Validity of a factorizable metric ansatz in string cosmology

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To support the validity of a factorizable metric ansatz used in string cosmology, we investigate a toy problem in the Randall-Sundrum I model. For this purpose, we revise the gradient expansion method to conform to the factorizable metric ansatz. By solving the five-dimensional equations of motion and substituting the results into the action, we obtain the four-dimensional effective action. It turns out that the resultant action is equivalent to that obtained by assuming the factorizable metric ansatz. Our analysis gives the support of the validity of the factorizable metric ansatz.

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I. INTRODUCTION

Results from the Wilkinson Microwave Anisotropy Probe strongly support the idea of the inflationary Universe. Hence, it is an urgent matter to construct an inflaton potential that agrees with observations based on a fundamental theory such as the string theory. Recent intense research on inflationary models in string theory have stemmed from the success in constructing brane inflation with moduli stabilization [1]. However, almost all studies computing potentials for moduli in type IIB string theory suppose a factorizable ansatz for the ten-dimensional metric:

$$ds^2 = e^{2\omega(y)} g_{\mu\nu}(x) dx^\mu dx^\nu + e^{-2\omega(y)+2u(x)} \tilde{g}_{ab}(y) dy^a dy^b, \tag{1}$$

where $\tilde{g}_{ab}(y)$ is the metric on the internal Calabi-Yau manifold, $\omega(y)$ is the warp factor, and $u(x)$ represents the volume modulus. This ansatz is based on the static solution [2]

$$ds^2 = e^{2\omega(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2\omega(y)} \tilde{g}_{ab}(y) dy^a dy^b. \tag{2}$$

It should be stressed that, in the presence of the moving branes, no proof for the factorizable ansatz exists even at low energy. If this ansatz is not correct, any conclusion derived using it is not reliable. In fact, recently, this ansatz was challenged by de Alwis [3]. As this factorizable ansatz for the metric is crucial in the discussion of the D-brane inflation, it is important to examine its validity.

In order to investigate the validity of this ansatz, as a modest step, we focus on the Randall-Sundrum I (RSI) model [4]. Here we also have a similar problem. In considering the cosmology, if we follow the factorizable metric ansatz, what we should do is to replace the Minkowski metric in a static solution

$$ds^2 = a^2(y) \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \qquad a(y) = e^{-y/\ell}, \tag{3}$$

with a spacetime dependent metric $g_{\mu\nu}(x)$ as

$$ds^2 = a^2(y) g_{\mu\nu}(x) dx^\mu dx^\nu + \mathcal{G}_{yy}(x) dy^2, \tag{4}$$

where we also included the modulus field $\mathcal{G}_{yy}$. The question is the validity of this assumption in the context of the brane cosmology. In this paper, we derive the four-dimensional low energy effective action on the brane without using this ansatz. Then we compare the result with the effective action derived using the factorizable ansatz to examine its validity.

Organization of this paper is as follows. In Sec. II, we present our strategy to attack the issue. In Sec. III, we solve the bulk equations of motion using the revised gradient expansion method. In the Sec. IV, we derive the four-dimensional effective action and discuss the validity of the factorizable metric ansatz. The final section is devoted to the conclusion.

II. HOW TO JUSTIFY THE METRIC ANSATZ?

We consider an S_1/Z_2 orbifold spacetime with the two branes as the fixed points. In the RSI model, the two flat 3-branes are embedded in the five-dimensional asymptotically anti-de Sitter bulk with the curvature radius ℓ with brane tensions given by $\sigma_+ = 6/(\kappa^2\ell)$ and $\sigma_- = -6/(\kappa^2\ell)$. The model is described by the action

$$\begin{aligned} S = {} & \frac{1}{2\kappa^2} \int d^5x \sqrt{-\mathcal{G}} \Big[\mathcal{R} + \frac{12}{\ell^2} \Big] - \frac{6}{\kappa^2 \ell} \int d^4x \sqrt{-g_+} \\ & + \frac{6}{\kappa^2 \ell} \int d^4x \sqrt{-g_-} + \frac{2}{\kappa^2} \int d^4x \sqrt{-g_+} \mathcal{K}_+ \\ & - \frac{2}{\kappa^2} \int d^4x \sqrt{-g_-} \mathcal{K}_-, \end{aligned} \tag{5}$$

where κ^2 is the five-dimensional gravitational coupling constant and $\mathcal{R}$ is the curvature scalar. We denoted the induced metric on the positive and negative tension branes by $g^+_{\mu\nu}$ and $g^-_{\mu\nu}$, respectively. In the last line, we have taken into account the Gibbons-Hawking boundary terms instead of introducing delta-function singularities in the curvature. The factor 2 in the Gibbons-Hawking term comes from the Z_2 symmetry of this spacetime. $\mathcal{K}_\pm$ is the trace part of the

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extrinsic curvature of the boundary near each one of the branes.

