Faddeev calculation of the hypertriton using the SU_6 quark-model nucleon-nucleon and hyperon-nucleon interactions

Y. Fujiwara*
Department of Physics, Kyoto University, Kyoto 606-8502, Japan

K. Miyagawa
Department of Applied Physics, Okayama Science University, Okayama 700-0005, Japan

M. Kohno
Physics Division, Kyushu Dental College, Kitakyushu 803-8580, Japan

Y. Suzuki
Department of Physics, Niigata University, Niigata 950-2181, Japan
(Received 22 March 2004; published 6 August 2004)

Quark-model nucleon-nucleon and hyperon-nucleon interactions by the Kyoto-Niigata group are applied to the hypertriton calculation in a three-cluster Faddeev formalism using the two-cluster resonating-group method kernels. The most recent model, fss2, gives a reasonable result similar to the Nijmegen soft-core model NSC89, except for an appreciable contribution of higher partial waves.

DOI: 10.1103/PhysRevC.70.024001 PACS number(s): 21.45.+v, 13.75.Ev, 21.80.+a, 21.30.−x

I. INTRODUCTION

The QCD-inspired spin-flavor SU_6 quark model for the baryon-baryon interaction, proposed by the Kyoto-Niigata group, is a unified model for the complete baryon octet ($B_8 = N$, Λ, Σ and Ξ), which has achieved very accurate descriptions of the nucleon-nucleon (NN) and hyperon-nucleon (YN) interactions [1–3]. In particular, the NN interaction of the most recent model fss2 [2] is accurate enough to compare with the modern realistic meson-exchange models. These quark-model interactions can be used for realistic calculations of few-baryon and few-cluster systems, once an appropriate three-body equation is formulated for the pairwise interactions described by the resonating-group method (RGM) kernels. The desired three-cluster equation should be able to deal with the nonlocality and the energy dependence intrinsically involved in the quark-exchange RGM kernel. Furthermore, the quark-model description of the YN and hyperon-hyperon (YY) interactions in the full coupled-channel formalism sometimes involves a Pauli forbidden state at the quark level, which excludes the most compact $(0s)^6$ spatial configuration, resulting in the strongly repulsive nature of the interactions in some particular channels. We have recently formulated a three-cluster equation which uses two-cluster RGM kernels explicitly [4]. This equation exactly eliminates three-cluster redundant components by requiring the orthogonality of the total wave function to the pairwise two-cluster Pauli-forbidden states. The explicit energy dependence inherent in the exchange RGM kernel is self-consistently treated. This equation is entirely equivalent to the Faddeev equation which uses a singularity-free T matrix (which we call the RGM T matrix) generated from the two-cluster RGM kernel.

We first applied this formalism to three di-neutron and three-alpha systems, and obtained complete equivalence between the Faddeev calculations and variational calculations using the translationally invariant harmonic-oscillator (h.o.) basis [4,5]. This formalism was also applied to the Faddeev calculation of the three-nucleon bound state [6], employing the off-shell T matrices which are derived from the nonlocal and energy-dependent RGM kernels for our quark-model NN interactions, fss2 and FSS. The model fss2 yields the triton binding energy $B_t = 8.519$ MeV in the 50 channel calculation, when the np interaction is employed for all the NN pairs in the isospin basis [7]. The effect of the charge dependence of the two-body NN interaction is estimated to be −0.19 MeV for the triton binding energy [8]. This implies that our result is not overbinding in comparison with the empirical value, $B_t^{\rm exp} = 8.482$ MeV. If we attribute the difference, 0.15 MeV, to the effect of the three-nucleon force, it is by far smaller than the generally accepted values, $0.5 \sim 1$ MeV [9], predicted by many Faddeev calculations employing modern realistic meson-theoretical NN interactions. The charge rms radii for ^{3}H and ^{3}He are also correctly reproduced. The nonlocal description of the short-range repulsion in the quark model is essential to reproduce the large binding energy and the correct size of the three-nucleon bound state without reducing the D-state probability of the deuteron.

Here we apply our quark-model NN and YN interactions to the hypertriton ($^3_\Lambda$H) with the small separation energy of the Λ particle, $B_\Lambda^{\rm exp} = 130 \pm 50$ keV. Since the Λ particle is far apart from the two-nucleon subsystem, the on-shell properties of the ΛN and ΣN interactions are expected to be well reflected in this system. In particular, this system is very useful to determine the relative strength of $^1S_0$ and $^3S_1$ interactions in our framework. We will be able to fine tune the quark model interaction to the hypertriton binding energy. This enables firmer quark-model predictions for the $^1S_0$ and

*Electronic address: fujiwara@ruby.scphys.kyoto-u.ac.jp

$^3S_1$ phase shifts. In fact, Refs. [10–12] showed that it is not at all trivial to bind the hypertriton as most meson-theoretical interactions fail to bind the hypertriton, except for the Nijmegen soft-core potentials NSC89 [13], NSC97f and NSC97e [14]. It is also pointed out in Refs. [10,12] that a small admixture of the ΣNN components less than 1% is very important for this binding.

In the next section, the Faddeev equation of the hypertriton system, using the quark-model *NN* and *YN* RGM kernels, is discussed, paying a special attention to the ΛN-ΣN $\tilde{T}$ matrix. The results are given in Sec. III and the summary in Sec. IV. The Appendix gives some essential points to derive the Faddeev equation, whose solution exactly satisfies the orthogonality conditions.

