Weak gravity in the Dvali-Gabadadze-Porrati braneworld model

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We analyze weak gravity in the braneworld model proposed by Dvali, Gabadadze, and Porrati, in which the unperturbed background spacetime is given by a five-dimensional Minkowski bulk with a brane which has an induced Einstein-Hilbert term. This model has a critical length scale r_c. Naively, we expect that four-dimensional general relativity (4D GR) is approximately recovered at a scale below r_c. However, the simple linear perturbation does not work in this regime. Only recently has the mechanism to recover 4D GR been clarified under the restriction to spherically symmetric configurations, and the leading correction to 4D GR been derived. Here, we develop an alternative formulation which can handle more general perturbations. We also generalize the model by adding a bulk cosmological constant and the brane tension.

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I. INTRODUCTION

A braneworld model, whose gravity behaves as four dimensional at a short distance scale but shows a higher dimensional nature at larger distances, was proposed by Dvali, Gabadadze, and Porrati (DGP) [1]. In this model, the brane, on which the fields of the standard model are confined, has an induced Einstein-Hilbert term [2,3]. This model has various cosmologically interesting features [4–11]. Particularly in the model with a five-dimensional bulk, an interesting cosmological solution was found, in which the accelerated expansion of the Universe at a late epoch is realized without introducing the cosmological constant [7]. Based on this model, a novel mechanism that dilutes the cosmological constant was also proposed [11].

Although we mentioned above that gravity in this model at short distances is expected to behave as four dimensional, it is not so transparent if the model actually mimics four-dimensional general relativity (4D GR). The linear analysis of this model shows that the tensor structure of the induced metric perturbations takes a five-dimensional form even at short distance [1]. The situation is analogous to the case of models with massive gravitons. In this case the deviation from 4D GR does not vanish even in the massless limit, which is known as the van Dam–Veltoman–Zakharov discontinuity [12–14]. In this context, the possibility that the 4D GR is recovered by nonlinear effect was suggested in Ref. [15]. There have been many discussions about this issue [16]. In particular, we have a clear statement that the discontinuity disappears when we introduce the cosmological constant [17,18]. Although the analysis with a cosmological constant is quite suggestive, the discontinuity is absent only when the limit is taken, keeping the length scale determined by the cosmological constant much smaller than the Compton wavelength of the massive graviton. The length scale determined by the cosmological constant must be longer than the Hubble horizon size. Hence, the recovery of 4D GR cannot be proven by introducing a negligibly small cosmological constant as far as the graviton mass is not completely negligibly small.

Also specialized to the context of the five-dimensional DGP model, there are various works aiming at answering the question whether 4D GR is recovered at short distances or not, and much evidence that indicates the recovery of 4D GR were reported [19–23]. It was shown that the evolution equation for a homogeneous isotropic universe becomes identical to that for 4D GR when the Hubble expansion rate is much larger than the inverse of the critical length scale, r_c^{-1} [19]. In Refs. [21,22] (also see Ref. [20]) it was clearly shown that the linear analysis breaks down at a scale shorter than $(r_c^2 r_g)^{1/3}$ since the brane bending becomes nonlinear there. Further, a consistent form of a black hole metric induced on the brane was presented. An approximate black hole solution including the bulk was constructed under the restriction to spherically symmetric configurations [21]. The same paper also gave the leading order correction to 4D GR, which is potentially observable by the future development of precision measurements of our solar system [24,23]. The results in Ref. [21] were extended to the case with the background of an expanding universe [23].

In this paper, we develop an alternative formalism which can handle general perturbations in a weak gravity regime. To handle general perturbations, we restrict our consideration to the case that the unperturbed metric on the brane is given by Minkowski space-time. We also make a further generalization to the model that also takes into account the bulk cosmological constant and the brane tension balanced with it. (Such a generalized DGP model was discussed before in Refs. [25–28].) We confirm the recovery of 4D GR at short distances and rederive the leading order correction to it.