Here the question is how to obtain the effective action for discussing the cosmology. One often takes the metric ansatz and substitutes the ansatz into the action to get the four-dimensional effective action. Let us assume that the metric is factorizable

$$ds^2 = a^2(y)g_{\mu\nu}(x)dx^\mu dx^\nu + \mathcal{G}_{yy}(x)dy^2 \tag{6}$$

and the branes are located at the fixed coordinate points. Substituting this metric into the action and integrating out the result with respect to y, we obtain the four-dimensional action. However, an inadequate restriction of the functional space in the variational problem yields the wrong result. The correct procedure to obtain the four-dimensional effective action is first to solve the bulk equations of motion and substitute the results into the original action. To solve the bulk equations, we can employ the gradient expansion method [5–8]. Our analysis using the gradient expansion method shows that the correct metric takes the form [5]:

$$ds^2 = a^2\left[y\sqrt{\mathcal{G}_{yy}(x)}\right]g_{\mu\nu}(x)dx^\mu dx^\nu + \mathcal{G}_{yy}(x)dy^2, \tag{7}$$

which clearly rejects the factorizable ansatz (6).

However, there is another possibility. One can assume the following factorizable metric:

$$ds^2 = a^2(y)g_{\mu\nu}(x)dx^\mu dx^\nu + dy^2, \tag{8}$$

and the branes are moving in the above coordinates. Namely, the positive and negative tension branes are, respectively, placed at

$$y = \phi_+(x), \qquad y = \phi_-(x), \tag{9}$$

which are often referred to as the moduli fields. This is another description of the RSI cosmology and often called as moduli approximation in the literature [9]. Although two scalar fields are introduced, one is the extra degree of freedom as we will see later in Eq. (46). The physical quantity is the difference of these moduli fields which corresponds to the radion in our previous work [5]. This ansatz leads to the action [9]

$$S = \frac{\ell}{2\kappa^2}\int d^4x\sqrt{-g}\bigg[\{a^2(\phi_+) - a^2(\phi_-)\}R(g) + \frac{6}{\ell^2}\{a^2(\phi_+)(\partial\phi_+)^2 - a^2(\phi_-)(\partial\phi_-)^2\}\bigg]. \tag{10}$$

In this case, we do not have the result to be compared, because the gradient expansion method is not prepared for this parametrization of the model. Our aim is to examine the validity of this ansatz by conforming the gradient expansion method to the factorizable metric ansatz.

III. REVISED GRADIENT EXPANSION METHOD

The metric we take in solving the bulk equations of motion is the one in the Gaussian normal coordinate system

$$ds^2 = \gamma_{\mu\nu}(y, x)dx^\mu dx^\nu + dy^2, \tag{11}$$

where the factorized metric is not assumed.

Now we give the basic equations in the bulk. When solving the bulk equations of motion, it is convenient to define the extrinsic curvature on the $y =$ constant slicing as $K_{\mu\nu} = -\frac{1}{2}\frac{\partial}{\partial y}\gamma_{\mu\nu}$. Decomposing this extrinsic curvature into the traceless part and the trace part

$$K_{\mu\nu} = \Sigma_{\mu\nu} + \frac{1}{4}\gamma_{\mu\nu}K, \qquad K = -\frac{\partial}{\partial y}\log\sqrt{-\gamma}, \tag{12}$$

we obtain the basic equations which hold in the bulk:

$$\Sigma^\mu{}_{\nu,y} - K\Sigma^\mu{}_\nu = -\left[R^\mu{}_\nu(\gamma) - \frac{1}{4}\delta^\mu_\nu R(\gamma)\right], \tag{13}$$

$$\frac{3}{4}K^2 - \Sigma^\alpha{}_\beta\Sigma^\beta{}_\alpha = R(\gamma) + \frac{12}{\ell^2}, \tag{14}$$

$$\nabla_\lambda\Sigma_\mu{}^\lambda - \frac{3}{4}\nabla_\mu K = 0, \tag{15}$$

where ∇_μ denotes the covariant derivative with respect to the metric $\gamma_{\mu\nu}$, and $R^\mu{}_\nu(\gamma)$ is the corresponding curvature.

