Magnetic anisotropy of the spin-ice compound Dy$_2$Ti$_2$O$_7$

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We report magnetization and ac susceptibility of single crystals of the spin-ice compound Dy$_2$Ti$_2$O$_7$. Saturated moments at 1.8 K along the characteristic axes [100] and [110] agree with the expected values for an effective ferromagnetic nearest-neighbor Ising pyrochlore with local (111) anisotropy, where each magnetic moment is constrained to obey the “ice rule.” At high enough magnetic fields along the [111] axis, the saturated moment exhibits a breaking of the ice rule; it agrees with the value expected for a three-in, one-out spin configuration. Assuming the realistic magnetic interaction between Dy ions given by the dipolar spin ice model, we completely reproduce the results at 2 K by Monte Carlo calculations. However, down to at least 60 mK, we have not found any experimental evidence of the long-range magnetic ordering predicted by this model to occur at around 180 mK. Instead, we confirm the spin freezing of the system below 0.5 K.

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I. INTRODUCTION

The observed absence of magnetic ordering in the Ising pyrochlore magnet Ho$_2$Ti$_2$O$_7$ had a strong impact on the present study of frustrated systems. This compound was known to have a ferromagnetic (FM) Curie-Weiss temperature $\theta_{\text{CW}} \sim 1.9$ K, and magnetic Ho$^{3+}$ ions which possess a strong Ising anisotropy along the local (111) axis. Previous to this discovery, it was expected that absence of ordering would take place in the pyrochlore lattice only with antiferromagnetic Heisenberg spins.$^{2,3}$

Harris $et$ $al$. showed the possibility of a macroscopically degenerate ground state in Ho$_2$Ti$_2$O$_7$ by explaining the multiplicity of the possible spin configurations. By analogy with proton disorder in ice $I_h$, the spin configuration with two spins pointing inward and two spins pointing outward for each tetrahedron corresponds to the “ice rule” for these compounds. Accordingly, Ho$_2$Ti$_2$O$_7$ and related compounds such as Dy$_2$Ti$_2$O$_7$, which also possess FM Ising spins on a pyrochlore lattice, were named “spin ices.”

The first naive theoretical model of the spin-ice materials placed Ising spins on a pyrochlore lattice, constrained to point along the local (111) direction, with only nearest-neighbor FM exchange interactions. However, it was soon deduced that the nearest-neighbor exchange interaction in these compounds is actually antiferromagnetic and that the effective nearest-neighbor interaction becomes ferromagnetic due to the long-range dipolar interaction arising from the large moments of the Ho$^{3+}$ and Dy$^{3+}$ ions. The physical model which correctly incorporated all of these interactions is called the dipolar spin ice model. Physical quantities (neutron-scattering cross sections and specific-heat curves) calculated from Monte Carlo simulations of this model are in excellent agreement with the corresponding experimental data.$^{8-10}$

Most experimental studies on the spin ices to date have been performed on polycrystalline samples.$^{1,7,9,11-13}$ Among these various experiments, the most dramatic and direct evidence for realization of the spin-ice state is the residual entropy equal to Pauling’s entropy: $\frac{1}{2} R \ln 3 = 1.68$ J/K/mol.$^4$ Ramirez $et$ $al.$ obtained an entropy very close to this value [$\sim 1.9(2)$ J/K$^2$mol/Dy] by integrating the experimental electronic specific heat $C_e/T$ of Dy$_2$Ti$_2$O$_7$ above 0.25 K.$^9$ This residual entropy can be removed by applying an external magnetic field, since the magnetic field can lift the macroscopic degeneracy by stabilizing a specific spin configuration with lower magnetic energy (Fig. 1), as proven by neutron-diffraction measurement on single crystals of Ho$_2$Ti$_2$O$_7$ (Ref. 10) and Dy$_2$Ti$_2$O$_7$ (Ref. 14).

