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A Distinction among Stable Equilibria in a Noncooperative Game

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1. Introduction

In the field of noncooperative game the Nash equilibrium point (NEP) is widely known as a concept of solution (Nash [7]). This is a point at which all players maintain the balance by aiming to maximize his own expected payoff (the individual rationality). But the NEP has two problems as follows:

(1) When a NEP doesn't satisfy the social efficiency (the Pareto optimality) , it happens often that a player selects various strategies except the Nash equilibrium strategy (NES). For example , in the prisoner's dilemma game (PDG) the NEP is (silence , silence) , which doesn't satisfy the Pareto optimality , but Minas et al. [5] shows that in many experiments of the PDG the 43% subjects take the strategy "confession".

(2) When there are many NESs , since a player can't distinguish a NEP which he should follow , NEPs can't become a suitable guidance for selecting a strategy.

As a method for getting rid of the gap between a NEP and social efficiency , the metagame analysis is known , which considers the selection of strategy to be a result of multistage thinking (Howard [3]). We call a response function to an opponent player in the original game by a metagame strategy. We can consider a new game in which its pure strategy is a metagame strategy in the original game. The new game is called a metagame of the original game. Moreover we can consider a new metagame for the above-mentioned metagame and then an infinite tree of metagames can be formed. The result of the original game determined from the NEP of a metagame on any step in the tree is called a metagame equilibrium of the original game. We can often obtain a metagame equilibrium satisfying the Pareto optimality. In such a case we can get rid of the gap between the individual optimality and the social one. It is known that a necessary and sufficient condition for a result of the original game to be a metagame equilibrium is that if any player intends to deviate from the result , another player can punish him (Howard [4]). But if the payoff of the punishing player is smaller than the payoff before the punishment , the punishment is not efficient. In the metagame analysis we don't demand the efficiency of punishment.

The conflict analysis improves the metagame analysis at the following two points:

(i)We demand strictly the condition that a punishment is efficient.

(ii)We allow simultaneous changes of strategies by many players.

In the next section the conflict analysis is explained and a stable equilibrium point (SEP) is defined. The SEP is a point from which any group of players doesn't intend to deviate (Fraser et al. [1],[2]). It is known that a SEP is a metagame equilibrium. But if there are many SEPs , they can't also become a suitable guidance for selecting a strategy.

In this paper we consider the attitude of each player for the conflict situation as a motive for selecting a strategy and consider several motives. The prediction by a certain player P for a motive of each player is represented by his motive distribution. We calculate a probability for each SEP to realize and distinguish SEPs each other by a realization probability. We propose that player P should use a strategy indicated by the SEP having the maximum realization probability.

In Section 2 the conflict analysis and a stable equilibrium point are explained. In Section 3 we propose a method of calculating a realization probability of each SEP by motive distributions. In Section 4 we give a numerical example.

2. Conflict Analysis and Stable Equilibrium

In this section we explain the conflict analysis and a stable equilibrium point (SEP).

We consider a situation in which n players are involved with a conflict. Let N be the set of players, namely, $N=\{1,\ldots,n\}$. An atomic alternative which can be selected by a player is called his option. Let O_i be the set of options for player i. Furthermore we put $O=\{O_1,\ldots,O_n\}$. A feasible subset of the set O_i, that is, a feasible combination of several options for player i is called a strategy of player i. Let T_i be the set of strategy of player i. We put $T=\{T_1,\ldots,T_n\}$. A feasible combination of strategies selected by all players is called a result for the situation of decision making. Let U be the set of results. Then any $u(\in U)$ can be represented by $u=\{u_1,\ldots,u_n\}$ where $u_i\in T_i\,(i=1,\ldots,n)$. Paying attention to a specific subset $S(\subseteq N)$, we may write $u=\{u_s,u_{-s}\}$ where $u_s=\{u_i\}_{i\in S}$ and $u_{-s}=\{u_i\}_{i\in N\setminus S}$. Let $U_s(U_{-s})$ be the set of $u_s(u_{-s})$.

We consider a preference relation R_i on the set U by player i. That is to say, for any two results u, $u'(\in U)$, the relation uR_iu' implies that player i prefers the result u more than or equal to the result u'. When the preference relation of player i depends on his payoff, letting $p_i(u)$ be the payoff of player i from the result u, the relation uR_iu' implies $p_i(u)\ge p_i(u')$. We put $R=\{R_1,\ldots,R_n\}$.

Definition 1. A conflict situation means a tuple (N,O,T,U,R).

