Interface superconductivity in the eutectic Sr$_2$RuO$_4$-Ru: 3-K phase of Sr$_2$RuO$_4$

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Enhanced superconductivity in the eutectic system Sr$_2$RuO$_4$-Ru is referred to as the 3-K phase of the spin-triplet superconductor Sr$_2$RuO$_4$ because of its enhanced superconducting transition temperature $T_c$ of $\sim$3 K. We have investigated the field-temperature ($H$-$T$) phase diagram of the 3-K phase for fields parallel and perpendicular to the $ab$ plane of Sr$_2$RuO$_4$, using out-of-plane resistivity measurements. We have found an upturn curvature in the $H_{c2}(T)$ curve for $H||c$, and a rather gradual temperature dependence of $H_{c2}$ close to $T_c$ for both $H||ab$ and $H||c$. We have also investigated the dependence of $H_{c2}$ on the angle between the field and the $ab$ plane at several temperatures. Fitting the Ginzburg-Landau effective-mass model apparently fails to reproduce the angle dependence, particularly near $H||c$ and at low temperatures. We propose that all of these characteristic features can be explained, at least in a qualitative fashion, on the basis of a theory by Sigrist and Monien that assumes surface superconductivity with a two-component order parameter occurring at the interface between Sr$_2$RuO$_4$ and Ru inclusions. This provides evidence of the chiral state postulated for the 1.5-K phase by several experiments.

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I. INTRODUCTION

Sr$_2$RuO$_4$ is the first layered perovskite superconductor without copper; it is isostructural to the cuprate high-temperature superconductor La$_2$CuO$_4$. This is one of the reasons that Sr$_2$RuO$_4$ has attracted great attention$^2$ despite its superconducting transition temperature $T_c$, being rather low (ideally 1.5 K)$^{3,4}$ In fact, a number of theoretical and experimental studies have revealed its unconventional nature. More importantly, it is now well established that Sr$_2$RuO$_4$ is a spin-triplet superconductor, in contrast to the spin-singlet $d$-wave pairing in high-$T_c$ cuprates.$^2$ This was first confirmed by$^{17}$O-NMR measurements.$^5$ The Knight shift is unaffected by the superconducting transition, strongly suggestive of spin-triplet pairing with the spin of Cooper pairs lying within the $ab$ plane.$^5$ Also, the observation of spontaneous magnetic moments accompanying the superconducting state indicates broken time-reversal symmetry,$^5$ suggesting a two-component order parameter with a relative phase of $\pi/2$. Subsequent experiments of Ru-NMR (Ref. 7) and polarized neutron scattering,$^8$ both of which measure the Knight shift, support the$^{17}$O-NMR measurements.$^5$ A detailed small-angle neutron scattering study has revealed the vortex field distribution, which cannot be understood without the two-component order parameter.$^5$ These results constrain the basic form of the vector order parameter to be $d(k)=\Delta_0(k_x+ik_y)$, which is called a chiral state.

Since the Fermi surface consists of three cylindrical (quasi-two-dimensional) sheets,$^{10,11}$ the above vector order parameter leads to an isotropic gap $\Delta(k)=\Delta_0\sqrt{k_x^2+k_y^2}$. However, a number of experimental results$^{12-18}$ have revealed the power-law temperature dependence of various thermodynamic quantities and thus strongly suggest lines of nodes in the superconducting gap. This fact postulates modifications to be made to the basic form $d(k)=\Delta_0(k_x+ik_y)$. In fact, several theories have been proposed to reconcile the discrepancy between those experimental facts and the vector order parameter $d(k)=\Delta_0(k_x+ik_y)$.$^{19,20}$ These theories take into account the orbital dependent superconductivity$^{21}$ and propose very strong in-plane anisotropy$^{20}$ or horizontal lines of nodes$^{19}$ in the superconducting gap to explain the power-law temperature dependence in thermodynamic data.