II. SETUP

The model that we consider is defined by the five-dimensional action

$$S=\frac{M_4^2}{4r_c}\int d^5x\sqrt{-g}\left(R^{(5)}+\frac{12}{\ell^2}\right)+\int d^4x\sqrt{-g^{(4)}}\left(\frac{M_4^2}{2}R^{(4)}-\frac{3M_4^2}{r_c\ell}+L_{matter}\right), \qquad (2.1)$$

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where M_4, r_c, and ℓ are constants. $R^{(5)}$ and $R^{(4)}$ are, respectively, the curvature scalars corresponding to the five-dimensional metric $g_{\mu\nu}$ and the four-dimensional one $g^{(4)}_{\mu\nu}$ induced on the brane. Here, we added both the bulk cosmological constant and the brane tension terms to the original DGP model. They are tuned to admit the Minkowski brane as a vacuum solution. The model is reduced to the original one by setting $\ell\to\infty$. The unperturbed background geometry is given by five-dimensional anti-de Sitter space-time,

$$ds^2=g^{(0)}_{ab}dx^a dx^b=dy^2+\gamma_{\mu\nu}(y)dx^\mu dx^\nu =dy^2+e^{-2y/\ell}\eta_{\mu\nu}dx^\mu dx^\nu, \tag{2.2}$$

with a brane located at $y=0$, where a Z_2 symmetry is imposed. Here $\eta_{\mu\nu}$ is a four-dimensional Minkowski metric.

III. SEMINONLINEAR PERTURBATIONS

We follow the method of Ref. [29] introduced for the purpose of analyzing weak gravity in the Randall-Sundrum model [30]. We prepare two coordinate systems. In the coordinates $\{x^a\}$, the gauge is chosen so that the metric perturbations h_{ab} can be easily computed in the five-dimensional bulk. That is, we use the Randall-Sundrum gauge,

$$h_{5a}=0,\quad h_\mu{}^\mu=0,\quad h_{\mu,\nu}^{\ \nu}=0. \tag{3.1}$$

In this paper the fifth direction is the direction of extra dimension. The Greek and Latin indices represent four- and five-dimensional coordinates, respectively. The other coordinate system $\{\overline{x^a}\}$ satisfies the Gaussian normal conditions

$$\overline{h}_{5a}=0, \tag{3.2}$$

and also keeps the location of the brane unperturbed at $\overline{y}=0$. Under the coordinate transformation $x^a=\overline{x^a}-\xi^a(\overline{x})$, the metric perturbation transforms as

$$\begin{aligned}\overline{h}_{ab}=&h_{ab}[\overline{x}-\xi(\overline{x})]+\left(-g^{(0)}_{ab,5}\xi^5+\frac{1}{2}g^{(0)}_{ab,55}(\xi^5)^2-\cdots\right)\\ &-\{\xi^c_{,a}[g^{(0)}_{cb}(\overline{x}-\xi)+h_{cb}(\overline{x}-\xi)]+(a\leftrightarrow b)\}\\ &+[g^{(0)}_{cd}(\overline{x}-\xi)+h_{cd}(\overline{x}-\xi)]\xi^c_{,a}\xi^d_{,b}.\end{aligned} \tag{3.3}$$

The argument of the variables is supposed to be $\overline{x}$ unless otherwise is specified, and "$_{,a}$" denotes a differentiation with respect to $\overline{x^a}$.

The conditions that the $\{00\}$ component and $\{0\mu\}$ components are zero in both coordinates provide equations for the gauge parameters, which are solved up to second order as

$$\begin{aligned}\xi^5&=\hat{\xi}^5+\overset{(2)}{\xi}{}^5,\\ \xi^\mu&=\frac{\ell}{2}(\eta^{\mu\nu}-\gamma^{\mu\nu})\hat{\xi}^5_{,\nu}+\hat{\xi}^\mu+\overset{(2)}{\xi}{}^\mu,\end{aligned} \tag{3.4}$$

where $\hat{\xi}^5(\overline{x^\rho})$ and $\hat{\xi}^\mu(\overline{x^\rho})$ are the values of the gauge parameters evaluated on the brane, and

$$\overset{(2)}{\xi}{}^5=\int_0^{\overline{y}}d\overline{y}\,\gamma^{\mu\nu}\hat{\xi}^5{}_{,\mu}\hat{\xi}^5{}_{,\nu}, \tag{3.5}$$

$$\overset{(2)}{\xi}{}^\mu=\int_0^{\overline{y}}d\overline{y}\,\gamma^{\mu\rho}\left[\hat{\xi}^5{}_{,\nu}\left(\overline{h}_\rho{}^\nu+\frac{2}{\ell}\delta_\rho{}^\nu\hat{\xi}^5\right)-\xi^\sigma{}_{,\rho}\hat{\xi}^5{}_{,\sigma}-\overset{(2)}{\xi}{}^5{}_{,\rho}\right].$$