Moreover, the electronic specific heat $C_e(T)/T$ of Dy$_2$Ti$_2$O$_7$ polycrystals exhibit three peaks at 0.34 K, 0.47 K, and 1.12 K independent of the magnitude of the magnetic...
field. Due to the freedom of the decoupled spins of Fig. 1(b), it was speculated by Ramirez et al. that these peaks are attributable to crystal grains with their [110] axis aligned parallel to the field.

The above experimental facts suggest that the spin-ice compounds have significant magnetic anisotropy, and therefore one must control the field direction carefully when studying their physical properties. In particular, the competition between the applied magnetic field and the spin-spin interactions that drive the ice rule obeying degenerate ground state is significantly affected by the crystal orientation in the field. However, to the best of our knowledge, to date there has been no experimental report detailing the behavior of this anisotropy using single crystals of Dy$_2$Ti$_2$O$_7$. In this paper, we report such measurements and compare our results with the results predicted by the dipolar spin ice model. We show experimental magnetization results at 1.8 K along three characteristic axes of this compound, namely, the [100], [110], and [111] axes, and the corresponding numerical results calculated by Monte Carlo simulations. As we will discuss, agreement between the two is excellent. In contrast, a similar set of measurements carried out by Cornelius and Gardner on Ho$_2$Ti$_2$O$_7$ failed to find the expected magnetic anisotropy associated with this material.

In addition, it has recently been theoretically predicted that for the dipolar spin ice model and, consequently, for Dy$_2$Ti$_2$O$_7$ and Ho$_2$Ti$_2$O$_7$, a first-order phase transition should occur at around 180 mK (=T_c) which fully releases Pauling’s ice entropy. Melko et al. predicted the transition by considering spin loop moves instead of single spin flips within their Monte Carlo simulation, as illustrated in Fig. 1 of Ref. 16. However, few explicit experimental searches for this transition have been reported. Hence, the second motivation for this paper is to investigate single crystals of Dy$_2$Ti$_2$O$_7$ using susceptibility measurements down to 60 mK, in order to search for the predicted magnetic ordering. It is hoped that this experimental and theoretical joint study on Dy$_2$Ti$_2$O$_7$ can help answer some current issues of debate, and help develop deeper understanding of the nature of the spin-ice materials.

**II. EXPERIMENTS AND CALCULATIONS**

Single crystals of Dy$_2$Ti$_2$O$_7$ were grown by the floating-zone method with an infrared furnace equipped with double-elliptical mirrors. Before the single-crystal growth we prepared a polycrystalline feed rod by thoroughly mixing Dy$_2$O$_3$ (99.99% ) and TiO$_2$ (99.99%) powders with the molar ratio of 1 to 2 and sintering the mixture in an alumina crucible at 1000 °C for 100 h in air. The typical growth conditions are 29.0 mm/h for the feed speed and 30.0 mm/h for the crystal-growth speed. To avoid oxygen deficiency, we grew the single crystals in 0.3 MPa of O$_2$ atmosphere. In addition, we annealed the as-grown crystals at 800 °C for 100 h in O$_2$ gas flow. The color of the annealed crystals is homogeneously translucent yellow, while that of the as-grown crystals is lighter yellow with some inhomogeneous color distribution. The sizes of samples used for magnetization and ac susceptibility measurements are 0.7×0.7×2.0 mm$^3$ and 3.0×1.0×0.2 mm$^3$, respectively. By performing powder x-ray-diffraction measurements on the annealed crystals, we found that the scattering pattern was consistent with a cubic unit cell with a = 10.124(5) Å without any extra peaks. We determined the principal axes using Laue pictures.

We measured magnetization between 1.8 and 40 K for fields up to 5 T with a superconducting quantum interference device magnetometer. We measured ac susceptibility by a mutual-inductance method down to 60 mK using a dilution refrigerator.

For magnetization measurements, we applied the magnetic field in the plane perpendicular to the longest side of the crystal, because all of the principal axes are contained within this plane. The field misalignment is estimated to be within a few degrees. Thus, we estimate the demagnetization factor N as 0.4 for this crystal. The demagnetization effect can be neglected for ac susceptibility measurements, since the ac field was applied parallel to the longest side, which is parallel to the [100] crystal axis.