Amongst several remarkable features related to Sr$_2$RuO$_4$, an enhancement of $T_c$ in the Sr$_2$RuO$_4$-Ru eutectic system, is rather surprising. This enhancement was found during the course of the optimization of sample growth. While the ideal $T_c$ of Sr$_2$RuO$_4$ turned out to be 1.5 K,$^{3,4}$ the ac susceptibility of certain batches was known to exhibit rather weak diamagnetism at a considerably higher onset temperature of about 3 K. A clear resistance drop, below which the resistance does not necessarily fall to zero, was also observed at a very close...
transition is followed by the original superconducting transition with an onset of about 3 K. On further cooling, this
of Sr$_2$RuO$_4$ and Ru. In the top panel of Fig. 1, lamellae
uniquely determined by the composition at the eutectic point
though the appearance of the Ru inclusions may depend on
thermodynamical direction often varies even
within a piece small of single crystal of Sr$_2$RuO$_4$.
Such a eutectic system shows a broad superconducting transition with an onset of about 3 K. On further cooling, this transition is followed by the original superconducting transi-
tion of Sr$_2$RuO$_4$ at 1.5 K. The higher-$T_c$ superconductivity is called the 3-K phase and the original lower-$T_c$ superconduc-
tivity is referred to as the 1.5-K phase. The manifestations of 3-K phase superconductivity in resistance and ac susceptibility suggest that the superconductivity is inhomogeneous and filamentary. In addition, 3-K phase superconductivity is con-
sidered to be essentially sustained in Sr$_2$RuO$_4$. Because the upper critical field of the 3-K phase is the highest (lowest)
when the applied field is parallel (perpendicular) to the $ab$
plane of Sr$_2$RuO$_4$. (That is, the anisotropy of the upper critical field reflects the crystallographical directions of Sr$_2$RuO$_4$.)

In addition to the enhancement of $T_c$ to $\sim$3 K, the field-
temperature phase diagram of the 3-K phase has intriguing properties. Earlier work of resistive measurements has re-
vealed clear hysteresis of the upper critical field $H_{c2}$ in mag-
etic fields parallel to the $ab$ plane at low temperatures: Two distinctly different $H_{c2}$’s are obtained when the applied mag-
etic field (or the temperature) is swept upwards and downwards. Also the $H_{c2}(T)$ curve for $H||c$ looks rather unusual, being nearly a straight line or possibly concave upwards.

Neither theoretically nor experimentally has very much been known about the 3-K phase thus far. However, recent tunneling measurements on $c$-axis junctions of the Sr$_2$RuO$_4$-Ru eutectic have observed zero bias conductance peaks, which is a hallmark of unconventional superconductivity. Therefore, the superconductivity in the 3-K phase is also unconventional and probably originates from the triplet pairing of Sr$_2$RuO$_4$. Also Sigrist and Monien (SM) have recently proposed a phenomenological theory that assumes surface spin-triplet superconductivity at the Sr$_2$RuO$_4$-Ru interface although the theory does not consider the mechanism of the enhanced superconductivity.

In the present work, we have studied the field-temperature phase diagram to higher precision than the previous work in Ref. 24 for a further discussion. We have also investigated the dependence of the upper critical field on the angle be-
tween the applied field and the $ab$ plane. We will discuss these results with the help of SM’s theory. In addition, we
have measured the specific heat to obtain thermodynamical evidence for nonbulk superconductivity, which supports an assumption of their theory.

II. EXPERIMENT

The eutectic samples of Sr$_2$RuO$_4$-Ru used in this study were grown by a floating zone method. The Ru inclusions were lamellate with typical dimensions of 1 $\mu$m $\times$ 10 $\mu$m $\times$ 30 $\mu$m. A surface parallel to the $ab$ plane of the sample used for resistive measurements is shown in the top panel of Fig. 1. There were no particular directions along which lamellae preferably line up throughout the whole sample. Details of the crystal growth are described in Ref. 22. We have measured the resistivity as a function of magnetic field or temperature to determine $H_{c2}(T)$ and $T_{c}(H)$.

We employed a lock-in technique at 137 Hz with a current of 0.5 mA along the $c$ axis. Low temperatures down to 60 mK.

FIG. 1. Top: Optical microscopy picture of a polished surface parallel to the RuO$_2$ plane (bright region: Ru, dark region: Sr$_2$RuO$_4$). Bottom: Schematic of the interface between Sr$_2$RuO$_4$ and a Ru inclusion modeled in Sigrist and Monien’s theory. The interface within the Sr$_2$RuO$_4$ part has a thin layer where a $p$-wave state nucleates at an enhanced transition temperature of $\sim$3 K; its wave function has lobes and nodes parallel and perpendicular to the interface, respectively.

