Interactions between octet baryons in the SU_6 quark model

Y. Fujiwara, 1 M. Kohno, 2 C. Nakamoto, 3 and Y. Suzuki 4
1 Department of Physics, Kyoto University, Kyoto 606-8502, Japan
2 Physics Division, Kyushu Dental College, Kitakyushu 803-8580, Japan
3 Suzuka National College of Technology, Suzuka 510-0294, Japan
4 Department of Physics, Niigata University, Niigata 950-2181, Japan

(Received 22 June 2001; published 27 September 2001)

Baryon-baryon interactions for the complete baryon octet (B_8) are investigated in a unified framework of the resonating-group method, in which the spin-flavor SU_6 quark-model wave functions are employed. Model parameters are determined to reproduce properties of the nucleon-nucleon system and the low-energy cross section data for the hyperon-nucleon interaction. We then proceed to explore B_8B_8 interactions in the strangeness S = −2, −3, and −4 sectors. The S-wave phase-shift behavior and total cross sections are systematically understood by (1) the spin-flavor SU_6 symmetry, (2) the special role of the pion exchange, and (3) the flavor symmetry breaking.

DOI: 10.1103/PhysRevC.64.054001 PACS number(s): 13.75.Cs, 12.39.Jh, 13.75.Ev, 24.85.+p

I. INTRODUCTION

In the quark model, the baryon-baryon interactions for the complete octet baryons (B_8 = N, Λ, Σ, and Ξ) are treated entirely equivalently with the well-known nucleon-nucleon (NN) interaction. Once the quark-model Hamiltonian is assumed in the framework of the resonating-group method (RGM), explicit evaluation of the spin-flavor factors leads to the stringent flavor dependence appearing in various interaction pieces. We can thus minimize the ambiguity of the model parameters by utilizing the rich knowledge of the NN interaction.

In this study we first upgrade our previous model [1] for the NN and hyperon-nucleon (YN) interactions by incorporating more complete effective meson-exchange potentials (EMEP’s) such as the vector mesons and some extra interaction pieces. This model is named fss2 [2] after the pioneering model FSS. Fixing the model parameters in the strangeness S = 0 and −1 sectors, we proceed to explore the B_8B_8 interactions in S = −2, −3, and −4 sectors. These include the ΛΛ and ΞN interactions, which have recently been attracting much interest in the rapidly developing fields of hypernuclear physics and strangeness nuclear matter.

In the next section, we recapitulate the formulation of the (3q)-(3q) Lippmann-Schwinger RGM [3]. In Sec. III, we summarize the essential features of the NN and YN interactions, in order to furnish the basic components to understand the phase-shift behavior of the B_8B_8 interactions in a unified way. The model predictions of the B_8B_8 interactions in the S = −2, −3, and −4 sectors are given in Sec. IV, with respect to the S-wave phase shifts and the total cross sections. The final section is devoted to a summary.

II. FORMULATION

The quark-model Hamiltonian H consists of the phenomenological confinement potential U^{ct}_{ij}, the colored version of the full Fermi-Breit (FB) interaction U^{FB}_{ij} with explicit quark-mass dependence, and the EMEP U^{b}_{ij} generated from the scalar (Ω = S), pseudoscalar (PS), and vector (V) meson-exchange potentials acting between quarks:

\[
H = \sum_{i=1}^{6} \left( m_i c^2 + \frac{P_i^2}{2m_i} - T_G \right) + \sum_{i<j} \left( U^{ct}_{ij} + U^{FB}_{ij} + \sum_{\beta} U^{b}_{ij}^{\beta} + \sum_{\beta} U^{pr\beta}_{ij} + \sum_{\beta} U^{r\beta}_{ij} \right).
\]

(2.1)