II. FORMULATION

We start from the three-cluster equation of the ΛNN-ΣNN system interacting via two-cluster RGM kernels

$$P[E - H_0 - V_{12}^{\rm RGM}(\varepsilon_{12}) - V_{31}^{\rm RGM}(\varepsilon_{31}) - V_{32}^{\rm RGM}(\varepsilon_{32})]P\Psi = 0, \tag{1}$$

where E is the negative three-body energy measured from the ΛNN threshold, and the free Hamiltonian $H_0 = H' + \Delta m_3$ is composed of the kinetic-energy operators $H' = h_{31} + h_2$, etc., and the mass term Δm_3. In the following, the two nucleons are numbered 1 and 2, the Λ or Σ is numbered 3. The equation actually implies the 2×2 matrix form and $P\Psi$ is the two-dimensional vectors composed of the upper component with ΛNN configuration and the lower one ΣNN. The mass term Δm_3 is, therefore, a diagonal matrix whose matrix elements are zero for the ΛNN channel and $\Delta M_{\Sigma\Lambda} = M_\Sigma c^2 - M_\Lambda c^2$ for the ΣNN channel. Here M_Λ and M_Σ are the Λ and Σ masses, respectively. The RGM kernel $V_{ij}^{\rm RGM}(\varepsilon_{ij}) = V_{{\rm D}ij} + G_{ij} + \varepsilon_{ij}K_{ij}$ consists of the direct potential $V_{{\rm D}ij}$, the sum of the exchange kinetic energy and interaction kernels, $G_{ij} = G_{ij}^{\rm K} + G_{ij}^{\rm V}$, and the exchange normalization kernel K_{ij} multiplied with the center-of-mass energy ε_{ij} of the ij subsystem for the relative motion. These are also 2×2 matrices. For example,

$$\varepsilon_{31} = \begin{pmatrix} \varepsilon_{\Lambda N} & 0 \\ 0 & \varepsilon_{\Sigma N} \end{pmatrix} = \begin{pmatrix} \varepsilon_{\Lambda N} & 0 \\ 0 & \varepsilon_{\Lambda N - \Delta M_{\Sigma\Lambda}} \end{pmatrix},$$

$$K_{31} = \begin{pmatrix} K_{\Lambda N,\Lambda N} & K_{\Lambda N,\Sigma N} \\ K_{\Sigma N,\Lambda N} & K_{\Sigma N,\Sigma N} \end{pmatrix}. \tag{2}$$

The two-cluster RGM equation is expressed as

$$[\varepsilon_{31} - h_{31} - V_{31}^{\rm RGM}(\varepsilon_{31})]\chi = 0. \tag{3}$$

The necessity of the projection operator P in Eq. (1) is related to the existence of the eigenstate of the ΛN-ΣN normalization kernel; $K_{31}|u_{31}\rangle = \gamma|u_{31}\rangle$ with the eigenvalue $\gamma = 1$. This is the most compact $(0s)^6$ spin-singlet configuration with the flavor SU_3 quantum number $(11)_s$; $|u_{31}\rangle = |(0s)^6; (11)_s^1 S_0\rangle$. We seek for the Pauli-allowed state of the ΛNN-ΣNN system by diagonalization

$$\sum_{i=1,2} |u_{3i}\rangle\langle u_{3i}|\Psi_\lambda\rangle = \lambda|\Psi_\lambda\rangle \tag{4}$$

in the antisymmetric model space, $\Psi_\lambda(123) = -\Psi_\lambda(213)$, which we denote as $\Psi_\lambda \in [11]$ in the following. The projection operator on the Pauli-allowed space, P, is defined by taking the model space spanned by the eigenvectors with $\lambda = 0$

$$P = \sum_{\lambda=0} |\Psi_\lambda\rangle\langle\Psi_\lambda|. \tag{5}$$

The three-cluster Faddeev equation, which is completely equivalent to Eq. (1), is derived by using some nice properties satisfied by P, which is briefly discussed in the Appendix. The total wave function of the hypertriton system is expressed as a superposition of two independent Faddeev components ψ and ϕ: $P\Psi = \psi + (1 - P_{12})\phi$ with $\psi \in [11]$. The coupled-channel Faddeev equation reads

$$\psi = G_0(E)T_{12}(E, \varepsilon_{12})(1 - P_{12})\phi,$$

$$\phi = G_0(E)\tilde{T}_{31}(E, \varepsilon_{31})(\psi - P_{12}\phi), \tag{6}$$

where P_{12} is the exchange operator of particles 1 and 2, $G_0(E) = 1/(E - H_0)$ is the three-body free Green function for the negative energy $E < 0$, $T_{12}(E, \varepsilon_{12})$ is the *NN* T matrix in the three-body space, and $\tilde{T}_{31}(E, \varepsilon_{31})$ is the redundancy-free ΛN-ΣN $\tilde{T}$ matrix in the coupled-channel formalism. These T matrices are generated from the *NN* and *YN* RGM kernels $V_{NN}^{\rm RGM}(\varepsilon_{NN})$ and $V_{YN}^{\rm RGM}(\varepsilon_{YN})$, respectively, according to the prescription essentially given in Ref. [4]. The energy dependence involved in these kernels is treated self-consistently by calculating the matrix elements of the two-cluster Hamiltonian, which will be discussed in some detail.

For the *NN* sector, $T_{12}(E, \varepsilon_{12})$ in the ΛNN and ΣNN spaces are given by the two-body T matrix $t(\omega, \varepsilon)$ through a simple replacement of the starting-energy argument

$$T_{12}(E, \varepsilon_{12}) = \begin{pmatrix} t_{NN}^\Lambda & 0 \\ 0 & t_{NN}^\Sigma \end{pmatrix},$$

$$t_{NN}^\Lambda = t_{NN}\left(E - \frac{\hbar^2}{M_N}\frac{\zeta_\Lambda + 2}{4\ \zeta_\Lambda}\boldsymbol{q}^2, \varepsilon_{NN}\right),$$

$$t_{NN}^\Sigma = t_{NN}\left(E - \Delta M_{\Sigma\Lambda} - \frac{\hbar^2}{M_N}\frac{\zeta_\Sigma + 2}{4\ \zeta_\Sigma}\boldsymbol{q}^2, \varepsilon_{NN}\right), \tag{7}$$

where M_N is the nucleon mass, $\zeta_\beta = M_\beta/M_N$ is the mass ratio for $\beta = \Lambda$ or Σ, and $\boldsymbol{q}$ is the momentum of the residual Λ or Σ. The *NN* relative energy in the three-body space, ε_{NN}, is determined from

$$\varepsilon_{NN} = \langle P\Psi|h_{NN} + V_{NN}^{\rm RGM}(\varepsilon_{NN})|P\Psi\rangle, \tag{8}$$

which is actually the sum of the ΛNN and ΣNN components.