The eutectic system, a two-phase composite structure of a single-crystalline Sr$_2$RuO$_4$ matrix and lamellar micro-
domains of ruthenium metal embedded in it, is obtained by
the same method as Sr$_2$RuO$_4$ but with excess of Ru and/or at
a faster growth speed. The top panel of Fig. 1 shows an
optical microscopy picture of a polished surface parallel to the RuO$_2$ plane. Typical dimensions of lamellae are 1 $\mu$m in thickness and 1–30 $\mu$m in length and width; the separation between adjacent lamellae is of the order of 10 $\mu$m. Although the appearance of the Ru inclusions may depend on growth conditions, the density of Ru inclusions should be uniquely determined by the composition at the eutectic point of Sr$_2$RuO$_4$ and Ru. In the top panel of Fig. 1, lamellae apparently line up along a certain direction, but empirically, there is no particular preferred orientation relative to the crystallographical axes. The direction often varies even
within a piece small of single crystal of Sr$_2$RuO$_4$.
were reached by means of a $^3$He cryostat or a dilution refrigerator. Magnetic fields of up to 5 T were generated by a superconducting solenoid. The sample was mounted in a rotator that enabled the angle between the $ab$ plane and the applied magnetic field to be changed. We have also measured the specific heat by a relaxation method applied magnetic field to be changed. We have also measured the specific heat by a relaxation method.

A. $H$-$T$ phase diagram

Figure 2 shows typical traces of the resistance as a function of magnetic field or temperature. The transition point has been defined as the inflection point associated with the superconducting transition to the 3-K phase. Figure 2 also demonstrates that the transition points determined from $H_{c2}(T)$ and $T_c(H)$ show a good agreement.

Figure 3 shows the resultant field-temperature ($H$-$T$) phase diagram of the 3-K phase of Sr$_2$RuO$_4$ for fields parallel to the $ab$ plane and the $c$ axis. This phase diagram contains a considerably larger number of transition points than those in Ref. 24, which makes possible a more detailed discussion. We, however, note that both phase diagrams appear to be very similar and consistent.

FIG. 2. Typical traces of the resistance of the 3-K phase (solid lines) and their derivatives with respect to magnetic field or temperature (dashed lines). These illustrate that the transition points (inflection points) from field sweep and temperature sweep show a good agreement.

FIG. 3. Field-temperature phase diagram of the 3-K phase. The transition points have been determined as the inflection point associated with the superconducting transition to the 3-K phase. The $H_{c2}$ curve for $H||c$ shows an upward curvature with an inflection of 2.32 K, as indicated by an arrow. The dashed curves represent fits of $(1-T/T_c)^n$ dependence to data close to $T_c$. (n=0.75 and 0.72 for $H||ab$ and $H||c$, respectively.)

As seen also in the phase diagram of Ref. 24, there are two branches, corresponding to up and down sweeps, below ~1.2 K for $H||ab$. This is a consequence of the hysteresis of $H_{c2}$ mentioned in the Introduction. As discussed in Ref. 24, two possibilities may be envisaged for the hysteresis. One is that the magnetic field effectively applied to the region responsible for the 3-K phase superconductivity is hysteretic. The second one is that the hysteresis of $H_{c2}$ is intrinsic (i.e., due to a first order transition). Obviously, the latter case is even more interesting as the superconducting transition (type II) in magnetic fields is normally second order. Possible interpretations for the latter case will include a first order transition due to spin depairing. However, we point out that this is irrelevant to the spin-triplet state we suggest (chiral state).

The present study has revealed that the lower branch (down sweep of field or temperature) of the $H_{c2}(T)$ curve for $H||ab$ is nearly flat, which seems to be rather unusual. This finding may contribute to a further understanding of the hysteretic behavior of $H_{c2}$. In addition to the hysteresis, we note two prominent features confirmed only in the $H$-$T$ phase diagram obtained in the present study. (1) The temperature dependence of $H_{c2}$ in the vicinity of $T_c$ is rather gradual. (2) An upward curvature is seen below an inflection point of 2.32 K in the $H_{c2}(T)$ line for $H||c$.

We will below propose that these two features may be explained, at least in a qualitative fashion, by SM’s recent theory. As stated in their original paper, they do not intend to consider the mechanism of the enhancement of $T_c$ in the eutectic system, but they have constructed a phenomenological theory.