It is important to include the momentum-dependent Bryan-Scott term [4] in the S- and V-meson contributions, in order to remedy the shortcoming of our previous model FSS; namely, the single-particle (s.p.) potential in nuclear matter is too attractive in the high-momentum region k ≥ 6 fm^{-1}. Another important feature of the present model is the introduction of vector mesons for improving the fit to the NN phase-shift parameters. Since the dominant effect of the ω-meson repulsion and the LS components of ρ, ω, and K^0 mesons are already accounted for by the FB interaction, only the quadratic LS component of the octet mesons is expected to play an important role in partially canceling the strong pion tensor force. Further details of the model fss2 are given in [2]. The model parameters are fixed to reproduce the most recent results of the phase-shift analysis SP99 [5] for np scattering with partial waves J≤2 and incident energies T_{lab} ≤ 350 MeV, under the constraint of the deuteron binding energy and the 1S_0 NN scattering length, as well as the low-energy YN total cross section data. Owing to the introduction of the vector mesons, the model fss2 in the NN sector has attained an accuracy almost comparable to that of one-boson-exchange-potential (OBEP) models. For example, the χ^2 values defined by χ^2 = \sum_{i=1}^{N} (a_i - \alpha_i)^2/N for the J=2 phase-shift parameters in the energy range T_{lab} = 2S–300 MeV are \sqrt{\chi^2} = 0.59°, 1.10°, 1.40°, and 1.32° for fss2, OBEP, Paris, and Bonn, respectively. The existing data for the YN scattering are well reproduced and the essential feature of the AN-SN coupling is almost unchanged from our previous models.
We ignore the EMEP terms $U_{ij}^{S,N}$, which is this a direct consequence of the fact that the glueons do not have the flavor degree of freedom. In the EMEP terms, the $SU_3$ scalar property of the interaction is not apparent since the mesons have the flavor degree of freedom. Nevertheless, one can easily show that the interaction Hamiltonian is actually $SU_3$ scalar if the masses of the octet mesons are all equal within each of the $S$, $PS$, and $V$ systems. A nice feature of the quark model is that the approximate $SU_3$-scalar property of the total Hamiltonian is automatically incorporated in the model. On the other hand, in the OBEP a similar situation is only realized by assuming the $SU_3$ relations for the many baryon-meson coupling constants. If the Hamiltonian is exactly $SU_3$ scalar, the $SU_3$ symmetry appears in several $Y$ states with a common $(\lambda \mu)$, is approximately $SU_3$ scalar. If we ignore the EMEP terms $U^{S,P,N}_{ij}$, this is fairly apparent since the flavor dependence appears only through the moderate mass difference of the up-down and strange quarks. In the $FB$ interaction $U^{FB}_{ij}$, this is a direct consequence of the fact that the glueons do not have the flavor degree of freedom. In the EMEP terms, the $SU_3$ scalar property of the interaction is not apparent since the mesons have the flavor degree of freedom. 

The two-baryon systems composed of the complete baryon octet are classified as

$$1/2(11) \times 1/2(11) = \{0, 1\} \{1(22) + (30) + (03) + (11)_{s} + (00)\},$$

where $S(\lambda \mu)$ stands for the spin value $S$ and the flavor $SU_3$ representation label $(\lambda \mu)$. If the space-spin states are classified by the flavor-exchange symmetry $P$, the correspondence between the $SU_3$ basis and the isospin basis becomes very transparent, as in Table I. This correspondence is essential in the following discussion. The key point is that the quark-model Hamiltonian, Eq. (2.1), is approximately $SU_3$ scalar. If we ignore the EMEP terms $U^{S,P,N}_{ij}$, this is fairly apparent since the flavor dependence appears only through the moderate mass difference of the up-down and strange quarks. In

