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Interactions between octet baryons in the SU$_6$ quark model

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Baryon-baryon interactions for the complete baryon octet ($B_8$) are investigated in a unified framework of the resonating-group method, in which the spin-flavor SU$_6$ quark-model wave functions are employed. Model parameters are determined to reproduce properties of the nucleon-nucleon system and the low-energy cross section data for the hyperon-nucleon interaction. We then proceed to explore $B_8B_8$ interactions in the strangeness $S=-2,-3$, and $-4$ sectors. The $S$-wave phase-shift behavior and total cross sections are systematically understood by (1) the spin-flavor SU$_6$ symmetry, (2) the special role of the pion exchange, and (3) the flavor symmetry breaking.

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I. INTRODUCTION

In the quark model, the baryon-baryon interactions for the complete octet baryons ($B_8 = N, \Lambda, \Sigma, \Xi$) are treated entirely equivalently with the well-known nucleon-nucleon ($NN$) interaction. Once the quark-model Hamiltonian is assumed in the framework of the resonating-group-method (RGM), explicit evaluation of the spin-flavor factors leads to the stringent flavor dependence appearing in various interaction pieces. We can thus minimize the ambiguity of the model parameters by utilizing the rich knowledge of the $NN$ interaction.

In the next section, we recapitulate the formulation of the $(3q)$-$\overline{(3q)}$ Lippmann-Schwinger RGM [3]. In Sec. III, we summarize the essential features of the $NN$ and $YN$ interactions, in order to furnish the basic components to understand the phase-shift behavior of the $B_8B_8$ interactions in a unified way. The model predictions of the $B_8B_8$ interactions in the $S=-2,-3$, and $-4$ sectors are given in Sec. IV, with respect to the $S$-wave phase shifts and the total cross sections. The final section is devoted to a summary.

II. FORMULATION

The quark-model Hamiltonian $H$ consists of the phenomenological confinement potential $U_{ij}^{\text{CF}}$, the colored version of the full Fermi-Breit (FB) interaction $U_{ij}^{\text{FB}}$ with explicit quark-mass dependence, and the EMEP $U_{ij}^{\text{EMEP}}$ generated from the scalar ($\Omega = S$), pseudoscalar (PS), and vector (V) meson-exchange potentials acting between quarks:

$$H = \sum_{i<j}^6 \left( m_i c^2 + \frac{p_{ij}^2}{2m_i} - T_G \right) + \sum_{i<j}^6 \left( U_{ij}^{\text{CF}} + U_{ij}^{\text{FB}} + \sum_{\beta} U_{ij}^{\text{PS} \beta} + \sum_{\beta} U_{ij}^{\text{V} \beta} \right).$$

It is important to include the momentum-dependent Bryan-Scott term [4] in the $S$- and $V$-meson contributions, in order to remedy the shortcoming of our previous model FSS; namely, the single-particle (s.p.) potential in nuclear matter is too attractive in the high-momentum region $k \approx 6$ fm$^{-1}$. Another important feature of the present model is the introduction of vector mesons for improving the fit to the $NN$ phase-shift parameters. Since the dominant effect of the $\omega$-meson repulsion and the $LS$ components of $\rho, \omega$, and $K^*$ mesons are already accounted for by the FB interaction, only the quadratic $LS$ component of the octet mesons is expected to play an important role in partially canceling the strong one-pion tensor force. Further details of the model fss2 are given in [2]. The model parameters are fixed to reproduce the most recent results of the phase-shift analysis SP99 [5] for $np$ scattering with partial waves $J \leq 2$ and incident energies $T_{\text{lab}} \approx 350$ MeV, under the constraint of the deuteron binding energy and the $1S_0$ $NN$ scattering length, as well as the low-energy $YN$ total cross section data. Owing to the introduction of the vector mesons, the model fss2 in the $NN$ sector has attained an accuracy almost comparable to that of one-boson-exchange-potential (OBEP) models. For example, the $\chi^2$ values defined by $\chi^2 = \sum_{n=1}^N (\delta_{ij} - \delta_n^{\text{exp}})^2/N$ for the $J \leq 2$ phase-shift parameters in the energy range $T_{\text{lab}} = 25$–300 MeV are $\sqrt{\chi^2} = 0.59^\circ$, $1.10^\circ$, $1.40^\circ$, and $1.32^\circ$ for fss2, OBEP, Paris, and Bonn, respectively. The existing data for the $YN$ scattering are well reproduced and the essential feature of the $AN$-$\Sigma N$ coupling is almost unchanged from our previous models.
TABLE I. The relationship between the isospin basis and the flavor-SU3 basis for the $B_p B_\beta$ systems. The flavor-SU3 symmetry is given by the Elliott notation ($\lambda\mu$). $P$ denotes the flavor-exchange symmetry and $I$ the isospin.