We assume the following order counting:

$$\hat{\xi}^5_{,\mu}\lesssim\epsilon,\qquad \frac{\hat{\xi}^5}{\ell},\frac{\hat{\xi}^5}{r_c},\hat{\xi}^\rho_{,\mu},\overline{h}^\rho{}_\mu\Big|_{\overline{y}=0}\lesssim\epsilon^2, \tag{3.6}$$

and keep the terms up to $O(\epsilon^2)$. Here ϵ^2 is the order of the Newton potential $\Phi=-\frac{1}{2}\overline{h}_{00}$. Later we will verify the consistency of this order counting. Then, the transformation for $\{\mu\nu\}$ components reduces to

$$\overline{h}_{\mu\nu}(\overline{x})=h_{\mu\nu}(\overline{x-\xi(x)})+\delta h_{\mu\nu}, \tag{3.7}$$

with

$$\delta h_{\mu\nu}=\frac{2}{\ell}\gamma_{\mu\nu}\xi^5-\xi_{\mu,\nu}-\xi_{\nu,\mu}+\hat{\xi}^5_{,\mu}\hat{\xi}^5_{,\nu}. \tag{3.8}$$

Hereafter, the Greek indices are lowered or raised by the metric $\gamma_{\mu\nu}$.

The brane location is given by $\overline{y}=0$. Hence in the $\{x^a\}$ coordinates the brane is bent. For simplicity, we impose the harmonic gauge condition, $\overline{h}^{\ \nu}_{\mu,\nu}=\frac{1}{2}\overline{h}_{,\mu}$ for the induced metric on the brane. To second order in ϵ, this condition gives

$$\hat{\xi}_\mu=\Box^{-1}\left[-\frac{2}{\ell}\hat{\xi}^5{}_{,\mu}+(\Box\hat{\xi}^5)\hat{\xi}^5_{,\mu}\right]. \tag{3.9}$$

From this relation, we find that the assumption $\hat{\xi}^\mu_{,\nu}=O(\epsilon^2)$ is consistent if the assumed order of $\hat{\xi}^5$ is correct. Substituting Eq. (3.9), the gauge transformation $\delta h_{\mu\nu}$ evaluated on the brane becomes

$$\begin{aligned}\delta h_{\mu\nu}|_{\overline{y}=0}=&\frac{2}{\ell}\gamma_{\mu\nu}\hat{\xi}^5\\ &+\Box^{-1}\left[\frac{4}{\ell}\hat{\xi}^5{}_{,\mu\nu}+2\gamma^{\rho\sigma}\hat{\xi}^5{}_{,\rho\mu}\hat{\xi}^5{}_{,\sigma\nu}-2(\Box\hat{\xi}^5)\hat{\xi}^5{}_{,\mu\nu}\right].\end{aligned} \tag{3.10}$$

Then, the trace of the induced metric is also evaluated as

$$\overline{h}=\frac{12}{\ell}\hat{\xi}^5+2\Box^{-1}[\gamma^{\mu\nu}\gamma^{\rho\sigma}\hat{\xi}^5{}_{,\mu\rho}\hat{\xi}^5{}_{,\nu\sigma}-(\Box\hat{\xi}^5)^2]. \tag{3.11}$$

Next we consider the junction condition. After a straightforward calculation, we can show

$$(\partial_{\bar{y}}+2\ell^{-1})\bar{h}_{\mu\nu}=(\partial_{\bar{y}}+2\ell^{-1})h_{\mu\nu}(\bar{x}-\xi)+2\hat{\xi}^5_{,\mu\nu}+\frac{2}{\ell}\Xi_{\mu\nu}\quad(\text{at }\bar{y}=0),\tag{3.12}$$

with

$$\Xi_{\mu\nu}:=\gamma_{\mu\nu}\gamma^{\rho\sigma}\hat{\xi}^5_{,\rho}\hat{\xi}^5_{,\sigma}+\hat{\xi}^5_{,\mu}\hat{\xi}^5_{,\nu}.\tag{3.13}$$