<table>
<thead>
<tr>
<th>$S$</th>
<th>$B_sB_s(I)$</th>
<th>$P=+1$ (symmetric)</th>
<th>$P=-1$ (antisymmetric)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$^{1}E$ or $^{3}O$</td>
<td>$^{3}E$ or $^{1}O$</td>
</tr>
<tr>
<td>0</td>
<td>$NN(I=0)$</td>
<td>–</td>
<td>(03)</td>
</tr>
<tr>
<td></td>
<td>$NN(I=1)$</td>
<td>(22)</td>
<td>–</td>
</tr>
<tr>
<td>$\Lambda N$</td>
<td>$\frac{1}{\sqrt{10}}[(11)_{s} + 3(22)]$</td>
<td>$\frac{1}{\sqrt{2}}[-(11)_{s} + (03)]$</td>
<td></td>
</tr>
<tr>
<td>$-1$</td>
<td>$\Sigma N(I=1/2)$</td>
<td>$\frac{1}{\sqrt{10}}[3(11)_{s} - (22)]$</td>
<td>$\frac{1}{\sqrt{2}}[(11)_{s} + (03)]$</td>
</tr>
<tr>
<td></td>
<td>$\Sigma N(I=3/2)$</td>
<td>(22)</td>
<td>(30)</td>
</tr>
<tr>
<td>$2$</td>
<td>$\Sigma \Lambda$</td>
<td>$\sqrt{3}(11)_{s} - 1/2 \sqrt{10}(22) - \sqrt{3}/2(00)$</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$\Sigma \Sigma (I=0)$</td>
<td>$\sqrt{3}(11)_{s} - 1/2 \sqrt{10}(22)$</td>
<td>$1/\sqrt{6} [2(11)_{s} + (30) + (03)]$</td>
</tr>
<tr>
<td></td>
<td>$\Sigma \Sigma (I=1)$</td>
<td>(22)</td>
<td>–</td>
</tr>
<tr>
<td>$3$</td>
<td>$\Xi \Lambda$</td>
<td>$\frac{1}{\sqrt{10}}[(11)_{s} + 3(22)]$</td>
<td>$\frac{1}{\sqrt{2}}[-(11)_{s} + (03)]$</td>
</tr>
<tr>
<td></td>
<td>$\Xi \Sigma (I=1/2)$</td>
<td>$\frac{1}{\sqrt{10}}[3(11)_{s} - (22)]$</td>
<td>$\frac{1}{\sqrt{2}}[(11)_{s} + (03)]$</td>
</tr>
<tr>
<td></td>
<td>$\Xi \Sigma (I=3/2)$</td>
<td>(22)</td>
<td>(03)</td>
</tr>
<tr>
<td>$4$</td>
<td>$\Xi \Xi (I=0)$</td>
<td>–</td>
<td>(30)</td>
</tr>
<tr>
<td></td>
<td>$\Xi \Xi (I=1)$</td>
<td>(22)</td>
<td>–</td>
</tr>
</tbody>
</table>
phase shifts of these channels have very similar behavior, as is shown in Fig. 1. In reality, the SU_3 symmetry is broken, but in a very specific way. The mechanism of the flavor symmetry breaking (FSB) depends on the details of the model. In the present framework, the following three factors cause FSB.

1. The strange to up-down quark mass ratio \( \lambda = m_s/m_{ud} = 1.551 \) (for fss2) > 1 in the kinetic-energy term and \( U^{FB}_{ij} \).
2. The singlet-octet meson mixing in \( U^{S,PS,V}_{ij} \).
3. The meson and baryon mass splitting in \( U^{S,PS,V}_{ij} \) and the kinetic-energy term, and the resultant difference of the threshold energies.