The effective action has to be derived by substituting the solution of Eqs. (13)–(15) into the action (5) and integrating out the result over the bulk coordinate y. In reality, it is difficult to perform this general procedure. However, what we need is the low energy effective theory. At low energy, the energy density of the matter, ρ, on a brane is smaller than the brane tension, i.e., $\rho/|\sigma| \ll 1$. In this regime, the four-dimensional curvature can be neglected compared with the extrinsic curvature. Thus, the anti-Newtonian or gradient expansion method used in the cosmological context [10] is applicable to our problem.

A. Zeroth order

At zeroth order, we can neglect the curvature term in Eqs. (13)–(15). Moreover, the tension term induces only the isotropic bending of the brane. Thus, an anisotropic term vanishes at this order, $\overset{(0)}{\Sigma}{}^\mu{}_\nu = 0$. As the result, we obtain

$$\overset{(0)}{K} = \frac{4}{\ell} \quad \text{or} \quad \overset{(0)}{K}{}^\mu{}_\nu = \frac{1}{\ell}\delta^\mu_\nu. \tag{16}$$

Using the definition of the extrinsic curvature

$$\overset{(0)}{K}_{\mu\nu} = -\frac{1}{2}\frac{\partial}{\partial y}\overset{(0)}{\gamma}_{\mu\nu}, \tag{17}$$

we get the zeroth order metric as

$$ds^2 = dy^2 + a^2(y)g_{\mu\nu}(x)dx^\mu dx^\nu, \quad a(y) = e^{-y/\ell}, \tag{18}$$

where the tensor $g_{\mu\nu}$ is the constant of integration which weakly depends on the brane coordinates x^μ.

B. First order

Our iteration scheme is to write the metric $\gamma_{\mu\nu}$ as a sum of local tensors built out of $g_{\mu\nu}$, with the number of derivatives increasing with the order of iteration, that is, $O[(\ell/L)^{2n}]$, $n = 0, 1, 2, \cdots$. Here L represents the characteristic length scale of the four-dimensional curvature. Hence, we seek the metric as a perturbative series

$$\gamma_{\mu\nu}(y, x) = a^2(y)[g_{\mu\nu}(x) + f_{\mu\nu}(y, x) + \cdots]. \tag{19}$$

The effective action can be constructed with the knowledge of the leading order metric $f_{\mu\nu}(y, x)$. Other quantities can also be expanded as

$$\begin{aligned} K^\mu{}_\nu &= \frac{1}{\ell}\delta^\mu_\nu + \overset{(1)}{K}{}^\mu{}_\nu + \overset{(2)}{K}{}^\mu{}_\nu + \cdots, \\ \Sigma^\mu{}_\nu &= 0 + \overset{(1)}{\Sigma}{}^\mu{}_\nu + \overset{(2)}{\Sigma}{}^\mu{}_\nu + \cdots. \end{aligned} \tag{20}$$

The first order solutions are obtained by taking into account the terms neglected at zeroth order. At first order, Eqs. (13)–(15) become

$$\overset{(1)}{\Sigma}{}^\mu{}_{\nu,y} - \frac{4}{\ell}\overset{(1)}{\Sigma}{}^\mu{}_\nu = -\left[R^\mu{}_\nu(\gamma) - \frac{1}{4}\delta^\mu_\nu R(\gamma)\right]^{(1)}, \tag{21}$$

$$\frac{6}{\ell}\overset{(1)}{K} = [R(\gamma)]^{(1)}, \tag{22}$$

$$\overset{(1)}{\Sigma}{}_\mu{}^\lambda{}_{|\lambda} - \frac{3}{4}\overset{(1)}{K}{}_{|\mu} = 0, \tag{23}$$

where the superscript (1) represents the order of the derivative expansion and | denotes the covariant derivative with respect to the metric $g_{\mu\nu}$. Here $[R^\mu{}_\nu(\gamma)]^{(1)}$ means that the curvature is approximated by taking the Ricci tensor of $a^2(y)g_{\mu\nu}(x)$ in place of $R^\mu{}_\nu(\gamma)$. It is also convenient to write it in terms of the Ricci tensor of $g_{\mu\nu}$, denoted by $R^\mu{}_\nu(g)$.