Using this relation, the junction condition becomes

$$\begin{aligned}T_{\mu\nu}-\frac{1}{2}\gamma_{\mu\nu}T+\frac{M_4^2}{2}\Box\bar{h}_{\mu\nu}&=-\frac{M_4^2}{2r_c}\bigg[(\partial_{\bar{y}}+2\ell^{-1})h_{\mu\nu}(\bar{x}-\xi)|_{\bar{y}=0}+2\hat{\xi}^5_{,\mu\nu}\\&\quad+\frac{2\Xi_{\mu\nu}}{\ell}+\gamma_{\mu\nu}\left(\Box\hat{\xi}^5+\frac{\Xi}{\ell}\right)\bigg].\end{aligned}\tag{3.14}$$

The equation that determines the brane bending is obtained from the trace of the above equation as

$$\begin{aligned}\Box^{-1}\frac{1}{M_4^2}T&=\frac{\bar{h}}{2}+\frac{3}{r_c}\left(\hat{\xi}^5+\Box^{-1}\frac{\Xi}{\ell}\right)\\&=\frac{3}{r_c^*}\hat{\xi}^5+\Box^{-1}(\gamma^{\mu\nu}\gamma^{\rho\sigma}\hat{\xi}^5_{,\mu\rho}\hat{\xi}^5_{,\nu\sigma}-(\Box\hat{\xi}^5)^2)\\&\quad+\frac{3}{r_c\ell}\Box^{-1}\Xi,\end{aligned}\tag{3.15}$$

where we have introduced $r_c^*:=[(1/r_c)+(2/\ell)]^{-1}$. We can neglect the last term on the right hand side of Eq. (3.15) because it is always higher order compared with the first term. The left hand side is something like the Newton potential Φ, hence we assume it to be $O(\epsilon^2)$. Outside the matter distribution with the total mass m, the left hand side can be expressed as $\approx r_g/r$, where $r_g:=m/4\pi M_4^2$. At large scale, $r\gtrsim(r_g r_c^2)^{1/3}$, the first term on the right hand side dominates, while at small scale, $r\lesssim(r_g r_c^2)^{1/3}$, the second term dominates. Therefore we have

$$O(\epsilon^2)=\max\left(\left|\frac{\hat{\xi}^5}{r_c^*}\right|,|\hat{\xi}^5_{,\mu}|^2\right).\tag{3.16}$$

Thus we find that our assumption as to the order counting for $\hat{\xi}^5$ is justified.

IV. MECHANISM FOR RECOVERING FOUR-DIMENSIONAL GENERAL RELATIVITY

The remaining task is to evaluate $(\partial_{\bar{y}}+2\ell^{-1})h_{\mu\nu}(\bar{x}-\xi(\bar{x}))|_{\bar{y}=0}$. Here we need to solve the bulk field equations. Different from the R-S case, we solve the bulk equations with the Dirichlet boundary condition (3.7). Here, we note that the location of the brane is not a straight sheet in the coordinates in the R-S gauge $\{x^a\}$.

We can give the general solution for the bulk field equations as a superposition of homogeneous mode solutions with a purely outgoing boundary condition:

$$\begin{aligned}h_{\mu\nu}(x)&=\int\mathcal{H}_{\mu\nu}(p)e^{ip_\mu x^\mu}K_2(p\ell e^{y/\ell})d^4p\\&=\int\mathcal{H}_{\mu\nu}(p)e^{ip_\mu(\bar{x}^\mu-\xi^\mu(\bar{x}))}K_2(p\ell e^{\bar{y}-\xi^5(\bar{x})/\ell})d^4p,\end{aligned}\tag{4.1}$$

where $K_2(p\ell)$ is the modified Bessel function and $\mathcal{H}_{\mu\nu}$ is the expansion coefficient. The coefficient $\mathcal{H}_{\mu\nu}$ is to be determined so as to satisfy the Dirichlet boundary condition

$$h_{\mu\nu}|_{\bar{y}=0}(\bar{x})=\int\mathcal{H}_{\mu\nu}(p)e^{ip_\mu(\bar{x}^\mu-\xi^\mu(\bar{x}))}K_2(p\ell e^{-\hat{\xi}^5(\bar{x})/\ell})d^4p.\tag{4.2}$$