For the *YN* sector, the situation is more complicated since the ΛN-ΣN coupling involves a complete Pauli forbidden state $|u_{31}\rangle$ in the three-body space and the difference of the momentum dependent starting energies in the ΛN and ΣN

channels is not constant because of the two types of reduced masses between YN and N. In fact, the equation satisfied by $\tilde{T}_{31}(E,\varepsilon_{31})$ can never be reduced to the two-cluster coupled-channel equation in the free space. We request

$$T_{31}(E,\varepsilon_{31}) = V_{31}^{\rm RGM}(\varepsilon_{31}) + V_{31}^{\rm RGM}(\varepsilon_{31})G_0(E)T_{31}(E,\varepsilon_{31}),$$

$$\tilde{T}_{31}(E,\varepsilon_{31}) = T_{31}(E,\varepsilon_{31}) - (E-H_0)|u_{31}\rangle \times[E-\varepsilon_{31}-h_2^-\Delta m_3]^{-1}\langle u_{31}|(E-H_0). \quad (9)$$

However, this equation is not actually satisfied, since the derivation is based on the unrealistic assumptions

(1) $[E-\varepsilon_{31}-h_2^-\Delta m_3]$ is channel independent,

(2) $[\varepsilon_{31}-h_{31}-V_{31}^{\rm RGM}(\varepsilon_{31})]|u_{31}\rangle=0$ is satisfied.

The second condition is only approximately satisfied since, in the strict RGM framework, it is not possible to use the empirical internal energies of clusters and reduced masses for the relative kinetic-energy operators. In fact, the correct expression for (2) is

$$[\varepsilon_0 - h_0 - V_{31}^{\rm RGM}(\varepsilon_0)]|u_{31}\rangle = 0, \quad (10)$$

where $\varepsilon_0=\varepsilon_{\Lambda N}-E_{\rm int}$ with $E_{\rm int}$ being the calculated internal energies, and h_0 uses the calculated reduced mass unlike h_{31}. We can choose $(E_{\rm int})_\Lambda=0$ for the ΛNN channel, but $(E_{\rm int})_\Sigma=\Delta M_{\Sigma\Lambda}$ is only approximately satisfied in our quark models, fss2 and FSS. This difficulty also takes place when we try to derive the basic orthogonality condition of the $\tilde{T}_{31}(E,\varepsilon_{31})$

$$\langle u_{31}|[\,1+G_0(\omega)\tilde{T}_{31}(E,\varepsilon_{31})] = 0, \quad (11)$$

which is essential to yield the orthogonality of the total wave function through the second equation, Eq. (6); $\langle u_{3i}|\psi+(1-P_{12})\phi\rangle=0$ $(i=1,2)$.

Fortunately, these problems are completely solved by simply adding a small correction term to the RGM kernel, which is a procedure developed in Ref. [15] for making it possible to use the empirical values of the internal energies and reduced masses in the RGM formalism. In Ref. [15], we have slightly modified the original two-cluster RGM equation $[\varepsilon_0-h_0-V_{31}^{\rm RGM}(\varepsilon_0)]\chi=0$ and considered the following RGM equation in the OCM (orthogonality condition model) form:

$$\Lambda[\varepsilon - h_0^{\rm exp} - V_{\rm RGM}(\varepsilon_0)]\Lambda\chi = 0. \quad (12)$$

From here on, we omit the subscript 31 or YN as much as possible, in order to simplify the notations. For example, $V_{\rm RGM}(\varepsilon_0)=V_{31}^{\rm RGM}(\varepsilon_0)$ and $\Lambda=\Lambda_{31}=1-|u\rangle\langle u|$ with $|u\rangle=|u_{31}\rangle$ in Eq. (12). Furthermore, $\varepsilon=\varepsilon_{\Lambda N}-E_{\rm int}^{\rm exp}$ in Eq. (12) uses the empirical internal energy, $E_{\rm int}^{\rm exp}=\Delta m_3$, and the relative kinetic-energy operator, $h_0{}^{\rm exp}=h_{31}=-(\hbar^2/2\mu^{\rm exp})\nabla^2$, uses the empirical reduced mass, $\mu^{\rm exp}=\mu_{\Lambda N}^{\rm exp}$ or $\mu_{\Sigma N}^{\rm exp}$. On the other hand, we need to use ε_0 with the calculated internal energies in the RGM kernel. It is shown in Ref. [15] that Eq. (12) is converted to Eq. (3) by simply adding ΔG to $V_{\rm RGM}(\varepsilon_0)$; i.e., $V_{31}^{\rm RGM}(\varepsilon_{31})=V_{\rm RGM}(\varepsilon_0)+\Delta G$. The explicit expression of ΔG is given in the paper [or $\Delta\varepsilon=0$ case in Eq. (16) below]. We use the same idea to eliminate the channel dependence of $[E-\varepsilon_{31}-h_2^-\Delta m_3]$ in (1). Let us start from the off-shell extension of the ΛN-ΣN coupled-channel RGM equation in the following OCM form:

$$\Lambda[\,\omega - h_0^{\rm exp} - V_{\rm RGM}(\varepsilon_0)]\Lambda\chi + |u\rangle(\omega-\varepsilon)\langle u|\chi\rangle = 0. \quad (13)$$

The solution of this equation satisfies the orthogonality, $\langle u|\chi\rangle=0$ for $\omega\neq\varepsilon$. The various energies in Eq. (13) are usually channel dependent; namely, if we use the label $\beta=\Lambda$ or Σ to specify the ΛN or ΣN channel, the diagonal matrix elements of these energies are given by

$$\omega = E - \frac{\hbar^2}{2M_N}\frac{\zeta_\beta+2}{\zeta_\beta+1}q^2 - E_{\rm int}^{\rm exp},$$

$$\varepsilon = \varepsilon_{\Lambda N} - \frac{\hbar^2}{2M_N}\left(\frac{\zeta_\beta+2}{\zeta_\beta+1} - \frac{\zeta_\Lambda+2}{\zeta_\Lambda+1}\right)\boldsymbol{q}^2 - E_{\rm int}^{\rm exp},$$

$$\varepsilon_0 = \varepsilon_{\Lambda N} - E_{\rm int}, \quad (14)$$

where E is the negative three-body energy, $\boldsymbol{q}$ this time is the momentum of the residual nucleon. We can prove that Eq. (13) is equivalent to the following Schrödinger-type RGM equation:

$$\omega\chi = [\,h_0^{\rm exp} + V_{\rm RGM}^{\rm mod}(\varepsilon)]\chi, \quad (15)$$

where a newly defined RGM kernel $V_{\rm RGM}^{\rm mod}(\varepsilon)$ is given by

$$V_{\rm RGM}^{\rm mod}(\varepsilon) = V_{\rm RGM}(\varepsilon_0) + \Delta G,$$

$$\Delta G = \Lambda(\Delta E_{\rm int} + \Delta\varepsilon + \Delta h_0)\Lambda - (\Delta E_{\rm int} + \Delta\varepsilon + \Delta h_0),$$

$$\Delta E_{\rm int} = E_{\rm int}^{\rm exp} - E_{\rm int}, \quad \Delta h_0 = h_0{}^{\rm exp} - h_0,$$

$$\Delta\varepsilon = \frac{\hbar^2}{2M_N}\left(\frac{1}{\zeta_\beta+1} - \frac{1}{\zeta_\Lambda+1}\right)\boldsymbol{q}^2. \quad (16)$$

The RGM T matrix $\tilde{T}_{31}(E,\varepsilon_{31})$ is therefore formulated for this modified RGM kernel $V_{\rm RGM}^{\rm mod}(\varepsilon)$. By repeating the same process as to derive Eq. (9) with respect to $V_{\rm RGM}^{\rm mod}(\varepsilon)$, we can find that Eq. (9) [and also Eq. (11)] is just valid if we replace (see Appendix for details)

$$V_{31}^{\rm RGM}(\varepsilon_{31}) \to V_{\rm RGM}^{\rm mod}(\varepsilon),$$

$$[E-\varepsilon_{31}-h_2^-\Delta m_3]^{-1} \to [E-\varepsilon_{\Lambda N}-(h_2^-)_\Lambda]^{-1}. \quad (17)$$

In order to determine $\varepsilon_{\Lambda N}$ or ε_0, we approximate ω in Eq. (15) as

$$\omega = \begin{pmatrix} E - \frac{\hbar^2}{2M_N}\frac{\zeta_\Lambda+2}{\zeta_\Lambda+1}\boldsymbol{q}^2 & 0 \\ 0 & E - \frac{\hbar^2}{2M_N}\frac{\zeta_\Sigma+2}{\zeta_\Sigma+1}\mathbf{q}^2 - \Delta M_{\Sigma\Lambda} \end{pmatrix} \to \begin{pmatrix} \varepsilon_{\Lambda N} & 0 \\ 0 & \varepsilon_{\Sigma N} \end{pmatrix} \quad (18)$$

with keeping the relationship $\varepsilon_{\Sigma N}=\varepsilon_{\Lambda N}-\Delta M_{\Sigma\Lambda}$ in the free space, and first calculate a ΛN-ΣN averaged value

$$\varepsilon_{YN} = \langle P\Psi |\ h_0^{\rm exp} + V_{\rm RGM}^{\rm mod}(\varepsilon)|P\Psi\rangle. \quad (19)$$

Then, from $\varepsilon_{YN}=\varepsilon_{\Lambda N}\langle\Psi|\Psi\rangle^{\Lambda}+\varepsilon_{\Sigma N}\langle\Psi|\Psi\rangle^{\Sigma}$ and $\Delta M_{\Sigma\Lambda}=\varepsilon_{\Lambda N}-\varepsilon_{\Sigma N}$, we find

$$\varepsilon_{\Lambda N} = \varepsilon_{YN} + \Delta M_{\Sigma\Lambda}\ P_{\Sigma},$$

$$\varepsilon_{\Sigma N} = \varepsilon_{YN} - \Delta M_{\Sigma\Lambda}(1 - P_{\Sigma}), \quad (20)$$

where $P_{\Sigma}=\langle\Psi|\Psi\rangle^{\Sigma}$ is the probability of the ΣNN component admixed in the hypertriton wave function.

III. RESULT

The angular-momentum states of the *NN*-*Y* channel are specified by $|[(\lambda s)I,(\ell\frac{1}{2})j]\frac{1}{2};[(\frac{1}{2}\frac{1}{2})t,t]0\rangle$, where $(\lambda s)I$ stands for the two-nucleon state, ${}^{2s+1}\lambda_I$, and t is the isospin value $t=0$ for Λ and $t=1$ for Σ. Due to the antisymmetric property of the two nucleons, $(-)^{\lambda+s+t}=-1$, we find that the ${}^3E$ and ${}^1O$ states contribute to the ΛNN channel and the ${}^1E$ and ${}^3O$ states contribute to the ΣNN channel. For the *YN*-*N* channel, the angular-momentum states are specified by $|[(\ell_1 s_1)I_1,(\ell_2\frac{1}{2})j_1]\frac{1}{2};[(t\frac{1}{2})\frac{1}{2},\frac{1}{2}]0\rangle$. Since the isospin of the hypertriton is zero, only the isospin $T=1/2$ sector of the ΛN-ΣN interaction contributes to the hypertriton calculation. All the partial waves of the orbital angular momentum are possible for each of the ΛNN and ΣNN channels, which makes the number of channels for a particular partial-wave truncation just three times larger than in the triton Faddeev calculation. The hyperon species of the *YNN* channels are uniquely specified by the isospin value $t=0$ or 1. For the orbital part, the parity conservation requires $(-1)^{\lambda+\ell}=(-1)^{\ell_1+\ell_2}=1$. The channel truncation is specified by the maximum value of the total angular momenta of the pairwise baryons, I and I_1, which we denote J. As an example, all the channels of the standard 15-channel calculation with S and D waves are listed in Table I.