In addition, we performed Monte Carlo simulations for magnetization at finite temperatures on the dipolar spin ice model. The detailed calculation techniques are described in Ref. 8. We adopted the following Hamiltonian:

$$H = -J \sum_{\langle i,j \rangle} S_i^{z_i} S_j^{z_j} - \sum_i H_{\text{eff}} \cdot S_i^{z_i} + D r^3 \sum_{j \neq i} \left[ \frac{S_i^{z_i} S_j^{z_j}}{|r_{ij}|^3} - \frac{3(S_i^{z_i} \cdot r_{ij})(S_j^{z_j} \cdot r_{ij})}{|r_{ij}|^5} \right].$$

Here $S_i^{z_i}$ is the Ising moment vector of magnitude of unity at site $i$ pointing along the local Ising axis $z_i$. The quantity $H_{\text{eff}}$ is the effective internal magnetic field which directly couples to each spin. For the Hamiltonian coefficients, we used $-1.24$ K for the nearest-neighbor exchange interaction $J_{nn} = J/3$ and $2.35$ K for the nearest-neighbor dipolar interaction $D_{nn} = 5D/3$. We took into account the long-range nature of the dipolar interactions through the Ewald summation technique. A standard Metropolis algorithm was used with a conventional cubic unit cell for the pyrochlore lattice containing...
netic field along the
are coupled to minimize their energy with the applied mag-
completely holds. In this state, three of the spins per tetrahedra
represent Landé’s $g$ factor, $J$ corresponds to the total angular
moment of the Dy$^{3+}$ ion, and $\mu_B$ represents the Bohr mag-
free Dy$^{3+}$ ion, $g_J$ is expected to be $10\, \mu_B$/Dy. Slight deviation from the expected values may be
due to a misalignment effect as we discuss below.

Below 0.5 T, the magnetization along the $[111]$ axis al-
ready reaches the value $g_J[(1+1/3\times1)/4]=3.33\, \mu_B$/Dy
expected for the saturated state in which the ice rule com-
pletely holds. In this state, three of the spins per tetrahedra
are coupled to minimize their energy with the applied mag-
netic field along the $[111]$ direction, but one spin chooses to
oppose the field and minimize its energy with respect to the
effective nearest-neighbor interaction, remaining in the ice
rule. For quite large fields ($H_{\text{eff}}\gg J_{\text{nn}}+D_{\text{nn}}$), this configura-
tion begins to cost a large amount of energy, and the field-
unopposed spin flips to minimize its energy with respect to the
applied magnetic field, hence breaking the ice rule. Accord-
ingly, the magnetization agrees with the value expected for
the three-in, one-out spin configuration [Fig. 1(c)], $g_J[(1+
1/3\times3)/4]=5.00\, \mu_B$/Dy. We infer that the gain of Zeem-
man energy exceeds that of the effective nearest-neighbor
FM interaction above 0.5 T: $(1/3)g_J\mu_B(\mu_B H_{\text{eff}})=1.12\, \text{K}$
in good accordance with $J_{\text{eff}}=J_{\text{nn}}+D_{\text{nn}}=1.11\, \text{K}$
For fields up to 0.5 T, the moments along all the axes
nearly reach the expected values for the corresponding spin-
ice configurations in a magnetic field. This experimental fact
indicates that the magnetic anisotropy in Dy$_2$Ti$_2$O$_7$ is indeed
compatible with the spin-ice model. In addition, the break-
ing of the ice rule observed along the $[111]$ axis confirms that the
strong Ising anisotropy along the local $[111]$ direction domi-
nates in this compound.

A corresponding field dependence of magnetization is re-
ported by Cornelius and Gardner for single crystals of
H$_2$Ti$_2$O$_7$.\textsuperscript{15} However, their data above 2 T is totally unex-
pected, because the magnetizations along all the directions
reach approximately 5.9 $\mu_B$/Ho. Their behavior cannot be
explained solely by a crystal misalignment effect, since a
similar effect of misalignment must also exist to some extent
in our experiment, but clearly results in a different behavior
that than observed here and in Ref. 21.