III. CHARACTERISTICS OF THE NN AND YN INTERACTIONS AND THE BASIC VIEWPOINT

The approximate SU_3-scalar property of the Hamiltonian implies that accurate knowledge of the NN and YN interactions is crucial to understand the \( B_S B_A \) interactions in the \( S = -2, -3, \) and \( -4 \) sectors. The following four points concern the qualitative behavior of the NN and YN interactions are essential [1,2]. First, in the NN system the \( ^1S_0 \) state with isospin \( I = 1 \) consists of the pure \( (22) \) state, and the phase shift shows a clear resonance behavior reaching more than 60° (see Fig. 1). On the other hand, the \( ^3S_1 \) state with \( I = 1 \) is composed of the pure \( (03) \) state, and the deuteron is bound in this channel owing to the strong one-pion tensor force. If we switch off this strong tensor force, the \( ^3S_1 \) phase shift rises to 20°–30° at most, indicating that the central attraction of the \( (11) \) state is not as strong as that of the \( (22) \) state (see crosses in Fig. 2). Detailed analysis of the YN interaction has clarified that the \( (11) \) state for the \( ^1S_0 \) state and the \( (30) \) state for the \( ^3S_1 \) state are both strongly repulsive, reflecting that the most compact \( (0S)^6 \) configuration is completely Pauli forbidden for the \( (11) \) state and almost forbidden for the \( (30) \) state [6]. The repulsive behavior of the \( \Sigma N(I = 3/2)^3S_1 \) state with the pure \( (30) \) symmetry should be observed as a strong isospin dependence of the \( \Sigma \) s.p. potential. On the other hand, the experimental evidence of the repulsion is not clear for the \( \Sigma N(I = 1/2)^1S_0 \) state, which contains 90% \( (11) \) component. This is because the observables are usually composed of the contributions both from the \( ^1S_0 \) and \( ^3S_1 \) states, and the \( ^3S_1 \) state in this channel involves a rather cumbersome \( \Lambda N - \Sigma N(I = 1/2) \) channel coupling. Nevertheless, the repulsive character of the \( (11) \) state is not inconsistent with the present experimental evidence, in the sense that FSS and fss2 reproduce the available low-energy cross section data of the \( \Lambda p \) and \( \Sigma p \) scatterings quite well. The strength of this repulsion depends on the detailed framework of the quark model. In the OBEP approach, even the qualitative features of these interactions are sometimes not reproduced. For example, in the \( \Sigma N(I = 3/2) \) \( ^3S_1 \) state almost all the Nijmegen soft-core models [7,8] predict a broad resonance around the intermediate-energy region of \( p_S \sim 400–600 \) MeV, although the low-energy behavior of the phase shifts is surely repulsive.

The following two additional features of the present

FIG. 1. \( ^1S_0 \) phase shifts for the \( B_S B_A \) interactions with the pure \( (22) \) state.

FIG. 2. \( ^3S_1 \) phase shifts for the \( (03) \) [\( \Sigma N, \Xi \Sigma(I = 3/2) \), \( (11) \)\( \Xi N(I = 0) \)], and \( (30) \) [\( \Sigma N(I = 3/2), \Xi \Xi \)] states. The \( ^3S_1 \) phase shift predicted only by the \( NN \) central interaction is also shown by crosses.

FIG. 3. Total elastic cross sections for the pure \( (22) \) state (\( pp, \Sigma^+ \Sigma^- \), \( \Xi^- \Xi^- \)) and for the \( (22) + (03) \) states (\( np, \Xi^- \Sigma^- \)).
involved in the system increases. The role of the pion is generally reduced if the strangeness $S$ interchange between $S = 1$ and $S = -1$ sector is moderately attractive, and for the $S = 3/2$ state becomes less effective due to the strong repulsion of the $(30)$ configuration, the most attractive $S = 1$ state is expected for the $\Xi \Sigma (I = 3/2)$ interaction. Table I shows that this interaction has a very interesting feature: the two SU$_3$ symmetries $(22)$ and $(03)$ in the $NN$ interaction appear in a common isospin state $I = 3/2$. Unlike the $(03)$ state in $NN$, the $(03)$ state in $\Xi \Sigma (I = 3/2)$ is only moderately attractive, since the $\Xi \Sigma$ system does not allow the strong one-pion exchange in the exchange Feynman diagram. Even in the direct Feynman diagram, the inclusion of the strangeness reduces the one-pion-exchange effect drastically through the SU$_3$ relations. The $\Xi^- \Sigma^-$ interaction thus gives the largest total cross sections in the strangeness sector, together with the $\Xi^0 \Sigma^0$ interaction, as seen in Fig. 3. The magnitudes of these total cross sections, however, are at most comparable with the $pp$ total cross sections. The $\Lambda \Lambda \Xi \Sigma (I = 1/2)$ coupled-channel problem, on the other hand, is less interesting, since the one-pion tensor force in the $3S_1 + 3D_1$ state becomes less effective due to the strong repulsion of the $(30)$ component. This is in contrast with the strong $\Lambda N \Sigma \Sigma (I = 1/2)$ channel coupling, which leads to the well-known cusp structure in the $\Lambda N$ total cross sections.