For the numerical calculation, we discretize the continuous momentum variable p (or q) for the Jacobi coordinate vectors, using the Gauss-Legendre n_1- (or n_2-) point quadrature formula, for each of the three intervals of $0-1$ fm^{-1}, $1-3$ fm^{-1} and $3-6$ fm^{-1}. The small contribution from the intermediate integral over p beyond $p_0=6$ fm^{-1} in the two-body T-matrix calculation is also taken into account by using the Gauss-Legendre n_3-point quadrature formula through the mapping $p=p_0+\tan\{\pi(1+x)/4\}$. [These n_3 points for p are not included for solving the Faddeev equation Eq. (6), since it causes a numerical inaccuracy for the interpolation.] The momentum region $q=6$ fm$^{-1}-\infty$ is also discretized by the n_3-point formula just as in the p discretization case. We take n_1-n_2-n_3=10-10-5 as is used for the triton Faddeev calculation in Refs. [6,7], for which well converged results are obtained. The partial-wave decomposition of the two-cluster RGM kernel is carried out numerically using the Gauss-Legendre 20-point quadrature formula. The modified spline interpolation technique developed in Ref. [16] is employed for simplifying the treatment of the rearrangement of the Jacobi momentum coordinates. The Faddeev formalism with two identical particles or clusters is discussed in Ref. [17], together with some formulas for calculating the matrix elements of the two-cluster Hamiltonian. For the diagonalization of the large nonsymmetric matrix appearing in solving Faddeev equations, the Arnoldi-Lanczos algorithm developed in the ARPACK subroutine package [18] is very useful.

TABLE I. The channel quantum numbers of the hypertriton included in 15-channel Faddeev calculation with S and D waves. For the *NN*-*Y* channels, I and j are coupled to the total angular momentum $\frac{1}{2}^+$ and the isospin coupling is $[(\frac{1}{2}\frac{1}{2})t;t]0$. For the *YN*-*N* channels, these are $[I_1 j_2]\frac{1}{2}$ and $[(t\frac{1}{2})\frac{1}{2};\frac{1}{2}]0$. The isospin value t specifies the hyperon species; i.e., $Y=\Lambda$ for $t=0$ and $Y=\Sigma$ for $t=1$.

NN-*Y*	${}^{2s+1}\lambda_I$	$(\ell\frac{1}{2})j$	t
1	${}^3S_1$	$(S\frac{1}{2})\frac{1}{2}$	0
2	${}^3S_1$	$(D\frac{1}{2})\frac{3}{2}$	0
3	${}^3D_1$	$(S\frac{1}{2})\frac{1}{2}$	0
4	${}^3D_1$	$(D\frac{1}{2})\frac{3}{2}$	0
5	${}^1S_0$	$(S\frac{1}{2})\frac{1}{2}$	1
YN-*N*	${}^{2s_1+1}(\ell_1)_{I_1}$	$(\ell_2\frac{1}{2})j_2$	t
1	${}^3S_1$	$(S\frac{1}{2})\frac{1}{2}$	0 (1)
2	${}^3S_1$	$(D\frac{1}{2})\frac{3}{2}$	0 (1)
3	${}^3D_1$	$(S\frac{1}{2})\frac{1}{2}$	0 (1)
4	${}^3D_1$	$(D\frac{1}{2})\frac{3}{2}$	0 (1)
5	${}^1S_0$	$(S\frac{1}{2})\frac{1}{2}$	0 (1)

Table II shows the results of the Faddeev calculations using fss2 and our previous model FSS. In the 15-channel calculation including the S and D waves of the *NN* and *YN* interactions, we have already obtained $B_{\Lambda}=-\varepsilon_d-E({}^3_{\Lambda}\mathrm{H})\sim 200$ keV for fss2. The convergence with the extension to the higher partial waves is very rapid, and the total angular momentum of the baryon pairs with $J\leqslant 4$ is good enough for 1 keV accuracy. As for the converged B_{Λ} values with 150-channel ΛNN and ΣNN configurations, we obtain $B_{\Lambda}=289$ keV and the ΣNN component $P_{\Sigma}=0.80\%$ for the fss2 prediction, and $B_{\Lambda}=878$ keV and $P_{\Sigma}=1.36\%$ for FSS. Table III shows the correlation between the Λ separation energies B_{Λ} and the ${}^1S_0$ and ${}^3S_1$ effective range parameters of FSS, fss2 and NSC89 ΛN interactions. Although all of these ΛN interactions reproduce the low-energy ΛN total cross section data below $p_{\Lambda}\sim 300$ MeV/c within the experimental error bars, our quark-model interactions seem to be more attractive than the Nijmegen soft-core potential NSC89 [13]. The model FSS gives a large overbinding since the ${}^1S_0$ ΛN interaction is strongly attractive. The phase-shift difference of the ${}^1S_0$ and ${}^3S_1$ states at $p_{\Lambda}\sim 200$ MeV/c is $\delta({}^1S_0)-\delta({}^3S_1)\sim 29°$ for FSS, while $\delta({}^1S_0)-\delta({}^3S_1)\sim 7°$ for fss2. Since the present fss2 result is still slightly overbound, this difference should be somewhat smaller in order to reproduce the correct experimental value $B_{\Lambda}^{\rm exp}=130\pm 50$ keV. From the two results given by fss2 and FSS, we can extrapolate that, in our quark model, the desired difference is $0°\sim 2°$.

In order to make sure that this extrapolation gives a good estimation, we modify the κ-meson mass of the model fss2

TABLE II. Results of the hypertriton Faddeev calculations by fss2 and FSS. The momentum discretization points are n_1-n_2-n_3 = 10-10-5 with q_1-q_2-q_3 = 1-3-6 fm^{-1}. The calculated deuteron binding energy is ε_d = 2.2247 MeV for fss2 and 2.2561 MeV for FSS ($\varepsilon_d^{\rm exp}$ = 2.2246 MeV). The heading E is the hypertriton energy measured from the ΛNN threshold; B_Λ is the Λ separation energy; ε_{NN} ($\varepsilon_{\Lambda N}$) is the NN (ΛN) expectation value determined self-consistently; and P_Σ is the ΣNN probability in percent. The norm of redundant components, $N_{\rm red}^2 = \langle u_{3i} | P\Psi \rangle^2$ (i = 1, 2), is less than 10^{-9}.