In Fig. 3 we show magnetizations at 1 and 2 K calculated
by the Monte Carlo simulations. Each solid symbol repre-
ses the magnetization along the corresponding field direc-
tion of the experimental results. The open squares are for
the magnetization at 2 K along the $[110]$ axis with intentional
0.5° misalignment toward the $[100]$ axis. The open diamonds
are for the magnetization at 1 K along the $[111]$ axis.

Clearly the Monte Carlo simulations at 2 K reproduce our
corresponding experimental magnetization curves very well,
though the absolute values do not completely coincide with
each other.\textsuperscript{52} This excellent agreement between experiments
and calculations implies that one may treat this compound as

![Image](84x559 to 264x733)

**FIG. 2.** Magnetization curves at 1.8 K along the $[100]$, $[110]$, and $[111]$ axes. Solid lines correspond to the expected saturated moments for the two-in, two-out spin configuration for all the directions: dotted line corresponds to that for the three-in, one-out spin configuration.

16 spins. Simulations of the specific heat obtained using
these values are in very good agreement with the experi-
tental specific heat reported for Dy$_2$Ti$_2$O$_7$\textsuperscript{8} The simulation size
was $L=3$ (432 spins). When calculating the magnetization
curves, a number of different simulation sizes were tried, and
little variation resulted.

**III. RESULTS AND DISCUSSION**

A. Magnetic anisotropy

In Fig. 2 we summarize the magnetization curves at 1.8 K
along the three characteristic axes. We took the horizontal
axis as the effective magnetic field, $\mu_0 H_{\text{eff}}=\mu_0 H_{\text{expt}}
-N M_{\text{exp}}$. We obtained nearly the same data at 2 K (not
shown). We note that no hysteresis is observed between the
upward and downward field sweeps.

Saturated moments along the $[100]$ and $[110]$ axes agree
with the expected values for Ising anisotropy along the local
$[111]$ direction and for effective FM coupling between the
nearest-neighbor Dy ions. The value for the $[100]$ axis is
$g_J(1/3)=5.77\, \mu_B$/Dy [two spins point in and two spins
point out: Fig. 1(a)] and that for the $[110]$ axis is
$g_J((\sqrt{2}/3\times2)/4)=4.08\, \mu_B$/Dy [one spin points in, one
spin points out, and two spins are free: Fig. 1(b)]. Here, $g_J$
represents Landé’s $g$ factor, $J$ corresponds to the total angular
moment of the Dy$^{3+}$ ion, and $\mu_B$ represents the Bohr mag-
neton. For the free Dy$^{3+}$ ion, $g_J$ is expected to be
$10\, \mu_B$/Dy. Slight deviation from the expected values may be
due to a misalignment effect as we discuss below.

Below 0.5 T, the magnetization along the $[111]$ axis al-
ready reaches the value $g_J[(1+1/3\times1)/4]=3.33\, \mu_B$/Dy
expected for the saturated state in which the ice rule com-
pletely holds. In this state, three of the spins per tetrahedra
are coupled to minimize their energy with the applied mag-
netic field along the $[111]$ direction, but one spin chooses to
oppose the field and minimize its energy with respect to the
effective nearest-neighbor interaction, remaining in the ice

![Image](348x559 to 528x733)

**FIG. 3.** Magnetization curves calculated by Monte Carlo simu-
lations of the dipolar spin ice model. The simulation size was $L
=3$ (432 spins), and parameters were chosen for Dy$_2$Ti$_2$O$_7$.
an ideal model system for the study of geometrical frustration in the pyrochlore lattice. Moreover, it shows that the assumption of an Ising doublet with leading wave functions \(|\psi_+\rangle\) and \(|\psi_-\rangle\) with \(|m_1=\pm 15/2\rangle\) is a very good approximation, in good agreement with crystal-field calculations for Dy\(_2\)Ti\(_2\)O\(_7\) (Ref. 23) adapted from the crystal-field parameters extracted for Ho\(_2\)Ti\(_2\)O\(_7\).