Substituting the zeroth order metric into $R(\gamma)$, we can write Eq. (22) as

$$\overset{(1)}{K} = \frac{\ell}{6a^2}R(g). \tag{24}$$

Hereafter, we omit the argument of the curvature for simplicity. Simple integration of Eq. (21) also gives the traceless part of the extrinsic curvature as

$$\overset{(1)}{\Sigma}{}^\mu{}_\nu = \frac{\ell}{2a^2}\left(R^\mu{}_\nu - \frac{1}{4}\delta^\mu_\nu R\right) + \frac{\chi^\mu{}_\nu(x)}{a^4}, \tag{25}$$

where $\chi^\mu{}_\nu$ is the constant of integration which satisfies

$$\chi^\mu{}_\mu = 0, \qquad \chi^\mu{}_{\nu|\mu} = 0. \tag{26}$$

Here the latter condition came from Eq. (23). In our previous work [5,6], we find this term corresponds to the dark radiation at this order. Because of the traceless property, $\chi^\mu{}_\nu$ is not relevant to the derivation of the effective action.

From Eqs. (24) and (25), the correction to the metric $g_{\mu\nu}$ at this order can be obtained as

$$f_{\mu\nu}(y, x^\mu) = -\frac{\ell^2}{2a^2}\left(R_{\mu\nu} - \frac{1}{6}g_{\mu\nu}R\right) - \frac{\ell}{2a^4}\chi_{\mu\nu} + C_{\mu\nu}(x), \tag{27}$$

where $C_{\mu\nu}$ is the constant of integration which will be fixed later in Eq. (45).

IV. EFFECTIVE ACTION

Now, up to the first order, we have

$$g_{\mu\nu}(y, x) = a^2(y)[g_{\mu\nu}(x) + f_{\mu\nu}(y, x)]. \tag{28}$$

In the following, we will calculate the bulk action $S_{\rm bulk}$, the actions for each brane $S_\pm$, and the Gibbons-Hawking term $S_{\rm GH}$ separately. After that, we collect all of them and obtain the four-dimensional effective action.

In order to calculate the bulk action, we need the determinant of the bulk metric

$$\begin{aligned} \sqrt{-\mathcal{G}} &= a^4(y)\sqrt{-g}\sqrt{1 - \frac{\ell}{6a^2}R + C^\mu{}_\mu} \\ &\approx a^4(y)\sqrt{-g}\left(1 - \frac{\ell}{12a^2}R\right)\left(1 + \frac{C^\mu{}_\mu}{2}\right), \end{aligned} \tag{29}$$

where we neglected the second order quantities. Then the bulk action becomes

$$\begin{aligned} S_{\rm bulk} &\equiv \frac{1}{2\kappa^2}\int d^5x\sqrt{-\mathcal{G}}\left[\mathcal{R} + \frac{12}{\ell^2}\right] \\ &= -\frac{8}{\kappa^2\ell^2}\int d^4x\sqrt{-g}\left[\frac{\ell}{4}\{a^4(\phi_+) - a^4(\phi_-)\}\right. \\ &\quad \left. - \frac{\ell^3}{24}\{a^2(\phi_+) - a^2(\phi_-)\}R\right]\left[1 + \frac{C^\mu{}_\mu}{2}\right], \end{aligned} \tag{30}$$

where we have used the equation $\mathcal{R} = -20/\ell^2$ which holds in the bulk. Notice that the Ricci scalar came from ${\rm tr}f_{\mu\nu}$ in $\sqrt{-\mathcal{G}}$.