Model	No. of channels	E (MeV)	B_Λ (keV)	ε_{NN} (MeV)	$\varepsilon_{\Lambda N}$ (MeV)	P_Σ (%)
	6 (S)	−2.362	137	−1.815	3.548	0.450
	15 (SD)	−2.423	198	−1.762	5.729	0.652
	30 ($J \leqslant 1$)	−2.403	178	−1.786	5.664	0.615
fss2	54 ($J \leqslant 2$)	−2.498	273	−1.673	5.974	0.777
	78 ($J \leqslant 3$)	−2.510	285	−1.660	6.014	0.800
	102 ($J \leqslant 4$)	−2.513	288	−1.658	6.022	0.804
	126 ($J \leqslant 5$)	−2.514	289	−1.657	6.024	0.805
	150 ($J \leqslant 6$)	−2.514	289	−1.657	6.024	0.805
	6 (S)	−2.910	653	−1.309	3.984	1.022
	15 (SD)	−2.967	710	−1.433	6.171	1.200
	30 ($J \leqslant 1$)	−2.947	691	−1.427	6.143	1.191
FSS	54 ($J \leqslant 2$)	−3.121	865	−1.323	6.467	1.348
	78 ($J \leqslant 3$)	−3.128	872	−1.320	6.480	1.357
	102 ($J \leqslant 4$)	−3.134	877	−1.317	6.488	1.360
	126 ($J \leqslant 5$)	−3.134	878	−1.317	6.488	1.361
	150 ($J \leqslant 6$)	−3.134	878	−1.317	6.489	1.361

from the original value, $m_\kappa c^2$ = 936 MeV [2], to 1000 MeV, and repeat the whole calculation. It is known that this modification makes the $^1S_0$ ΛN interaction less attractive and the $^3S_1$ more attractive. We obtain B_Λ = 145 keV with P_Σ = 0.53%. The effective range parameters of this modified fss2 interaction are a_s = −2.15 fm, r_s = 3.05 fm, and a_t = −1.80 fm, r_t = 2.87 fm. The phase-shift difference is only 1.3° and the total cross section of the ΛN scattering increases at most 10 mb at p_Λ = 100 MeV/c from 286 mb to 296 mb, which is still within the experimental error bars.

It should be kept in mind that the effective range parameters or the S-wave phase-shift values determined in this way are very much model dependent, since the B_Λ value is not solely determined by these quantities. It depends on how higher partial waves contribute and also on the details of the

TABLE III. $^1S_0$ and $^3S_1$ effective range parameters of FSS [1], fss2 [2,3], and NSC89 [13] ΛN interactions (Λp for NSC89) and the Λ separation energies B_Λ of the hypertriton. The values for NSC89 are taken from Ref. [12].

Model	a_s (fm)	r_s (fm)	a_t (fm)	r_t (fm)	B_Λ (keV)
FSS	−5.41	2.26	−1.03	4.20	878
fss2	−2.59	2.83	−1.60	3.01	289
NSC89	−2.59	2.90	−1.38	3.17	143

TABLE IV. Decomposition of the NN and YN expectation values (ε_{NN} and ε_{YN}), the deuteron energy ($-\varepsilon_d$) and the total three-body energy E to the kinetic-energy and potential-energy contributions. The unit is in MeV. The results for NSC89 are from Ref. [10].

Model	$h_{NN} + V_{NN} = \varepsilon_{NN}$
FSS	19.986 − 21.303 = −1.317
fss2	19.376 − 21.032 = −1.657
NSC89	20.48 − 22.25 = −1.77
Model	$h_d + V_d = -\varepsilon_d$ (deuteron)
FSS	16.982 − 19.238 = −2.256
fss2	17.495 − 19.720 = −2.225
NSC89	19.304 − 21.528 = −2.224
Model	$h_{YN} + V_{YN} = \varepsilon_{YN}$
FSS	10.036 − 4.602 = 5.435
fss2	8.071 − 2.671 = 5.401
NSC89	7.44 − 3.54 = 3.90
Model	$\langle H_0 \rangle + \langle V \rangle = E$
FSS	27.372 − 30.506 = −3.134
fss2	23.860 − 26.374 = −2.514
NSC89	23.45 − 25.79 = −2.34

ΛN-ΣN coupling of a particular model. A nice extrapolation shown here is based on the similarity of the models fss2 and FSS, which have a common framework for the quark sector and the effective meson-exchange potentials.

Table II also shows that the expectation value of the NN Hamiltonian, ε_{NN}, determined self-consistently is rather close to the deuteron energy $-\varepsilon_d$, especially in fss2. This feature is even marked if we decompose these energies to the kinetic-energy and potential-energy contributions. Table IV shows this decomposition with respect to fss2, FSS and NSC89. (For this comparison, we use the definition of the kinetic-energy part of the deuteron by $h_d = \langle \chi_d | h_{NN} | \chi_d \rangle / \langle \chi_d | \chi_d \rangle$, where χ_d is the RGM relative wave function between the neutron and the proton.) In fss2, the kinetic energy of the NN subsystem is 1.88 MeV larger than that of the deuteron, which implies that the NN subsystem shrinks by the effect of the outer Λ particle, in comparison with the deuteron in the free space. In NSC89, this difference is even smaller; i.e., 1.18 MeV. These results are consistent with the fact that the hypertriton in NSC89 is more loosely bound (B_Λ = 143 keV [12]) than in fss2 (289 keV), and the Λ particle is very far apart from the NN cluster. The ΣNN probability in NSC89 is P_Σ = 0.5% [10,12]. Table IV also lists the kinetic-energy and potential-energy decompositions for the averaged YN expectation value ε_{YN} and the total energy E. The kinetic energies of ε_{YN} are much smaller than those of ε_{NN}, which indicates that the relative wave functions between the hyperon and the nucleon are widely spread in the configuration space. The comparison of the total-energy decomposition shows that the wave functions of fss2 and NSC89 may be very similar. A clear difference between fss2 and NSC89 appears in the roles

of higher partial waves. The energy increase due to the higher partial waves than the *S* and *D* waves is 91 keV in fss2 and 168 keV in FSS, respectively. On the other hand, the results in Ref. [10] imply that this is only 20–30 keV in the case of NSC89. This difference can originate from both of the *NN* and *YN* interactions. Since the characteristics of the meson-theoretical *YN* interactions in higher partial waves are *a priori* unknown, more detailed analysis of the fss2 results might shed light on the adequacy of the quark-model baryon-baryon interactions.