The calculated magnetization with the field along the [110] axis with the intentional 0.5° misalignment deviates substantially from the aligned value at higher fields. This is due to the acute angular sensitivity specific to field application along the [110] axis, since slight deviation of the field from the [110] axis can fix the spin directions of the two free spins normal to the [110] axis in every tetrahedron [Fig. 1(b)]. The increase in magnetization is attributable to the field-coupled component of these spins still satisfying the ice rule. Therefore, the experimental deviation from the expected value, 4.08 \(\mu_B/\text{Dy}\), can be explained by assuming a crystal misalignment within 0.5° from the [110] axis.

The derivative of the experimental and calculated magnetizations at 1.8 K with \(\mu_0H_{\text{eff}}\) along the [111] axis is shown in Fig. 4. The derivative of the calculated magnetization, \(dM_{\text{cal}}/dH_{\text{eff}}\), also exhibits a local minimum at around 0.5 T and a broad peak at around 1 T. In addition, the behavior below 0.5 T nearly coincides with the experimental results. This good agreement between experiments and calculations indicates that the behavior of the dipolar spin ice model in the spin-interacting regime (i.e., at low fields) behaves almost the same as the real materials in the same regime. The difference between them is most likely due to the experimental misalignment effect, since the difference became smaller when we measured another single crystal by adjusting the alignment more accurately.  

\[ \chi(T) = \frac{N_A\mu_{\text{eff}}^2\mu_0^2/3k_B}{T-\theta_{\text{CW}}} \]  

where \(\chi=M/H\) is magnetic susceptibility, \(N_A\) is the Avogadro number, \(\mu_{\text{eff}}\) is the effective Bohr magneton, and \(k_B\) is the Boltzmann constant. Again, \(H\) has been corrected for the demagnetization factor.

For a given temperature, the ratio \(H/M\) increases with an increasing magnetic field. This is because the magnetization at higher fields saturates compared with that at lower fields especially at lower temperatures.

We obtained positive Curie-Weiss temperatures 1.16(5), 1.31(5), and 1.28(5) K from the data at 0.01 T along the [100], [110], and [111] axes, respectively. This is consistent with the existence of the effective nearest-neighbor FM interaction, mainly originating from large dipolar interaction between Dy ions, and consistent with the previous results using polycrystalline samples.  

The calculated broad peak becomes sharper at 1 K and correspondingly a clear plateau between 0.25 and 0.45 T emerges in the magnetization curve at 1 K. The plateau definitely suggests that an ice rule obeying spin configuration is more stable than the three-in, one-out spin configuration at lower temperatures and at lower fields. Such a plateau is also reported in Ho\(_2\)Ti\(_2\)O\(_7\) at 1.8 K between 0.4 and 1.6 T. In Ho\(_2\)Ti\(_2\)O\(_7\), the appearance of the plateau clearly corresponds to the temperature of the maximum in the specific heat. This characteristic temperature is 1.97 K in Ho\(_2\)Ti\(_2\)O\(_7\),  and corresponds to the onset of strong spin-ice correlations in the material. The corresponding characteristic temperature in Dy\(_2\)Ti\(_2\)O\(_7\) is 1.2 K. The results that a clear plateau appears at 1 K in our calculations but not above 1.8 K in our experiments as well as our calculations are reasonable compared with the characteristic temperature for Dy\(_2\)Ti\(_2\)O\(_7\).

In Fig. 5 we show the temperature dependence of reciprocal dc susceptibility for various magnetic fields along the [100] axis. The solid line denotes the Curie-Weiss fitting between 16 and 40 K for the data at 0.01 T.
Further validity in using the value for the effective FM interaction used in our calculations, $J_{\text{eff}} = J_{nn} + D_{nn} = 1.11$ K.\textsuperscript{26} We also obtained the effective Bohr magneton $10.2(1)$, $10.0(1)$, and $10.0(1)$ $\mu_B$/Dy for the [100], [110], and [111] axes, respectively, from the Curie-Weiss fitting. These values are nearly equal to the value $p_{\text{eff}} = g_J \sqrt{J(J+1)} = 10.6 \mu_B$/Dy expected for the completely localized spin of the Dy$^{3+}$ ion.