### IV. RESULTS IN S = −2, −3, AND −4 SECTORS

Table I shows a "reflection" symmetry with respect to the interchange between $S = 0$ and $S = −4$ sectors and, also, between $S = −1$ and $S = −3$ sectors. Just like the particle-hole symmetry in the nuclear shell model, the SU$_3$ state on one side is obtained from the $(\lambda \mu)$ state on the other side by simply interchanging the $\lambda$ and $\mu$. Since the $(22)$ [and also $(11)$] symmetry in the $3S_0$ state returns to itself, the $NN$, $\Lambda N$, and $\Sigma N$ interactions with $N$ being replaced by $\Xi$ should be very similar to the original ones. On the other hand, in the $3S_1$ state the $(03)$ symmetry changes into the $(30)$ symmetry and the attractive interaction turns to the repulsive one. For example, the $\Xi \Xi (I = 0)$ interaction with the pure $(30)$ symmetry is repulsive. Figure 3 shows the $\Xi \Xi$ total cross sections are about $1/4–1/5$ of the $NN$ cross sections. This result is different from the very large prediction of the Nijmegen soft-core potentials in [10], which have strong attractions in all the $(22)$ channels except for the $NN$ and $\Sigma^+ p$ ones. Among the strangeness $B_s B_s$ channels having the $1S_0$ $(22)$ configuration, the most attractive $S = 1$ state is expected for the $\Xi \Sigma (I = 3/2)$ interaction. Table I shows that this interaction has a very interesting feature: the two SU$_3$ symmetries $(22)$ and $(03)$ in the $NN$ interaction appear in a common isospin state $I = 3/2$. Unlike the $(03)$ state in $NN$, the $(03)$ state in $\Xi \Sigma (I = 3/2)$ is only moderately attractive, since the $\Xi \Sigma$ system does not allow the strong one-pion exchange in the exchange Feynman diagram. Even in the direct Feynman diagram, the inclusion of the strangeness reduces the one-pion-exchange effect drastically through the SU$_3$ relations. The $\Xi^- \Sigma^-$ interaction thus gives the largest total cross sections in the strangeness sector, together with the $\Xi^0 \Sigma^0$ interaction, as seen in Fig. 3. The magnitudes of these total cross sections, however, are at most comparable with the $pp$ total cross sections. The $\Lambda \Lambda \Xi \Sigma (I = 1/2)$ coupled-channel problem, on the other hand, is less interesting, since the one-pion tensor force in the $3S_1 + 3D_1$ state becomes less effective due to the strong repulsion of the $(30)$ component. This is in contrast with the strong $\Lambda N \Sigma \Sigma (I = 1/2)$ channel coupling, which leads to the well-known cusp structure in the $\Lambda N$ total cross sections.

The baryon-baryon interactions in the $S = −2$ sector constitute the most difficult case to analyze, involving three different types of two-baryon configurations: $\Lambda \Lambda \Xi \Sigma \Sigma$ for $I = 0$ and $\Xi \Sigma \Sigma \Sigma \Lambda \Sigma$ for $I = 1$. In this case, the isospin dependence of the interaction is very important, just as in the $\Sigma N (I)$ interactions with $I = 1/2$ and $3/2$. Figure 4 shows the $1S_0$ phase-shift behavior of the full $\Lambda \Lambda \Xi \Sigma \Sigma$ coupled-channel system with $I = 0$, in which the $H$-dibaryon bound state might exist. In the previous model FSS the $\Lambda \Lambda$ phase shift rises to $40°$ [11], while in the present fss2 it rises only to $20°$ at most. The situation is the same as in the $\Xi \Sigma (I$
However, the recently discovered “Demachi-Yanagi event” phase-shift rise on the order of 40° is at least necessary to have the fss2 interaction, it has been claimed that a phase-shift rise on the order of 40° is at least necessary to explain the known three events of the double Λ hypernuclei. However, the recently discovered “Demachi-Yanagi event” [12] for $^{10}_{\Lambda\Lambda}$Be and “Nagara event” [13] for $^{6}_{\Lambda\Lambda}$He indicate that the $\Lambda\Lambda$ interaction is less attractive. A rough estimate of $\Delta B_{\Lambda\Lambda}$ for $^{6}_{\Lambda\Lambda}$He in terms of the $G$-matrix calculation using fss2 is about 1 MeV, which is consistent with this experimental observation.