Next, let us calculate the action for the brane tension. The induced metric on each brane is written by

$$g^{\pm}_{\mu\nu}(\phi_\pm, x) = a^2 g_{\mu\nu}(x) + a^2 f_{\mu\nu}(\phi_\pm, x) + \partial_\mu\phi_\pm\partial_\nu\phi_\pm. \tag{31}$$

The determinant of the induced metric can be calculated as

$$\sqrt{-g_\pm} = a^4(\phi_+)\sqrt{-g}\sqrt{1 + \frac{1}{a^2}(\partial\phi_\pm)^2 - \frac{\ell}{6a^2}R + C^\mu{}_\mu} \approx a^4(\phi_+)\sqrt{-g}\left[1 + \frac{1}{2a^2}(\partial\phi_\pm)^2 - \frac{\ell}{12a^2}R\right] \times \left(1 + \frac{C^\mu{}_\mu}{2}\right), \quad (32)$$

where $(\partial\phi_\pm)^2$ means $\partial^\alpha\phi_\pm\partial_\alpha\phi_\pm$. Thus, the action for each brane becomes

$$S_\pm \equiv \mp\frac{6}{\kappa^2\ell}\int d^4x\sqrt{-g_\pm} = \mp\frac{6}{\kappa^2\ell}\int d^4x\sqrt{-g}\left[a^4(\phi_\pm) + \frac{a^2(\phi_\pm)}{2}(\partial\phi_\pm)^2 - \frac{\ell^2}{12}a^2(\phi_\pm)R\right]\left[1 + \frac{C^\mu{}_\mu}{2}\right]. \quad (33)$$

Note that the Ricci scalar came from $\mathrm{tr} f_{\mu\nu}$ in $\sqrt{-g_\pm}$.

In order to calculate the Gibbons-Hawking term, we need the extrinsic curvature defined by

$$\mathcal{K}_{\mu\nu} \equiv n_A\left(\frac{\partial^2 x^A}{\partial\xi^\mu\partial\xi^\nu}\right) + \Gamma^A_{BD}\frac{\partial x^B}{\partial\xi^\mu}\frac{\partial x^D}{\partial\xi^\nu}, \quad (34)$$

where x^A is the coordinate of the brane, $\xi^\mu = x^\mu$ is the one on the brane, and n_A is the normal vector to the brane. Note that $\mathcal{K}_{\mu\nu}$ is different from $K_{\mu\nu}$ in Eq. (12). The Christoffel symbols we need are

$$\Gamma^y_{\mu\nu} = \frac{1}{\ell}a^2(g_{\mu\nu} + f_{\mu\nu}) - \frac{1}{2}a^2 f_{\mu\nu,y}, \quad (35)$$

$$\Gamma^\alpha_{y\mu} = -\frac{1}{\ell}\delta^\alpha_\mu + \frac{1}{2}g^{\alpha\beta}f_{\beta\mu,y}. \quad (36)$$

The tangent basis on the brane are given by

$$\frac{\partial x^A}{\partial\xi^\mu} = (\delta^\alpha_\mu, \partial_\mu\phi_\pm). \quad (37)$$

Thus, the normal vector takes the form

$$n_A = (-n_y\partial_\alpha\phi_\pm, n_y). \quad (38)$$

From the normalization condition $n_A n^A = 1$, we have

$$n_y = \frac{1}{\sqrt{1 + (1/a^2)(\partial\phi_\pm)^2}}. \quad (39)$$

Then the extrinsic curvature is calculated as

$$\mathcal{K}^\pm_{\mu\nu} = n_y\left[\nabla_\mu\nabla_\nu\phi_\pm + \frac{a^2}{\ell}\left(g_{\mu\nu} + f_{\mu\nu} - \frac{\ell}{2}f_{\mu\nu,y}\right) + \frac{2}{\ell}\partial_\mu\phi_\pm\partial_\nu\phi_\pm\right]. \quad (40)$$

The trace part of extrinsic curvature on each brane is

$$\mathcal{K}_\pm = g^{\mu\nu}_\pm\mathcal{K}^\pm_{\mu\nu} = n_y\left[\frac{4}{\ell} + \frac{1}{a^2}\Box\phi_\pm + \frac{1}{\ell a^2}(\partial\phi_\pm)^2 + \frac{\ell}{6a^2}R\right]. \quad (41)$$

Therefore, the Gibbons-Hawking term is obtained as

$$S_{\mathrm{GH}} \equiv \frac{2}{\kappa^2}\int d^4x\sqrt{-g_+}\mathcal{K}_+ - \frac{2}{\kappa^2}\int d^4x\sqrt{-g_-}\mathcal{K}_- = \frac{2}{\kappa^2}\int d^4x\sqrt{-g}\left[\frac{4}{\ell}a^4(\phi_+) + \frac{3}{\ell}a^2(\phi_+)(\partial\phi_+)^2 - \frac{\ell}{6}a^2(\phi_+)R\right]\left[1 + \frac{C^\mu{}_\mu}{2}\right] - (\phi_+ \to \phi_-). \quad (42)$$

Note that the Ricci scalar came from $\mathrm{tr} f_{\mu\nu}$ in $\sqrt{-g_\pm}$ and $\mathrm{tr} f_{\mu\nu,y}$ in $\mathcal{K}_\pm$.