**B. Ac susceptibility and search for the 180 mK transition**

Considering the above excellent agreement between our experiments and calculations, we may expect the realization of the theoretically predicted first-order phase transition in the real material.\textsuperscript{16} In Fig. 6, we show the ac susceptibility $\chi'$ down to 60 mK. In order to keep the sample in zero dc field, we surrounded the sample coils with a high-permeability paramagnetic tube. An ac field of 0.05 mT was applied along the [100] axis. This amplitude satisfies the condition, $H_{ac}/D_{nn} \approx 2 \times 10^{-4} \ll 0.08 = T_1/D_{nn}$, suggested in Ref. 16. We measured $\chi'$ at 1 kHz on cooling and at 17 Hz on warming after keeping the crystal at 60 mK for 1 h. We normalized the data by dividing the raw data by the measuring frequency.

The in-phase component $\chi'$ measured at 17 Hz exhibits a broad peak at around 1.2 K and steep decrease below this temperature. In addition, it diminishes below about 0.5 K. $\chi'$ at 1 kHz and the out-of-phase, dissipative components $\chi''$ at both frequencies also exhibit the similar temperature dependence. These data are consistent with those obtained from polycrystalline Dy$_2$Ti$_2$O$_7$.\textsuperscript{17}

The absence of any change in the susceptibilities below 0.5 K indicates that the spins on each tetrahedron in the system become firmly locked in a random ice-rule configuration at about 0.5 K, below which the specific heat has also essentially vanished as shown in the previous study by Ramirez et al.\textsuperscript{9} Further, our data suggest that the predicted first-order phase transition does not occur, at least within the limited experimental measurement time scale. Because the “freeze-in” temperature is well above the predicted ordering temperature of 0.18 K, it is plausible that there is insufficient local spin dynamics left in the system that can promote the ordering associated with the nonlocal dynamics of a spin loop move in the Monte Carlo simulations of Ref. 16. We therefore speculate that the system has essentially completely fallen out of equilibrium at around 0.5 K; below this temperature one does not expect to recover equilibrium in a time that is reasonably accessible by experiment.

The temperatures of the maxima in $\chi'$ and $\chi''$ increase with increasing frequencies. The dependence is ascribable to a long magnetic relaxation time of the spin lock-in formation as Matsuhira et al. discussed in their previous polycrystalline study.\textsuperscript{17} This also supports our interpretation of the absence of ordering down to 60 mK. In further studies, it may be possible that slight dilution of Dy with nonmagnetic elements such as Y may bring the static ordering, in analogy to the transition in water ice from the $I_h$ phase to the long-range ordered crystal $I_{\text{XI}}$ phase via substitution with KOH.\textsuperscript{27}

**IV. CONCLUSION**

In conclusion, we reported magnetization and ac susceptibility of Dy$_2$Ti$_2$O$_7$ using single crystals. The clear magnetic anisotropy indicates the strong Ising anisotropy and the effective FM interaction between the nearest-neighbor Dy ions, as well as the dictating ice rule at low temperatures. These effects have been quantitatively confirmed by both experimental and theoretical approaches in the present study.

No indication of the predicted long-range magnetic ordering was observed. This may be due to differences in time scale between the dynamics of the real system and the simulation. Further investigations, such as specific-heat measurements, or dilution with nonmagnetic impurities, may clarify the attainability of the real ground state of Dy$_2$Ti$_2$O$_7$.

*Note added in proof.* Interestingly, it has recently been reported that the freezing process in Dy$_2$Ti$_2$O$_7$ may also display a signature in the 10–20 K temperature range.\textsuperscript{17,28}

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For a recent review on spin ice systems, see S.T. Bramwell and M.J.P. Gingras, Science 294, 1495 (2001).


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