The theory includes the following reasonable assumptions. First, 3-K superconductivity occurs at interfaces between Sr$_2$RuO$_4$ and Ru inclusions. For simplicity, they treat the interface as a single flat plane, as depicted in the lower panel of Fig. 1.) Second, the superconducting order parameter is represented by a two-component order parameter with
a relative phase of $\pi/2$, similar to Sr$_2$RuO$_4$. They used a Ginzburg-Landau (GL) free energy for tetragonal symmetry, with the two-component order parameter $\eta = \eta_x + i \eta_y$, corresponding to $d(k) = z\Delta_0(\eta_x k_x + i \eta_y k_y)$, which is attributed to the original superconducting transition in Sr$_2$RuO$_4$. However, a signature of the transition to the 3-K phase is barely observed in the specific heat. This thermodynamically supports the first assumption above. In contrast, the imaginary part of the ac susceptibility, displayed in the inset of SM's paper, has a finite width. Our exponents quoted recently improved the calculations in SM's paper and have obtained an exponent of about 0.7 for the temperature range used for our fitting; this exponent is very close to our results.

The formalism and assumptions Matsumoto and Sigrist have used are in principle based on those of SM's theory. Their important improvement is that Matsumoto and Sigrist use a more realistic wave function of the order parameter in magnetic fields than SM's calculations. Whereas SM used an exponentially decaying wave function as in Ref. 32, Matsumoto and Sigrist have pointed out that this functional form is appropriate only when the field is very low. In fact, Matsumoto and Sigrist's numerical results indicate that the exponent tends to 0.5 with approaching $H = H_c$, which is attributed to the original superconducting transition in Sr$_2$RuO$_4$. However, a signature of the transition to the 3-K phase is barely observed in the specific heat. This thermodynamically supports the first assumption above. In contrast, the imaginary part of the ac susceptibility, displayed in the inset of SM's theory, has a finite width. Our exponents quoted recently improved the calculations in SM's paper and have obtained an exponent of about 0.7 for the temperature range used for our fitting; this exponent is very close to our results.

While we intend to discuss the exponent in the vicinity of $H_{c2}$, the range of temperature over which the fit has been applied inevitably has a finite width. Our exponents quoted above are from a temperature range of approximately 0.9$T_c < T < T_c$. The exponent from fitting seems not to significantly depend on the temperature range over which the fitting was done, unless the lower temperature limiting the fitting range is too low or the number of the data used for the fit are too few.

SM's theory also provides a qualitative explanation for the anomalous behavior of the $H_{c2}(T)$ curve for $H || c$ (Ref. 27) (i.e., upward curvature at low temperatures and high fields). They predict that only one of the two components of a superconducting order parameter, such as $k_x$ or $k_y$, is stabilized at $T_c$ in zero applied field and that the other component with a relative phase of $\pi/2$ arises at a slightly lower temperature. However, the application of a magnetic field not perpendicular to the c axis will induce simultaneously the two components with a relative phase of $\pi/2$ at $T_c$. Since the triplet state represented by the order parameter $k_x + i k_y (0 < z = 1)$ has an orbital magnetic moment along
the $c$ axis, the state is energetically stabilized by a finite magnetic-field component parallel to the $c$ axis.] As a consequence of both components being stabilized, the coupling between the two components results in an enhancement of $H_{c2}$. In addition, the coupling becomes stronger at lower temperatures, leading to an upward curvature in the $H_{c2}(T)$ curve for $H||c$.

In addition to the mechanism described in the last paragraph, Matsumoto and Sigrist suggest that there is another mechanism for the low-temperature high-field enhancement of $H_{c2}$ for $H||c$.38 They have recently raised that the region of the enhanced superconductivity (3-K phase) has a finite width in an actual eutectic system although SM adopted the GL free energy with a $\delta$-function potential enhancing the $T_c$ at the interface. At sufficiently high fields, the spacial extension of the wave function of the order parameter will be confined within the region where the enhanced superconductivity nucleates, leading to an additional enhancement of $H_{c2}$.

**B. Angle dependence of the upper critical field**

Figure 5(a) shows the angle $\theta$ dependence of the upper critical field $H_{c2}$ at 0.29, 1.32, and 2.45 K. ($\theta$ is the angle between the $ab$ plane and the $c$ axis; $\theta=0$ corresponds to $H||ab$.) Only at 0.29 K of these three temperatures, does $H_{c2}$ show hysteresis close to $H||ab$. For 0.29 K, the hysteresis of $H_{c2}$ persists to $|\theta|=10^\circ$. [For 60 mK, the hysteresis is observed for $|\theta|\approx20^\circ$. The angle range for which the hysteresis can be seen decreases with increasing temperature. As mentioned in Sec. III A, the hysteresis of $H_{c2}$ disappears at $\sim1.2$ K even for $\theta=0^\circ$ ($H||ab$).] While the lower branch (down sweep of field) for 0.29 K is plotted with solid circles in Fig. 5(a), the upper branch (up sweep of field) is used for the fitting below in this subsection. This is because whether the up sweep or down sweep is used hardly affects the discussion below. Also shown in Fig. 5(a) are fits of the GL effective mass model

$$H_{c2}(\theta) = \frac{H_{c2||c}}{\sqrt{\sin^2 \theta + \Gamma^2 \cos^2 \theta}}.$$  