If we are allowed to approximate the above expression by setting $\xi^a=0$, we have $\tilde{h}_{\mu\nu}(p):=(2\pi)^{-4}\int d^4\bar{x}e^{-ip_\rho\bar{x}^\rho}\times h_{\mu\nu}(\bar{x})=\mathcal{H}_{\mu\nu}(p)$, and therefore we have

$$\begin{aligned}(2\pi)^{-4}\int d^4\bar{x}e^{-ip_\rho\bar{x}^\rho}(\partial_{\bar{y}}+2\ell^{-1})h_{\mu\nu}|_{\bar{y}=0}(\bar{x})\\=-\frac{pK_1(p\ell)}{K_2(p\ell)}\tilde{h}_{\mu\nu}(p)=-\frac{pK_1(p\ell)}{K_2(p\ell)}(\tilde{\bar{h}}_{\mu\nu}-\delta\tilde{h}_{\mu\nu}).\end{aligned}\tag{4.3}$$

We think that the errors caused by this naive approximation are not large, although any rigorous proof is not ready yet. If the leading errors are simply proportional to $h_{\mu\nu}\xi$, we can neglect them since they are of higher order in ϵ. Such a naive expansion with respect to ξ will be justified for small p. But for large p, we will not be allowed to expand the combination $p\xi$ in the exponent. However, as we will see below, even the leading correction to the gravitational potential coming from the contribution of this part is suppressed to be irrelevantly small at small scale $r\lesssim r_c$. Hence, the errors due to this naive approximation can be crucial only if this approximation significantly underestimate the magnitude of $(\partial_{\bar{y}}+2\ell^{-1})h_{\mu\nu}|_{\bar{y}=0}$, which is quite unlikely.

Using Eq. (4.3), the junction condition (3.14) is written down explicitly as

$$\begin{aligned}\tilde{\bar{h}}_{\mu\nu}-\frac{2}{M_4^2}\mathcal{D}\left(\tilde{T}_{\mu\nu}-\frac{1}{2}\gamma_{\mu\nu}\tilde{T}\right)&=\mathcal{D}\bigg[\left(-\frac{p^2}{r_c}+\frac{2pK_1(p\ell)}{r_c\ell K_2(p\ell)}\right)\gamma_{\mu\nu}\tilde{\hat{\xi}}^5-\frac{2}{r_c}p_\mu p_\nu\tilde{\hat{\xi}}^5\\&\quad+\frac{pK_1(p\ell)}{r_cK_2(p\ell)}\delta\tilde{h}^{(2)}_{\mu\nu}+\frac{1}{r_c\ell}\left(\tilde{\Xi}_{\mu\nu}+\frac{1}{2}\gamma_{\mu\nu}\tilde{\Xi}\right)\bigg]\\&\quad(\text{at }\bar{y}=0),\end{aligned}\tag{4.4}$$

where

$$\mathcal{D}=\frac{1}{p^{2}+\dfrac{pK_{1}(p\ell)}{r_{c}K_{2}(p\ell)}} \tag{4.5}$$

and

$$\delta h_{\mu\nu}^{(2)}=\Box^{-1}[2\gamma^{\rho\sigma}\hat{\xi}^{5}_{,\rho\mu}\hat{\xi}^{5}_{,\sigma\nu}-2(\Box\hat{\xi}^{5})\hat{\xi}^{5}_{,\mu\nu}]. \tag{4.6}$$

Here the quantity with $\tilde{}$ represents the Fourier coefficient of the corresponding variable as before. We show that in the square brackets on the right hand side of Eq. (4.4) the first term gives the dominant contribution. We can drop the last two terms simply because they are always higher order in ϵ compared with the first term. The second term is irrelevant since it can be eliminated by a four-dimensional gauge transformation. As a result, the equation that determines the metric induced on the brane $\overline{h}_{\mu\nu}$ is reduced to the one for the linear theory. The only difference is in the equation that determines the brane bending [Eq. (3.15)].