Substituting the results Eqs. (30), (33), and (42) into the five-dimensional action Eq. (5), we get the four-dimensional effective action

$$S = S_{\mathrm{bulk}} + S_+ + S_- + S_{\mathrm{GH}} = \frac{\ell}{2\kappa^2}\int d^4x\sqrt{-g}\left[\{a^2(\phi_+) - a^2(\phi_-)\}R + \frac{6}{\ell^2}\{a^2(\phi_+) \times (\partial\phi_+)^2 - a^2(\phi_-)(\partial\phi_-)^2\}\right]\left[1 + \frac{C^\mu{}_\mu}{2}\right]. \quad (43)$$

Here $C^\mu{}_\mu$ is the first order quantity, so we can ignore this term at leading order. We see that this effective action (43) is indistinguishable from Eq. (10) obtained by assuming the factorizable metric. Thus, we have shown that the action obtained from the factorizable ansatz is correct at the leading order.

Here it should be stressed that the Einstein-Hilbert term is originated from the contributions of $f_{\mu\nu}$ in each S_{bulk}, $S_\pm$, and S_{GH}, so the correction $f_{\mu\nu}$ to the metric $g_{\mu\nu}$ plays an important role.

Note that the induced metric on the positive tension brane is

$$g^+_{\mu\nu}(\phi_+, x) = a^2(\phi_+)g_{\mu\nu}(x), \quad (44)$$

where we have chosen the constant of integration $C_{\mu\nu}$ to be

$$f_{\mu\nu}(\phi_+, x) = -\frac{1}{a^2(\phi_+)}\partial_\mu\phi_+\partial_\nu\phi_+. \quad (45)$$

We see that the induced metric on the positive tension brane is different from the factorized metric $g_{\mu\nu}$. Using a conformal transformation $g_{\mu\nu} = [1/a^2(\phi_+)]g^+_{\mu\nu}$ to rewrite the effective action in terms of the induced metric, we finally get

$$S = \frac{\ell}{2\kappa^2}\int d^4x\sqrt{-g_+}\left(\left\{1 - \left[\frac{a(\phi_-)}{a(\phi_+)}\right]^2\right\}R(g_+) - 6\partial_\mu\left[\frac{a(\phi_-)}{a(\phi_+)}\right]\partial^\mu\left[\frac{a(\phi_-)}{a(\phi_+)}\right]\right), \quad (46)$$

where

$$\frac{a(\phi_-)}{a(\phi_+)} = \exp\left[-\frac{1}{\ell}(\phi_- - \phi_+)\right]. \quad (47)$$

Two moduli fields appear only in the form of the difference, which corresponds to the radion field. In the physical frame, the extra degree of freedom disappears.

V. CONCLUSION

To support the validity of the factorizable metric ansatz used in string cosmology, we investigated a toy problem in the RSI model. For this purpose, we have revised the gradient expansion method to conform to the factorizable metric ansatz. We have solved the five-dimensional equations of motion and substituted the results into the action. Consequently, we have obtained the four-dimensional effective action which is equivalent to that obtained by assuming the factorizable metric ansatz. Hence, our calculation supports the factorizable metric ansatz.

In the higher order analysis including Kaluza-Klein corrections, the factorizable metric ansatz cannot be correct anymore [8]. However, in string cosmology, what we want is the leading order action. Then the factorizable metric ansatz becomes a useful method when discussing the cosmology without solving the bulk equations of motion at least at the leading order.

Finally, we should mention the limitation of our approach. Although we have shown the validity of the factorizable ansatz in five dimensions, the issue is still unclear in the case of higher codimension. This is because the higher codimension objects is difficult to treat in a relativistic manner. To give a more strong support of the factorizable metric ansatz, we need to settle this issue.

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