Definition 2. We consider any subset $S(\subseteq N)$ and any result $u=(u_s,u_{-s})\subseteq U$. For any $u_s'\in U_s$, we call (u_s',u_{-s}) the movement from the result u by the subset S. Let $M_s(u)$ be the set of movements.

$$M_s(u)=\{(u_s',u_{-s})\in U \mid u_s'\in U_s\}. \tag{1}$$

Let $m_s^+(u)$ be the set of results which guarantee more payoff than the result u to all players in the subset S, that is,

$$m_s^+(u)=\{u'\in U \mid p_i(u)<p_i(u') \quad for \quad any \quad i\in S\}. \tag{2}$$

Let $m_s^-(u)$ be the set of results which can't guarantee more payoff than the result u to at least one player in S, that is,

$$m_s^-(u)=\{u' \in U \mid p_i(u') \le p_i(u) \quad for \quad \text{some} \quad i \in S\}. \tag{3}$$

Definition 3. If a movement from a result $u(\in U)$ by a subset $S(\subseteq N)$ guarantees more payoff than the result u to all players in S , then the movement is called an improvement from the result u by the subset S. Let $M_s^+(u)$ be the set of improvements from u by S , that is,

$$M_s^+(u)=M_s(u)\cap m_s^+(u). \tag{4}$$

Definition 4. When an improvement $u'(\in M_s^+(u))$ is given, if there exists an improvement $u''(\in M_{s'}^+(u'))$ from u' by another subset $S'(\ne S)$, which gives at least one player in S a payoff less than or equal to a payoff from the result u , we call u'' a punishment for u' by S'. Let $\overline{M}_{s,s'}(u,u')$ be the set of punishments, that is,

$$\overline{M}_{s,s'}(u,u')=M_{S'}^+(u')\cap m_s^-(u). \tag{5}$$

Definition 5. A result $u(\in U)$ is a stable equilibrium point (SEP) if and only if either of the following two conditions is satisfied :

(1) There is no improvement from the result u by any subset $S(\subseteq N)$, that is,

$$M_s^+(u)=\phi \quad for \quad any \quad S(\subseteq N). \tag{6}$$

(2) Even if there is an improvement u' from the result u by the subset $S(\subseteq N)$, there exists a punishment for u' by some subset $S'(\ne S)$, that is,

$$\overline{M}_{s,s'}(u,u')\ne\phi \quad for \quad \text{some} \quad S'(\subseteq N). \tag{7}$$

All players are balanced at a SEP since any group of players has no incentive to deviate from the SEP.

3. Distinction among Stable Equilibria

When there are many SEPs, a player is at a loss which to aim. In this section we consider a distinction among stable equilibria from a viewpoint of a possibility of occurrence of each SEP. Several motives for selecting a strategy are considered. We represent the uncertainty for a motive of a player by a motive distribution, and obtain a probability for each SEP to realize. We consider that a player should follow the SEP having the maximum realization probability.

Let G be the original game,

$$G=[a_i(j_1,\dots,j_n) \mid j_k \in T_k(k=1,\dots,n), i=1,\dots,n] \tag{8}$$

where $a_i(j_1,\dots,j_n)$ is a payoff of player i given that player $1,\dots,n$ use pure strategies $j_1,\dots,j_n$ respectively. We consider l motives $m_1,\dots,m_l$ for a player to select a strategy and put $M=\{m_1,\dots,m_l\}$. Let $a_i^k(j_1,\dots,j_n)$ be a payoff of player i

following a motive m_k , which can be represented by the original game G , for example ,

m_1 : maximization of his own payoff (selfish motive)

$$a_i^1(j_1,\dots,j_n)=a_i(j_1,\dots,j_n)\longrightarrow\max_{j_i} \tag{9}$$

m_2 : maximization of the social payoff , that is , the expected payoff of all players (coexistence motive)

$$a_i^2(j_1,\dots,j_n)=\frac{1}{n}\sum_{j=1}^{n}a_j(j_1,\dots,j_n)\longrightarrow\max_{j_i} \tag{10}$$

m_3 : maximization of the expected payoff in the group $S(\subseteq N)$

$$a_i^3(j_1,\dots,j_n)=\frac{1}{\|S\|}\sum_{j\in S}a_j(j_1,\dots,j_n)\longrightarrow\max_{j_i} \tag{11}$$

where $\|S\|$ denotes the number of elements of a subset S . Player i following the motive m_3 makes self-sacrifices for the sake of the group S . The motive $m_1(m_2)$ is equivalent to the case of $S=\{i\}(S=N)$ in the motive m_3 .

m_4 : minimization of the expected payoff in the group $S(\subseteq N)$

$$a_i^4(j_1,\dots,j_n)=-\frac{1}{\|S\|}\sum_{j\in S}a_j(j_1,\dots,j_n)\longrightarrow\max_{j_i} \tag{12}$$