In the isospin $I = 1$ channel, the lowest incident baryon channel in the $S = -2$ sector is the $\Xi N$ channel. Figure 5 shows the phase-shift behavior of the $^{1}S_{0}$ and $^{3}S_{1}$ states, calculated for the full coupled-channel system $\Xi N-\Sigma\Lambda-\Sigma\Sigma$ with $I = 1$. In the $\Xi N(I = 1)$ single-channel calculation, both of these phase shifts show monotonic repulsive behavior, originating from the main contributions of the $(11)$ and $(30)$ components, respectively [9]. In the full-coupled-channel calculation, however, the channel-coupling effect between $\Xi N(I = 1)$ and $\Sigma\Lambda$ channels is enhanced by the cooperative role of the FB contribution $U^{(\beta)}_{ij}$ and EMEP contribution $U^{(\beta)}_{ij}$ in the strangeness-exchange process. As a result, the $\Xi N(I = 1)$ phase shifts show very prominent cusp structure at the $\Sigma\Lambda$ threshold, as seen in Fig. 5. Below the $\Sigma\Lambda$ threshold, the phase-shift values are almost zero. Subsequently, the $\Xi^{-}p$ (and $\Xi^{-}n$) total cross sections with the pure $I = 1$ component are predicted to be very small below the $\Sigma\Lambda$ threshold around $p_{\Xi} \sim 600$ MeV/c. This behavior of the $\Xi^{-}p$ total cross sections, illustrated in Fig. 6(b), is essentially the same as the Nijmegen result in [10]. On the other hand, the $\Xi^{-}n$ total cross sections, shown in Fig. 6(a), exhibit a typical channel-coupling behavior similar to that of the $\Sigma^{-}p$ total cross sections. These features demonstrate that the $\Sigma\Lambda$ channel-coupling effect is very important for the correct description of scattering observables, resulting in the strong isospin dependence of the $\Xi N$ interaction.

V. SUMMARY

The conversion processes among octet baryons $B_{8}$ are most straightforwardly incorporated in the coupled-channel formalism. Since our quark model Hamiltonian is approximately SU3 scalar, it is crucial to clarify the characteristics of the $B_{8}B_{8}$ interaction for each of the SU3 states, rather than for each of the two-baryon systems. An interbaryon potential in a single-baryon channel is sometimes used for the study of hypernuclei and strangeness nuclear matter. However, such effective interactions are very much model dependent and linkage to the bare interactions like the ones discussed here is sometimes obscured by inherent ambiguities originating from the many-body calculations. The widespread argument that the hypernuclear structure reflects the hyperon-nucleon (YN) interaction rather faithfully since the YN interaction is weak should not be overemphasized.

In this study we have upgraded our previous quark model [1] for the nucleon-nucleon (NN) and YN interactions by incorporating more complete effective meson-exchange potentials such as the vector mesons and some extra interaction pieces. This model, fss2 [2], reproduces the existing data of the NN and YN interactions quite well. We have then proceeded to predict all the $B_{8}B_{8}$ interactions in the strangeness $S = -2, -3$, and $-4$ sectors, without adding any extra parameters. We have discussed some characteristic features of the $B_{8}B_{8}$ interactions, focusing on the qualitative aspect. Among these features are the following: (1) There is no bound state in the $B_{8}B_{8}$ systems, except for the deuteron. (2) The $\Xi N$ total cross sections are far smaller than the $NN$ cross sections. (3) The $\Xi N$ interaction has a strong isospin dependence similar to the $\Sigma N$ system. (4) The $\Xi^{-}\Sigma^{-} [\Xi\Sigma(I = 3/2)]$ interaction is moderately attractive. The $S$-wave phase-shift behavior yielding these qualitative features of the $B_{8}B_{8}$ interactions is systematically understood by (1) the spin-flavor SU3 symmetry, (2) the special role of pion exchange, and (3) the flavor symmetry breaking. A more detailed analysis of the $B_{8}B_{8}$ interactions consistent with the available experimental data will be published in a forthcoming paper.

ACKNOWLEDGMENTS

This research was supported by Japan Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports and Culture (12640265).


[5] Scattering Analysis Interactive Dial-up (SAID), Virginia Polytechnic Institute, Blacksburg, Virginia; R. A. Arndt (private communication).


