Temperature dependencies of elastic constants and ultrasonic attenuation of a polycrystalline MgB$_2$ showing high-$T_c$ superconductivity are reported. An electromagnetic acoustic resonance method detected the specimen’s resonance frequencies, from which we derived the elastic constants for nonporous MgB$_2$ using micromechanics calculation. The bulk and shear moduli extrapolated to 0 K are determined to be 96.6 GPa and 74.2 GPa, respectively, which gives the Debye temperature $\Theta_D=819$ K and the coupling constant $\lambda\sim0.76$–0.89 with McMillan formula. In the temperature behavior of ultrasonic attenuation, two anomalous peaks are observed near 30 K.

Here we have examined the elastic and anelastic properties of MgB$_2$ through the ultrasonic characteristics. This paper presents the temperature dependence of elastic constants of nonporous MgB$_2$ deduced using a micromechanics model, the temperature behavior of ultrasonic attenuation, and observation of two anomalous attenuation peaks below 30 K.

A sample was prepared from commercially available powder by pseudo-HIP sintering at 1273 K for 12 h (referring to the method described in Ref. 1), and we checked that $T_c$ of the sample was about 39 K by an eddy-current method. The x-ray-diffraction spectrum indicated that the sample included a very small amount of MgO. The sample was machined into a rectangular parallelepiped, measuring 5.125($x_1$)$\times$3.940($x_2$)$\times$2.349($x_3$) mm$^3$, where $x_i$ denotes the coordinate axis of the specimen. The mass density is $\rho=2301$ kg/m$^3$, 87.7% of the theoretical value.

The remarkable discovery of intermetallic superconductor MgB$_2$ with a high $T_c$ of 39 K (Ref. 1) has triggered related extensive studies to elucidate the mechanism of its superconductivity. A significant boron isotope effect$^6$ implies that MgB$_2$ is a phonon-mediated superconductor. The $ab$ initio calculations,$^{3,4}$ tunneling-spectroscopy studies,$^5$ nuclear spin-lattice relaxation,$^6$ and photoemission measurements$^7$ have concluded that MgB$_2$ is a conventional s-wave superconductor of “strong” or “intermediate” electron-phonon coupling, although an essential uncertainty still remains in whether MgB$_2$ is a multiple-gap$^{8-11}$ or anisotropic-gap$^{12,13}$ superconductor.

The elastic constants are important parameters for knowing the Debye temperature and the electron-phonon coupling constant in a phonon-mediated superconductor. Several related studies appeared: first-principles calculations predicted the single-crystal elastic constants,$^{14,15}$ x-ray measurements under high pressure gave the isothermal bulk modulus,$^{16}$ the flexural resonant method gave Young’s modulus,$^{17}$ and other results were given in sound-velocity measurements.$^{18}$ However, these are inconsistent with each other. Also the temperature dependence has not been systematically measured yet.

Figure 1 shows the resonance spectrum, the resonance peak of the $B_{2g}$-2 vibration mode (-2 denotes the order of the resonance), and the free-decay amplitude.

At low temperatures, we used only the EMAR measurement and focused on the two vibration groups: $B_{2g}$ (shear modes around the $x_2$ axis) and $A_{g}$ (dilatation modes). The ultrasonic attenuation was measured by the free-decay method, and the attenuation coefficient $\alpha$ was determined by fitting $\exp(-\alpha t)$ to the measured ring-down with time. The normalized attenuation coefficient (internal friction) is obtained by $Q^{-1}=\alpha/\pi f$.

In the RUS procedure, the elastic constants are determined by comparing the measured and calculated resonance frequencies, from which we derived the elastic constants for nonporous MgB$_2$ using micromechanics calculation. The bulk and shear moduli extrapolated to 0 K are determined to be 96.6 GPa and 74.2 GPa, respectively, which gives the Debye temperature $\Theta_D=819$ K and the coupling constant $\lambda\sim0.76$–0.89 with McMillan formula.
and the normalized attenuation coefficient $\tilde{Q}$ is a quantity with a dimension of the sound velocity.