This is the case that player i is opposed to the group S .

m_5 : maximization of the difference between the payoff of the group S_1 and one of the group S_2 $(S_1\cap S_2=\phi)$

$$a_i^5(j_1,\dots,j_n)=\sum_{j\in S_1}a_j(j_1,\dots,j_n)-\sum_{j\in S_2}a_j(j_1,\dots,j_n)\longrightarrow\max_{j_i} \tag{13}$$

The case of $m_1,\dots,m_4$ are equivalent to special cases of the motive m_5 .

m_6 : maximization of the minimum payoff in the group S

$$a_i^6(j_1,\dots,j_n)=\min_{j\in S}a_j(j_1,\dots,j_n)\longrightarrow\max_{j_i} \tag{14}$$

The player following the motive m_6 comes to the aid of the least fortunate member of society S .

m_7 : maximization of a probabrility that player $i(\in S)$ takes the maximum payoff in a group S

$$a_i^7(j_1,\dots,j_n)=\begin{cases}1 & if \quad a_i(j_1,\dots,j_n)=\max_{j\in S}a_j(j_1,\dots,j_n)\\ 0 & otherwise\end{cases} \tag{15}$$

When player $1,\dots,n$ follow motives $s_1,\dots,s_n(\in M)$ respectively , we represent this game by $G(s_1,\dots,s_n)$, that is ,

$$G(s_1,\dots,s_n)=\left[a_i^{s_i}(j_1,\dots,j_n)(i=1,\dots,n)\mid s_k\in M,j_k\in T_k(k=1,\dots,n)\right]. \tag{16}$$

Let $\tilde{x}(s_1,\ldots,s_n)$ be a SEP of the game $G(s_1,\ldots,s_n)$, that is ,

$$\tilde{x}(s_1,\ldots,s_n)=\left\{\tilde{x}_1(s_1,\ldots,s_n),\ldots,\tilde{x}_n(s_1,\ldots,s_n)\right\} \tag{17}$$

where $\tilde{x}_i(s_1,\ldots,s_n)$ is the stable equilibrium strategy (SES) of player i. Let $E(s_1,\ldots,s_n)$ is the set of all SEPs of the game $G(s_1,\ldots,s_n)$ and E be the union of all $E(s_1,\ldots,s_n)$.

$$E=\bigcup_{(s_1,\ldots,s_n)}E(s_1,\ldots,s_n) \tag{18}$$

where the symbol $\bigcup_{(s_1,\ldots,s_n)}$ denotes the union over all combinations of motives of all players. Enumerating all elements of the set E , we put

$$E=\left\{x^1,\ldots,x^r\right\} \tag{19}$$

where r is the number of elements of the set E.

We consider a certain player $P(\in N)$. Let $\lambda^i=\left\langle\lambda^i{}_1,\ldots,\lambda^i{}_l\right\rangle$ be the motive distribution of player i which player P anticipates , where $\lambda^i{}_k$ is a probability that player P thinks that player i follows the motive m_k. We put

$$\alpha^P\left(x^k\right)=\sum_{(s_1,\ldots,s_n)\in M(k)}\lambda^1{}_{s_1}\bullet\cdots\bullet\lambda^n{}_{s_n} \tag{20}$$

where

$$M(k)=\left\{(s_1,\ldots,s_n)\in M^n \mid \tilde{x}(s_1,\ldots,s_n)=x^k\right\}. \tag{21}$$

That is to say , $\alpha^P\left(x^k\right)$ is a realization probability of the SEP x^k for player P. Then it is desirable that player P selects a strategy indicated by the SEP having the maximum realization probability , that is , if

$$\alpha^P\left(x^*\right)=\max_{x^k(\in M)}\alpha^P\left(x^k\right) , \tag{22}$$

then player P should aim at the SEP x^*.

4. Discussion

In order to explain the variety of strategy selected by a player we have considered the subjective feeling of a player for a conflict situation. We represent uncertainties about motives of other players for a strategy selection by motive distributions. We calculate a payoff of each player under his motive , formulate a new noncooperative game and obtain its stable equilibrium points (SEP) by carrying out the conflict analysis. Using the motive distributions of all players we calculate the realization probability of each SEP. We propose that a player should follow a strategy indicated by the SEP having the maximum realization probability. We can explain the diversity in strategy selection and distinguish SEPs each other considerably.

We have proposed another method to take into consideration subjective feelings of players. Namely we formulate a subjective game by motive distributions and follow its SEP. But by this method we can't always indicate a unique SEP which a player should aim at.

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