<table>
<thead>
<tr>
<th>$S$</th>
<th>$B_p B_\beta(I)$</th>
<th>$P=+1$ (symmetric)</th>
<th>$P=-1$ (antisymmetric)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$^1E$ or $^3O$</td>
<td>$^1E$ or $^3O$</td>
</tr>
<tr>
<td>0</td>
<td>$NN(I=0)$</td>
<td>–</td>
<td>(03)</td>
</tr>
<tr>
<td></td>
<td>$NN(I=1)$</td>
<td>(22)</td>
<td>–</td>
</tr>
<tr>
<td>1</td>
<td>$\Lambda N$</td>
<td>$\frac{1}{\sqrt{10}}[(11)_{a}+3(22)]$</td>
<td>$\frac{1}{\sqrt{10}}[(-11)_{a}+(03)]$</td>
</tr>
<tr>
<td>1</td>
<td>$\Sigma N(I=1/2)$</td>
<td>–</td>
<td>(11$\bar{a}$)</td>
</tr>
<tr>
<td>1</td>
<td>$\Sigma N(I=3/2)$</td>
<td>(22)</td>
<td>(30)</td>
</tr>
<tr>
<td>2</td>
<td>$\Xi N(I=0)$</td>
<td>$\frac{1}{\sqrt{5}}[(11)_{a}-2\sqrt{30}(22)+\frac{1}{\sqrt{2}}(00)]$</td>
<td>$\frac{1}{\sqrt{5}}[(-11)_{a}+(30)+(03)]$</td>
</tr>
<tr>
<td>2</td>
<td>$\Xi N(I=1)$</td>
<td>$(22)$</td>
<td>(30)</td>
</tr>
<tr>
<td>3</td>
<td>$\Xi\Xi (I=0)$</td>
<td>–</td>
<td>(30)</td>
</tr>
<tr>
<td>3</td>
<td>$\Xi\Xi (I=1)$</td>
<td>(22)</td>
<td>–</td>
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<td></td>
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</tr>
</tbody>
</table>

The two-baryon systems composed of the complete baryon octet are classified as

$$1/2(11)\times 1/2(11) = \{0, 1\} \{(22) + (30) + (03) + (11)_{a} + (11)_{s} + (00)\},$$

where $S(\lambda\mu)$ stands for the spin value $S$ and the flavor SU3 representation label $(\lambda\mu)$. If the space-spin states are classified by the flavor-exchange symmetry $P$, the correspondence between the SU3 basis and the isospin basis becomes very transparent, as in Table I. This correspondence is essential in the following discussion. The key point is that the quark-model Hamiltonian, Eq. (2.1), is approximately SU3 scalar. If we ignore the EMEP terms $U_{ij}^{\beta}$, this is fairly apparent since the flavor dependence appears only through the moderate mass difference of the up-down and strange quarks. In the FB interaction $U_{ij}^{FB}$, this is a direct consequence of the fact that the gluons do not have the flavor degree of freedom. In the EMEP terms, the SU3 scalar property of the interaction is not apparent since the mesons have the flavor degree of freedom. Nevertheless, one can easily show that the interaction Hamiltonian is actually SU3 scalar if the masses of the octet mesons are all equal within each of the $S$, $PS$, and $V$ mesons. A nice feature of the quark model is that the approximate SU3 scalar property of the total Hamiltonian is automatically incorporated in the model. On the other hand, in the OBE a similar situation is only realized by assuming the SU3 relations for the many baryon-meson coupling constants. If the Hamiltonian is exactly SU3 scalar, the SU3 states with a common $(\lambda\mu)_{s}$, this is fairly apparent.
phase shifts of these channels have very similar behavior, as
is shown in Fig. 1. In reality, the SU\(_3\) symmetry is broken,
but in a very specific way. The mechanism of the flavor
symmetry breaking (FSB) depends on the details of the
model. In the present framework, the following three factors
cause FSB.

(1) The strange to up-down quark mass ratio \(\lambda = m_s/m_{ud} = 1.551\) (for fss2)>1 in the kinetic-energy term
and \(U_{ij}^{FB}\).

(2) The singlet-octet meson mixing in \(U_{ij}^{S,PS,V}\).

(3) The meson and baryon mass splitting in \(U_{ij}^{S,PS,V}\) and
the kinetic-energy term, and the resultant difference of the
threshold energies.