The order of magnitude of the first term on the right hand side of Eq. (3.15) is estimated as $\hat{\xi}^{5}/r_{c}\sim\Phi^{1/2}r/r_{c}$ at small scale, and hence it is suppressed by the factor $\Phi^{-1/2}r/r_{c}$ compared with the Newton potential Φ. The leading behavior of the induced metric is therefore determined by setting the left hand side of Eq. (4.4) to zero. Thus we conclude that 4D GR is recovered by taking into account the nonlinear brane bending for weak gravity at small scale $r\lesssim(r_{c}^{2}r_{g})^{1/3}$. If we take the limit $r_{c}\to\infty$, all length scales come into this regime. Hence, we have confirmed the absence of van Dam–Veltman–Zakharov discontinuity. As first pointed out in Ref. [21], however, because of the factor $\Phi^{-1/2}$ the leading order deviation from 4D GR at small scale is less strongly suppressed than the naively expected suppression r/r_{c}.

At large scale, this term becomes more and more important. For $r\gtrsim(r_{c}^{2}r_{g})^{1/3}$, we have

$$\tilde{\hat{\xi}}^{5}\approx-\frac{r_{c}^{*}\tilde{T}}{3M_{4}^{2}p^{2}}. \tag{4.7}$$

This is nothing but the result for the linearized case. In Sec. V, we discuss the regime where the linear theory is valid. After that, in the succeeding section, we discuss the leading order correction to the 4D GR at short distance scale assuming static and spherically symmetric configurations.

V. LINEAR REGIME

In this section we consider perturbations at large scale $r\gg(r_{c}^{2}r_{g})^{1/3}$, where the linear theory is valid. Substituting Eq. (4.7) into Eq. (4.4), we obtain

$$\tilde{\bar{h}}_{\mu\nu}\approx\frac{2}{M_{4}^{2}}\mathcal{D}\left(\tilde{T}_{\mu\nu}-\frac{1}{2}\alpha\gamma_{\mu\nu}\tilde{T}\right), \tag{5.1}$$

with

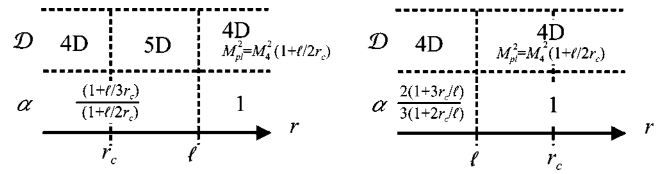

FIG. 1. Summary of the results of the linear analysis, which is valid when $r\gg(r_{c}^{2}r_{g})^{1/3}$. The horizontal axis represents the length scale. The raws labeled $\mathcal{D}$ and α, respectively, explain the property of the propagator $\mathcal{D}$ and the indicator of the tensor structure α.

$$\alpha=\frac{r_{c}+\dfrac{\ell}{3}\left(1+\dfrac{1}{\ell p}\dfrac{K_{1}(p\ell)}{K_{2}(p\ell)}\right)}{r_{c}+\dfrac{\ell}{2}}. \tag{5.2}$$

First we look at the behavior of the propagator D, which was already discussed in Ref. [28]. When we consider the length scale much smaller than ℓ, we have

$$\mathcal{D}\to\frac{1}{p^{2}+r_{c}^{-1}p}\qquad(p\ell\gg1). \tag{5.3}$$

At a length scale smaller than $r_{c}(pr_{c}\gg1)$, the propagator $\mathcal{D}$ behaves as that for a four dimensional field. On the other hand, at the intermediate scale between r_{c} and ℓ $(r_{c}\gg p^{-1}\gg\ell)$, the propagator behaves as that for a five-dimensional field. When the length scale is much larger than ℓ, $K_{1}(p\ell)/K_{2}(p\ell)$ goes to $p\ell/2$. Thus we have

$$\mathcal{D}\to\frac{2r_{c}}{(\ell+2r_{c})p^{2}}\qquad(p\ell\ll1). \tag{5.4}$$

Hence again the propagator $\mathcal{D}$ behaves as a four-dimensional field, but Newton's constant is not given by $2M_{4}^{-2}$ but by $2M_{4}^{-2}/(1+\ell/2r_{c})$.