IV. SUMMARY

In this study, we have carried out the Faddeev calculations of the hypertriton, using the recent quark-model *NN* and *YN* interactions, FSS [1] and fss2 [2,3]. These are realistic interactions which describe all the available *NN* and *YN* data, by incorporating the effective meson-exchange potentials at the quark level. Since these quark-model baryon-baryon interactions are formulated in the RGM framework, they are nonlocal, energy dependent, and sometimes involve the Pauli-forbidden component at the quark level. The hypertriton is an appropriate place to investigate the roles of the compact SU_3 $(11)_s$ component, which is completely Pauli forbidden in the $^1S_0$ ΛN-ΣN channel coupling with the isospin $T=1/2$. In order to deal with this off-shell effect of the quark-model interaction precisely, we have formulated a type of the Faddeev equation which explicitly employs the two-cluster RGM kernels [4,5]. The energy dependence of the RGM kernel is self-consistently treated, by calculating the expectation values of the two-cluster Hamiltonian with respect to the obtained solutions of the Faddeev equation [17]. We have especially paid attention to how to extend the microscopic description of the ΛN-ΣN coupling in the hypertriton system without spoiling the effect of the Pauli principle at the quark level. The present study is the second application of this formalism to the few-baryon systems interacting via the quark-model baryon-baryon interactions, following our previous one to the triton system [6,7]. The hypertriton is well suited to investigate the on-shell properties of the ΛN and ΣN interactions, since the hyperon is very far apart from the two-nucleon cluster.

We have found that our quark-model interaction fss2 gives a reasonable result for the hypertriton properties, which is rather similar to the result of the Nijmegen soft-core model NSC89 [13]. The Λ separation energy given by fss2 is $B_\Lambda = 289$ keV, which is a little too large in comparison with the experimental value $B_\Lambda{}^{\rm exp} = 130 \pm 50$ keV. The admixture of the ΣNN component is $P_\Sigma = 0.80\%$. Modifying the κ-meson mass of fss2 from the original value, $m_\kappa c^2 = 936$ MeV, to 1000 MeV leads to the almost correct Λ-separation energy 145 keV with $P_\Sigma = 0.53\%$. Unlike the NSC89 result, the effects of higher partial waves up to the *G* wave are rather important in the quark-model *NN* and *YN* interactions. If we use the dominant *S*-wave character of the ΛN interaction in the hypertriton system, the $^1S_0$ ΛN interaction of the model fss2 is slightly too attractive. It is a future problem to investigate whether or not a reduction in the $^1S_0$ attraction like the modification to $m_\kappa c^2 = 1000$ MeV produces a favorable feature for the level spacing of the 0^+ and 1^+ states of $^4_\Lambda$H and $^4_\Lambda$He systems. The fairly large charge symmetry breaking in these systems is also an important issue to understand the hyperon-nucleon interactions in detail.

ACKNOWLEDGMENT

This work was supported by Grants-in-Aid for Scientific Research (C) from the Japan Society for the Promotion of Science (JSPS) (Grant Nos. 15540270, 15540284 and 15540292).

APPENDIX: DERIVATION OF THE THREE-CLUSTER FADDEEV EQUATION FOR THE ΛNN-ΣNN SYSTEM

In this Appendix, we discuss some essential points to derive the Faddeev equation Eq. (6) from the three-cluster equation Eq. (1). In the derivation, we extensively use the following properties of the projection operator *P* defined in Eq. (5):

(i) $\Lambda_{3i} P = P \Lambda_{3i} = P \quad (i = 1,2)$,

$$\text{where } \Lambda_{3i} = 1 - |u_{3i}\rangle\langle u_{3i}|,$$

(ii) when $\Psi \in [11]$, $\quad \forall \langle u_{3i}|\Psi\rangle = 0 \leftrightarrow P\Psi = \Psi$,

(iii) when $\Psi \in [11]$, $\quad P\Psi = 0 \leftrightarrow \exists\, |f\rangle$,

$$\Psi = |u_{31} f_2\rangle - |u_{32} f_1\rangle. \tag{A1}$$

Using the property (i), we can replace $V_{3i}^{\rm RGM}(\varepsilon_{3i})$ in Eq. (1) by $v_{3i}(\varepsilon_{3i}) = \Lambda_{3i} V_{3i}^{\rm RGM}(\varepsilon_{3i}) \Lambda_{3i}$ or

$$\mathcal{V}_{3i}(E, \varepsilon_{3i}) = (E - H_0) - \Lambda_{3i}(E - H_0)\Lambda_{3i} + v_{3i}(\varepsilon_{3i}). \tag{A2}$$

We further use the property (iii) for the whole equation and introduce the ansatz for the Faddeev components, $P\Psi = \psi + (1 - P_{12})\phi$, to derive a pair of equations

$$[E - H_0 - V_{12}^{\rm RGM}(\varepsilon_{12})]\psi = V_{12}^{\rm RGM}(\varepsilon_{12})(1 - P_{12})\phi,$$

$$[E - H_0 - \mathcal{V}_{31}(E, \varepsilon_{31})]\phi = \mathcal{V}_{31}(E, \varepsilon_{31})(\psi - P_{12}\phi) + |u_{31} f_2\rangle. \tag{A3}$$

In the second equation, we note that

$$E - H_0 - \mathcal{V}_{31}(E, \varepsilon_{31}) = \Lambda_{31}[E - H_0 - V_{3i}^{\rm RGM}(\varepsilon_{3i})]\Lambda_{31}, \tag{A4}$$

and introduce the projected two-body Green function in the three-body space, $G_{\Lambda_{31}}(E, \varepsilon_{31})$, which satisfies

$$G_{\Lambda_{31}}(E, \varepsilon_{31})\Lambda_{31}[E - H_0 - v_{31}(\varepsilon_{31})]\Lambda_{31} = \Lambda_{31}. \tag{A5}$$