III. CHARACTERISTICS OF THE NN AND YN
INTERACTIONS AND THE BASIC VIEWPOINT

The approximate SU\(_3\)-scalar property of the Hamiltonian
implies that accurate knowledge of the NN and YN interac-
tions is crucial to understand the \(B_S B_d\) interactions in the \(S = -2, -3, \) and \(-4\) sectors. The following four points con-
cerning the qualitative behavior of the NN and YN interac-
tions are essential [1,2]. First, in the NN system the \(^1S_0\) state
with isospin \(I = 1\) consists of the pure \((22)\) state, and the
phase shift shows a clear resonance behavior reaching more
than 60° (see Fig. 1). On the other hand, the \(^3S_1\) state with
\(I = 1\) is composed of the pure \((03)\) state, and the deu-
teron is bound in this channel owing to the strong one-pion tensor
force. If we switch off this strong tensor force, the \(^3S\) phase
shift rises to 20°–30° at most, indicating that the central
attraction of the \((03)\) state is not as strong as that of the \((22)\)
state (see crosses in Fig. 2). Detailed analysis of the YN
interaction has clarified that the \((11)\) state for the \(^1S_0\) state
and the \((30)\) state for the \(^3S_1\) state are both strongly repul-
sive, reflecting that the most compact \((0s)^6\) configuration is
completely Pauli forbidden for the \((11)\) state and almost
forbidden for the \((30)\) state [6]. The repulsive behavior of the
\(\Sigma N(I = 3/2)^3 S_1\) state with the pure \((30)\) symmetry should be
observed as a strong isospin dependence of the \(\Sigma\) s.p.
potential. On the other hand, the experimental evidence of the
repulsion is not clear for the \(\Sigma N(I = 1/2)\) \(^1S_0\) state, which
contains 90% \((11)\) component. This is because the observ-
ables are usually composed of the contributions both from

\[ \delta_{S_0} \]

\[ \delta_{S_1} \]

\[ \delta_{S_2} \]

FIG. 1. \(^1S_0\) phase shifts for the \(B_S B_d\) interactions with the pure
\((22)\) state.

FIG. 2. \(^3S_1\) phase shifts for the \((03)\) \([N N, \Sigma \Sigma(I = 3/2)\), \((11)\]
\([\Xi N(I = 0)]\), and \((30)\) \([\Sigma N(I = 3/2), \Xi \Xi]\) states. The \(^3S\) phase
shift predicted only by the NN central interaction is also shown by
crosses.

FIG. 3. Total elastic cross sections for the pure \((22)\) state
\((pp, \Sigma^- \Sigma^-, \Xi^- \Xi^-)\) and for the \((22)+(03)\) states \((np, \Xi^- \Sigma^-)\).

The following two additional features of the present

\[ \delta_{S_0} \]

\[ \delta_{S_1} \]

\[ \delta_{S_2} \]
The phase-shift behavior of the $3^1S_1$ state is only moderately attractive, since the $\Xi \Sigma$ system does not allow the strong one-pion exchange in the exchange Feynman diagram. In the direct Feynman diagram, the inclusion of the strangeness reduces the one-pion-exchange effect drastically through the $\Sigma_3$ relations. The $\Xi^- \Sigma^-$ interaction thus gives the largest total cross sections in the strangeness sector, together with the $\Sigma^- \Sigma^-$ interaction, as seen in Fig. 3. The magnitudes of these total cross sections, however, are at most comparable with the $pp$ total cross sections. The $\Xi N \Sigma (I=1/2)$ coupled-channel problem, on the other hand, is less interesting, since the one-pion tensor force in the $3^3S_1+3^3D_1$ state becomes less effective due to the strong repulsion of the (30) component. This is in contrast with the strong $\Lambda N \Sigma (I=1/2)$ channel coupling, which leads to the well-known cusp structure in the $\Lambda N$ total cross sections.

The baryon-baryon interactions in the $S=-2$ sector constitute the most difficult case to analyze, involving three different types of two-baryon configurations: $\Lambda\Lambda-\Xi N-\Sigma\Sigma$ for $I=0$ and $\Xi N-\Sigma\Lambda-\Sigma\Sigma$ for $I=1$. In this case, the isospin dependence of the interaction is very important, just as in the $\Sigma N$ interactions with $I=1/2$ and $3/2$. Figure 4 shows the $1^1S_0$ phase-shift behavior of the full $\Lambda\Lambda-\Xi N-\Sigma\Sigma$ coupled-channel system with $I=0$, in which the $H$-dibaryon bound state might exist. In the previous model FSS the $\Lambda\Lambda$ phase shift rises to $40^\circ$ [11], while in the present fss2 it rises only to $\sim 20^\circ$ at most. The situation is the same as in the $\Xi N(I$}

![Image](image_url)
However, the recently discovered "Demachi-Yanagi event" explains the known three events of the double hypernuclei. As to the interaction, it has been claimed that a phase-shift rise on the order of 40° is at least necessary to explain the known three events of the double hypernuclei. However, the recently discovered "Demachi-Yanagi event" [12] for $^{10}_{\Lambda\Lambda}$Be and "Nagara event" [13] for $^6_{\Lambda\Lambda}$He indicate that the $\Lambda\Lambda$ interaction is less attractive. A rough estimate of $\Delta B_{\Lambda\Lambda}$ for $^6_{\Lambda\Lambda}$He in terms of the $G$-matrix calculation using fss2 is about 1 MeV, which is consistent with this experimental observation.