Next we turn to the tensor structure specified by α. For a four-dimensional massless graviton we have $\alpha=1$, while $\alpha=\frac{2}{3}$ for the case of a massive graviton. For $p\ell\gg1$, we have

$$\alpha\to\frac{1+\dfrac{\ell}{3r_{c}}}{1+\dfrac{\ell}{2r_{c}}}. \tag{5.5}$$

We have $\alpha\to1$ for $\ell\ll r_{c}$, while $\alpha\to2/3$ for $\ell\ll r_{c}$. On the other hand, for $p\ell\ll1$, we have $\alpha\to1$ irrespective of the ratio between ℓ and r_{c}.

The results are summarized in Fig. 1. When $r\gtrsim\ell$, the 4D GR is realized by the Randall-Sundrum mechanism. The effective Planck mass differs from M_{4} in this case. On the other hand, when $r\lesssim r_{c}$, the gravity becomes four dimensional again, but the tensor structure differs from 4D GR.

VI. STATIC SPHERICAL SYMMETRIC NONRELATIVISTIC STAR

We consider a static spherical symmetric nonrelativistic star. Assuming that the energy momentum tensor is dominated by the $\{00\}$ component $T_{00}=\rho$, we neglect the effect of pressure when we solve the metric perturbation. First we solve the nonlinear equation for $\hat{\xi}^5$ [Eq. (3.15)]. Under the present assumptions, Eq. (3.15) is simplified as

$$-\frac{1}{M_4^2}\rho=\frac{1}{r^2}\partial_r\left(\frac{3r^2}{r_c^*}\hat{\xi}^5_{,r}-2r(\hat{\xi}^5_{,r})^2\right). \tag{6.1}$$

This equation can be immediately integrated once, and we obtain

$$2(\hat{\xi}^5_{,r})^2-\frac{3r}{r_c^*}\hat{\xi}^5_{,r}-\frac{r_g(r)}{r}=0, \tag{6.2}$$

where

$$r_g(r)=\frac{1}{M_4^2}\int_0^r dr r^2\rho. \tag{6.3}$$

Outside the star, we have $r_g(r)=r_g=m/4\pi M_4^2$. Solving the above equation with respect to $\hat{\xi}^5_{,r}$, we have

$$\hat{\xi}^5=\int dr\left[\frac{3r}{4r_c^*}-\frac{1}{4}\sqrt{\left(\frac{3r}{r_c^*}\right)^2+\frac{8r_g(r)}{r}}\right]. \tag{6.4}$$

Here we have chosen the signature in front of the square root imposing the condition that $\hat{\xi}^5_{,r}$ does not become large at $r\to\infty$. The other branch with the "+" sign is outside the scope of the present formalism since we have assumed a Minkowski brane background. At small scale, this expression reduces to $\hat{\xi}^5\approx-\int dr\sqrt{r_g(r)/2r}$. Outside the matter distribution, we simply have $\hat{\xi}^5=-\sqrt{2rr_g}$. Hence, the correction to the Newton potential is given by

$$\delta\Phi\approx\sqrt{\frac{rr_g}{2r_c^2}}, \tag{6.5}$$

which recovers the result obtained in Ref. [21].

VII. CONCLUSION

We developed a formalism to calculate the metric perturbations induced by the matter localized on the brane in the generalized DGP model, in which the bulk cosmological constant and the brane tension terms are added. Here we clarified the mechanism for the disappearing van Dam–Veltman–Zakharov discontinuity in this model. In our approach, the crucial point was to take into account a part of the second order perturbations of the brane bending. This method was largely motivated by recent works [31,32]. Our scheme almost completely controls the order of magnitude of the neglected higher order correction terms. In this sense, we think that this work gives an alternative sufficiently satisfactory proof of the absence of the van Dam–Veltman–Zakharov discontinuity in the DGP model. Under the restriction to the static and spherically symmetric source, we confirmed that our formulation correctly reproduces the leading order correction to 4D GR at short distances obtained in Ref. [21].

Our new formulation has an advantage compared with previous works on the following point. Here basic equations were perturbatively derived without assuming static or spherical symmetric configurations. Analyzing solutions of these equations in general cases is not so simple because we need to partly take into account the second order perturbation. The equations derived here, however, will be still useful to understand how 4D GR is approximately recovered in general and to study dynamics of a weakly self-gravitating system such as a star binary.

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