This can be easily constructed through

$$G_{\Lambda_{31}}(E,\varepsilon_{31}) = G_{v_{31}}(E,\varepsilon_{31}) - G_{v_{31}}(E,\varepsilon_{31})|u_{31}\rangle \times \frac{1}{\langle u_{31}|G_{v_{31}}(E,\varepsilon_{31})|u_{31}\rangle}\langle u_{31}|G_{v_{31}}(E,\varepsilon_{31}) \tag{A6}$$

by using the two-body Green function $G_{v_{31}}(E,\varepsilon_{31})=[E-H_0-v_{31}(\varepsilon_{31})+i0]^{-1}$ in the three-body space. The essential equation we need for deriving the full Green function $G_{31}(E,\varepsilon_{31})=[E-H_0-V_{31}^{\mathrm{RGM}}(\varepsilon_{31})+i0]^{-1}$ is the decomposition

$$\begin{aligned} E - H_0 - V_{31}^{\mathrm{RGM}}(\varepsilon_{31}) &= (E - \varepsilon_{31} - h_2^- - \Delta m_3) \\ &\quad - \Lambda_{31}(E - \varepsilon_{31} - h_2^- - \Delta m_3)\Lambda_{31} \\ &\quad + \Lambda_{31}[E - H_0 - V_{31}^{\mathrm{RGM}}(\varepsilon_{31})]\Lambda_{31} \\ &= |u_{31}\rangle(E - \varepsilon_{31} - h_2^- - \Delta m_3)\langle u_{31}| \\ &\quad + \Lambda_{31}[E - H_0 - V_{31}^{\mathrm{RGM}}(\varepsilon_{31})]\Lambda_{31}, \end{aligned} \tag{A7}$$

but the last equality is actually not satisfied since $(E-\varepsilon_{31}-h_2^- - \Delta m_3)$ is channel dependent. This difficulty is avoided by using $V_{\mathrm{RGM}}^{\mathrm{mod}}(\varepsilon)$ in Eq. (16), in place of $V_{31}^{\mathrm{RGM}}(\varepsilon_{31})$. In fact, we find that

$$\begin{aligned} &(E - \varepsilon_0 - h_2^- - \Delta m_3) - (h_{31} - h_0) + (\Delta E_{\mathrm{int}} + \Delta\varepsilon + \Delta h_0) \\ &\quad = E - \varepsilon_{\Lambda N} - (h_2^-)_\Lambda \end{aligned} \tag{A8}$$

is channel independent. Here ε_0 and $(h_{31}-h_0)$ terms appear since $|u_{31}\rangle$ actually satisfies Eq. (10) and not Eq. (3). This makes it possible to derive our basic relationship

$$\begin{aligned} G_0(E)T_{31}(E,\varepsilon) &= G_{31}(E,\varepsilon)V_{\mathrm{RGM}}^{\mathrm{mod}}(\varepsilon) \\ &= G_{\Lambda_{31}}(E,\varepsilon)\mathcal{V}_{31}(E,\varepsilon) - |u_{31}\rangle\langle u_{31}| + |u_{31}\rangle \\ &\quad \times \frac{1}{E - \varepsilon_{\Lambda N} - (h_2^-)_\Lambda}\langle u_{31}|(E - H_0) \\ &= G_0(E)\tilde{T}_{31}(E,\varepsilon) + |u_{31}\rangle\frac{1}{E - \varepsilon_{\Lambda N} - (h_2^-)_\Lambda} \\ &\quad \times \langle u_{31}|(E - H_0), \end{aligned} \tag{A9}$$

where all the kernels are defined by using $V_{\mathrm{RGM}}^{\mathrm{mod}}(\varepsilon)$. From Eq. (A9) we can easily prove the second Faddeev equation, Eq. (6), and the orthogonality condition, Eq. (11).

[1] Y. Fujiwara, C. Nakamoto, and Y. Suzuki, Phys. Rev. Lett. **76**, 2242 (1996); Phys. Rev. C **54**, 2180 (1996).
[2] Y. Fujiwara, T. Fujita, M. Kohno, C. Nakamoto, and Y. Suzuki, Phys. Rev. C **65**, 014002 (2002).
[3] Y. Fujiwara, M. Kohno, C. Nakamoto, and Y. Suzuki, Phys. Rev. C **64**, 054001 (2001).
[4] Y. Fujiwara, H. Nemura, Y. Suzuki, K. Miyagawa, and M. Kohno, Prog. Theor. Phys. **107**, 745 (2002).
[5] Y. Fujiwara, Y. Suzuki, K. Miyagawa, M. Kohno, and H. Nemura, Prog. Theor. Phys. **107**, 993 (2002).
[6] Y. Fujiwara, K. Miyagawa, M. Kohno, Y. Suzuki, and H. Nemura, Phys. Rev. C **66**, 021001(R) (2002).
[7] Y. Fujiwara, K. Miyagawa, Y. Suzuki, M. Kohno, and H. Nemura, Nucl. Phys. **A721**, 983c (2003).
[8] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
[9] A. Nogga, H. Kamada, and W. Glöckle, Phys. Rev. Lett. **85**, 944 (2000).
[10] K. Miyagawa, H. Kamada, W. Glöckle, and V. Stoks, Phys. Rev. C **51**, 2905 (1995).
[11] K. Miyagawa, H. Kamada, W. Glöckle, H. Yamamura, T. Mart, and C. Bennhold, Few-Body Syst., Suppl. **12**, 324 (2000).
[12] A. Nogga, H. Kamada, and W. Glöckle, Phys. Rev. Lett. **88**, 172501 (2002); A. Nogga, Ph.D. thesis, University of Bochum, 2001.
[13] P. M. M. Maessen, Th. A. Rijken, and J. J. de Swart, Phys. Rev. C **40**, 2226 (1989).
[14] Th. A. Rijken, V. G. J. Stoks, and Y. Yamamoto, Phys. Rev. C **59**, 21 (1999).
[15] Y. Fujiwara, M. Kohno, C. Nakamoto, and Y. Suzuki, Prog. Theor. Phys. **104**, 1025 (2000).
[16] W. Glöckle, G. Hasberg, and A. R. Neghabian, Z. Phys. A **305**, 217 (1982).
[17] Y. Fujiwara, K. Miyagawa, M. Kohno, Y. Suzuki, D. Baye, and J.-M. Sparenberg, nucl-th/0404071, Phys. Rev. C (in press).
[18] See ARPACK homepage, http://www.caam.rice.edu/software/ARPACK/