In the isospin $I = 1$ channel, the lowest incident baryon channel in the $S = -2$ sector is the $\Xi^-N$ channel. Figure 5 shows the phase-shift behavior of the $^1S_0$ and $^3S_1$ states, calculated for the full coupled-channel system $\Xi^-\Sigma\Lambda\Sigma\Sigma$ with $I = 1$. In the $\Xi^-N(I = 1)$ single-channel calculation, both of these phase shifts show monotonic repulsive behavior, originating from the main contributions of the $(11)_0$ and $(30)$ components, respectively [9]. In the full coupled-channel calculation, however, the channel-coupling effect between $\Xi^-N(I = 1)$ and $\Sigma\Lambda$ channels is enhanced by the cooperative role of the FB contribution $U^{(1)}_{ij}$ and EMEP contribution $U^{(1')}_{ij}$ in the strangeness-exchange process. As a result, the $\Xi^-N(I = 1)$ phase shifts show very prominent cusp structure at the $\Sigma\Lambda$ threshold, as seen in Fig. 5. Below the $\Sigma\Lambda$ threshold, the phase-shift values are almost zero. Subsequently, the $\Xi^-p$ (and $\Xi^-n$) total cross sections with the pure $I = 1$ component are predicted to be very small below the $\Sigma\Lambda$ threshold around $p_{\Xi} \sim 600$ MeV/c. This behavior of the $\Xi^-n$ total cross sections, illustrated in Fig. 6(b), is essentially the same as the Nijmegen result in [10]. On the other hand, the $\Xi^-p$ total cross sections, shown in Fig. 6(a), exhibit a typical channel-coupling behavior similar to that of the $\Sigma^-p$ total cross sections. These features demonstrate that the $\Sigma\Lambda$ channel-coupling effect is very important for the correct description of scattering observables, resulting in the strong isospin dependence of the $\Xi^-N$ interaction.

V. SUMMARY

The conversion processes among octet baryons $B_8$ are most straightforwardly incorporated in the coupled-channel formalism. Since our quark model Hamiltonian is approximately SU$_3$ scalar, it is crucial to clarify the characteristics of the $B_8B_8$ interaction for each of the SU$_3$ states, rather than for each of the two-baryon systems. An interbaryon potential in a single-baryon channel is sometimes used for the study of hypernuclei and strangeness nuclear matter. However, such effective interactions are very much model dependent and linkage to the bare interactions like the ones discussed here is sometimes obscured by inherent ambiguities originating from the many-body calculations. The widespread argument that the hypernuclear structure reflects the hyperon-nucleon ($YN$) interaction rather faithfully since the $YN$ interaction is weak should not be overemphasized.

In this study we have upgraded our previous quark model [1] for the nucleon-nucleon ($NN$) and $YN$ interactions by incorporating more complete effective meson-exchange potentials such as the vector mesons and some extra interaction pieces. This model, fss2 [2], reproduces the existing data of the $NN$ and $YN$ interactions quite well. We have then proceeded to predict all the $B_8B_8$ interactions in the strangeness $S = -2, -3,$ and $-4$ sectors, without adding any extra parameters. We have discussed some characteristic features of the $B_8B_8$ interactions, focusing on the qualitative aspect. Among these features are the following: (1) There is no bound state in the $B_8B_8$ systems, except for the deuteron. (2) The $\Xi^-N$ total cross sections are far smaller than the $NN$ cross sections. (3) The $\Xi^-N$ interaction has a strong isospin dependence similar to the $\Sigma\Lambda$ system. (4) The $\Xi^-\Sigma^-\Sigma^+(I = 3/2)$ interaction is moderately attractive. The $S$-wave phase-shift behavior yielding these qualitative features of the $B_8B_8$ interactions is systematically understood by (1) the spin-flavor SU$_3$ symmetry, (2) the special role of pion exchange, and (3) the flavor symmetry breaking. A more detailed analysis of the $B_8B_8$ interactions consistent with the available experimental data will be published in a forthcoming paper.

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[5] Scattering Analysis Interactive Dial-up (SAID), Virginia Polytechnic Institute, Blacksburg, Virginia; R. A. Arndt (private communication).