Here $\Gamma$ is the square root of the ratio of the effective mass for in-plane motion to that for inter-plane motion, $\Gamma = H_{c2||ab}/H_{c2||c}$.42 We have taken $H_{c2||ab}$ and $H_{c2||c}$ to be the adjustable parameters for the fitting. The resultant values of $(H_{c2||ab}, H_{c2||c})$ shown in Fig. 5(a) are (3.52 T, 0.92 T), (3.14 T, 0.50 T), and (1.57 T, 0.11 T) for 0.29 K (up sweep), 1.32 K and 2.45 K, respectively. [Those for 0.29 K (down sweep) is (3.33, 0.97 T); the curve is not shown.]

Although this model is known to best fit for temperatures close to $T_c$, it reproduces as a whole the $\theta$ dependence of $H_{c2}$ for the 1.5-K phase (pure $\text{Sr}_2\text{RuO}_4$) even at 60 mK.43 It should be noted here that a region close to $H||ab$ (e.g., $\Delta \theta \approx 5^\circ$) for the 1.5-K phase is exceptional due to the unusual suppression of the upper critical field; this is probably related to (or caused by) a double superconducting transition.34,43 In contrast, Fig. 5(a) exhibits that the model apparently fails to reproduce experimental results of the 3-K phase in a very wide angle range. This discrepancy between the data and the model is particularly evident for low-temperature data.

SM’s theory27 can be extended to the case of the applied field pointing to arbitrary directions within the $\text{Sr}_2\text{RuO}_4$-Ru interface plane.27,44 A discussion with a minor simplification yields an analytic functional form identical to the GL effective mass model [Eq. (1)].42 However, this analytic expression is valid for the present system only when the temperature $T$ is close to $T_c$ and/or the coupling of the two components is small (i.e., $H$ is nearly parallel to the $ab$ plane). In fact, in the framework of the GL formalism, SM resorted numerical means to investigate the behavior of $H_{c2}(T)$ for $H||c$ at low temperatures.

Consequently, fitting Eq. (1) to data for a certain angle range close to $H||ab$ will show a reasonable agreement. In this context, comparison of the data with the effective-mass model will reveal how the discrepancy becomes evident and thus will enable the enhancement of $H_{c2}$ due to the coupling of the two components to be discussed. As Fig. 5(a) indicates that fitting Eq. (1) to the whole data for $0^\circ \leq \theta \leq 90^\circ$ does not yield satisfactory results, we have fitted Eq. (1) to the data for $0^\circ \leq \theta \leq 5^\circ$ (at 0.29 and 1.32 K) and $0^\circ \leq \theta \leq 10^\circ$ (at 2.45 K), yielding $(H_{c2||ab}, H_{c2||c})$ of (3.62 T, 0.45 T), (3.16 T, 0.41 T).
The fitting described in the last paragraph are used for the deviation becomes evident is irrespective of the choice of values for the angle dependence of \( \theta \) for 2.45 K (for 0.29 and 1.32 K) and \( \theta = 10^\circ \) (for 2.45 K). (Note that the angle \( \theta \) at which the deviation becomes evident is irrespective of the choice of values for \( H_{c2}[ab] \) or \( H_{c2}[c] \).

The deviation is obviously large at low temperatures and large angles (close to \( H||c \)). In other words, \( H_{c2} \) is enhanced at low temperatures and large angles. Similarly, \( H_{c2}[c] \) from the fitting for the whole data (0° ≤ θ ≤ 90°) is larger than that from the limited range (0° ≤ θ ≤ 5° or 10°). This tendency is in very good agreement with SM’s theory. They suggest that the coupling between the two components of the order parameter, which enhances \( H_{c2} \), becomes stronger with decreasing temperature and increasing magnetic field component parallel to the c axis.

