefficient $w = w(f(t,\cdot);t,x)$ are defined as follows:

$$W = W(f(t,\cdot);t;x|y) \equiv v(t)\exp\{U(f(t,x),g(t,x))-U(f(t,y),g(t,y))-C(|x-y|)\}, \quad (25)$$

$$w = w(f(t,\cdot);t,x) \equiv \Sigma_{j=1,\dots,N^2} W(f(t,\cdot);t;X_j|x)(ab/N^2), \quad (26)$$

where $v = v(t)$ is a function of $t \geq 0$ defined as follows (see (18-19) for $A = A(\cdot)$ and $h = h(t)$):

$$v = v(t) \equiv (k/2ab)\exp\{-A(h(t))\}. \quad (27)$$

See Assumption 4 for $U = U(\cdot,\cdot)$. See Assumption 6 for $C = C(\cdot)$. See Assumption 3 and (9) for $g = g(t,x)$. See (13) for k. Note that (24) and (26) contain the factor ab/N^2, which is equal to the area of each section. In Weidlich&Haag (1988) and Helbing (1998), $\nu = \nu(t)$ is called the *flexibility*, and is employed in order to describe the activity of population. The system of equations (24) is exactly the same as *the master equation* treated in Helbing (1995, (1.5a)) (recall Remark 1, (ii)). Consider the initial value problem for (24) whose initial function is equal to the initial density of agents of **M**, i.e., to $\mathbf{f} = \mathbf{f}(0,\mathrm{x})$. It will be proved in the last section that the initial value problem has a unique solution for each initial data in each time interval of the form $[0,T]$, $T > 0$, i.e., that the initial value problem has a unique *global* solution for each initial data. Therefore, we see that the discrete master equation can define a continuous model. We denote the model by M. It is known in quantitative sociodynamics that the model M can describe the interregional migration (see, e.g., Helbing (1995), Weidlich (2000), and Weidlich & Haag (1983, 1988)).

The main result

In the preceding section we construct the stochastic agent-based model $\mathbf{M}$ and the continuous model M. The main result of the present paper is as follows:

Main Result. If we describe the interregional migration in terms of the stochastic agent-based model $\mathbf{M}$, then the description is almost the same as that given by the continuous model M.

The following theorem will be proved in the mathematical level of rigor in the last section:

Theorem 1. If the total number of agents tends to infinity and if the least unit of discrete time variable converges to 0+0, then the stochastic agent-based model $\mathbf{M}$ converges to the continuous model M with a probability converging to 1.

Noting that the probability mentioned in Theorem 1 converges to 1 as the total number of agents tends to infinity and as the least unit of discrete time variable converges to 0+0, and combining Assumption 1 and Theorem 1, we obtain Main Result.

Remark 4. (i) It follows from Main Result that a description given by the model $\mathbf{M}$ is almost the same as that done by the model M. Therefore we see that the discrete master equation can play the same role in the theory of agent-based models as that played by *replicator dynamics* in *the theory of* *evolutionary games.*

(ii) In the present paper we make the total number of sections sufficiently large, but we let it be fixed. In Theorem 1 we do not make the total number of sections tend to infinity. However, if we investigate the behavior of the agent-based model **M** when the total number of sections becomes larger and larger, then we need to make the total number of sections tend to infinity. If we make the total number of sections tend to infinity in Theorem 1, then we can prove that the stochastic agent-based model **M** converges to a continuous model described by the following nonlinear integro-partial differential equation:

$$\partial f(t,x)/\partial t = -w(f(t,\cdot);t,x)f(t,x) + \int_{y\in D} W(f(t,\cdot);t;x|y)f(t,y)dy, \qquad (28)$$

where we denote the unknown function by $f = f(t,x)$. The coefficient $w = w(f(t,\cdot);t,x)$ and the kernel $W = W(f(t,\cdot);t;x|y)$ are the same as (25-26). The equation (28) is called *the continuous master equation*, and is exactly the same as the master equation treated in Helbing (1995, (1.5b)). However, this subject is too mathematical. Hence, we will discuss it in another paper. In the present paper we do not treat the case where the total number of sections tends to infinity.

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