Before finishing this subsection, we here make a remark on the angle dependence of \( H_{c2} \) from another viewpoint. The deviation from the GL effective model (1) becomes larger with decreasing temperature and \( H_{c2}(\theta) \) becomes peaked in the vicinity of \( H||ab \) at low temperatures. The latter behavior is somewhat reminiscent of the two-dimensional thin film model

\[
\left( \frac{H_{c2}(\theta) \cos \theta}{H_{c2}[ab]} \right)^2 + \left( \frac{H_{c2}(\theta) \sin \theta}{H_{c2}[c]} \right)^2 = 1, \quad (2)
\]

the functional form of Eq. (1) is represented by a straight line connecting (0,1) and (1,0) in this plot. Figure 5(b) indeed illustrates that Eq. (1) fits well the data at each temperature for a limited angle range close to \( H||ab \). The data start to deviate from the functional form of Eq. (1) [i.e., the dashed straight line in Fig. 5(b)] at about \( \theta = 5^\circ \) (for 0.29 and 1.32 K) and \( \theta = 10^\circ \) (for 2.45 K). (Note that the angle \( \theta \) at which the deviation becomes evident is irrespective of the choice of values for \( H_{c2}[ab] \) or \( H_{c2}[c] \).

In summary, we have investigated the field-temperature phase diagram of the 3-K phase of Sr$_2$RuO$_4$ in detail using resistivity measurements. We have found a rather gradual temperature dependence of the upper critical field \( H_{c2} \) close to \( T_c \) and an enhancement of \( H_{c2} \) for \( H||c \) at low temperatures. We have also investigated the dependence of \( H_{c2} \) on the angle between the field and the ab plane at several temperatures. Fitting of the GL effective-mass model apparently fails to reproduce the angle dependence. All of these experimental results, with the help of the theory of SM may be interpreted in a consistent manner with other existing experimental facts. Taken together with the phenomenological theory by SM, these observations support that the 3-K phase is surface spin-triplet superconductivity with a two-component order parameter occurring at Sr$_2$RuO$_4$-Ru interfaces. This, although indirectly, supports the basic form of \( d(k) = z\Delta_0(k_x + ik_y) \) for bulk Sr$_2$RuO$_4$ as well.

Note added in proof. Recently, Matsumoto et al. have written a paper which includes part of the private communication with Matsumoto and Sigrist in Ref. 38.

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Recently, magnetization measurements on CeCoIn$_5$ have observed hysteresis of $H_{c2}$; a first order transition is discussed in T. Tayama, A. Harita, T. Sakakibara, Y. Haga, H. Shishido, R. Settai, and Y. Onuki, Phys. Rev. B 65, 180504(R) (2002).


The tetragonal symmetry of the crystal structure of Sr$_2$RuO$_4$ is conserved down to temperatures as low as 110 mK [J.S. Gardner, G. Balakrishnan, D.McK. Paul, and C. Haworth, Physica C 265, 251 (1996)].

The estimation of the 3-K phase volume fraction is based on the ratio of $\Delta C_p/\gamma_n T_c$ of the 3-K phase to that of the 1.5-K phase, with the value of $\gamma_n$ taken to be common.


33 It seems that the exponents do not strongly depend on definition of $T_c$ (or $H_{c2}$). We have also used another definition for $T_c$ (or $H_{c2}$): the onset of the superconducting transition. The exponents $n$ from this definition are $n=0.67$ and $n=0.70$ for $H||ab$ and for $H||c$, respectively.


K. Deguchi (private communication).

The fittings have been done for a temperature range of approximately 0.9$T_c$ to $T_c$, similar to the case of the 3-K phase in the present work.


M. Matsumoto and M. Sigrist (private communication).

This is particularly obvious for $H||c$ as there is an inflection point at $T=2.32$ K.

When the lower temperature limiting the fitting range is between 2.4 K (0.85$T_c$) and 2.6 K (0.93$T_c$), the resultant exponent hardly changes. The exponent for $H||ab$ stays 0.75; that for $H||c$ lies between 0.72 and 0.77. The calculations by Matsumoto and Sigrist suggest that the exponent is between 0.61 and 0.63 for $H||ab$, and is between 0.75 and 0.77 for $H||c$.

This is due to the lowered symmetry at the Sr$_2$RuO$_4$-Ru interface. The situation is identical to bulk Sr$_2$RuO$_4$ in a symmetry breaking field that lifts the degeneracy in energy; the two components $k_x$ and $k_y$ are degenerate in bulk Sr$_2$RuO$_4$. In fact, Agterberg envisages successive superconducting transitions in magnetic fields parallel to the $ab$ plane [D. F. Agterberg, Phys. Rev. Lett. 80, 5184 (1998)].


M. Sigrist (private communication).