![Graph showing temperature dependence of elastic constants, internal friction, and resonance frequency.](image)

FIG. 2. Temperature dependence of elastic constants (upper) of the nonporous MgB$_2$, internal friction, and resonance frequency (lower).

spectra. The resonance frequencies are first calculated by the Rayleigh-Ritz method, assuming the elastic constants and using the dimensions and mass density of the specimen. Then, attaining good agreement between the measured and calculated spectra after iterations, the assumed elastic constants can be regarded as the true values.

No texture was found from the three-directional longitudinal-wave velocities by pulse-echo measurements. We then supposed a polycrystalline MgB$_2$ to be elastically isotropic. The elastic constants of porous MgB$_2$ are determined to be $c_{11}^{(p)} = 136$ and $c_{44}^{(p)} = 51.7$ GPa (at room temperature). We confirmed the validity with two more specimens of different sizes and virtually the same porosity.

From the determined elastic constants of the porous MgB$_2$, we deduced those for the nonporous material through a micromechanics model. We assume that (i) the matrix is elastically isotropic, and (ii) the pores (inclusions) are spherical. The elastic constants $C_{\text{mat}}$ of the nonporous MgB$_2$ are obtained by solving Eqs. (1)–(3) self-consistently:

$$C_{\text{mat}} = [I - c_{1}A]^{-1}(C_{\text{com}} - c_{1}C_{\text{inc}}A),$$

$$A = [c_{0}I + c_{1}B]^{-1}B,$$

$$B = [I + SC_{\text{mat}}^{-1}(C_{\text{inc}} - C_{\text{mat}})]^{-1}.$$ (3)

Here, $C_{\text{com}}$ are the measured elastic constants of the porous MgB$_2$, $C_{\text{inc}}$ are those of the inclusion (now $C_{\text{inc}} = 0$), $I$ is the unit matrix, $c_{0}$ and $c_{1}$ are the volume fractions of the matrix and the pores, respectively, and $S$ is the Eshelby tensor for the spherical inclusion, which depends on $C_{\text{mat}}$ and the pore shape.

Figure 2 shows the temperature dependence of the elastic constants, $c_{11} = B + 4G/3$, $c_{44} = G$, Young’s modulus $E$, and the bulk modulus $B$, of nonporous MgB$_2$ (upper part) and the normalized attenuation coefficient $\tilde{Q}^{-1}$ of the $B_{2g}$-3 mode (lower part). In the behavior of the elastic constants, we found no anomaly such as lattice instability as seen in the A15 superconductors. The polycrystalline elastic constants extrapolated to 0 K are $c_{11} = 196$, $c_{44} = 74.2$, $E = 177$, and $B = 96.6$ GPa. Our Young’s modulus is close to 167 GPa (uncorrected for porosity, impurities, and temperature) acoustically measured by Cordero et al., and the longitudinal and transverse sound velocities at 77 K (Ref. 18) are very consistent with our measurements. However, the bulk modulus appears to be very different from the isotothermal bulk modulus by x-ray measurements and the ab initio calculations. $B$ ranges from 120 to 160 GPa in the literature.

According to the calculation, the bulk and Young’s moduli are 158 and 299 GPa for the isotropic MgB$_2$, respectively. Probably, the ab initio calculations are based on the elastic property of TiB$_2$, which has the AlB$_2$ crystallographic structure as MgB$_2$. Table I shows the lattice properties and bulk moduli of AlB$_2$-type compounds at room temperature. $\sqrt{B/\rho}$ is a quantity with a dimension of the sound velocity.

<table>
<thead>
<tr>
<th>Compound</th>
<th>$c/a$</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$B$ (GPa)</th>
<th>$\sqrt{B/\rho}$ (m/s)</th>
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<tr>
<td>TiB$_2$</td>
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<tr>
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<td>6139</td>
<td>239</td>
<td>6239</td>
<td>25</td>
</tr>
<tr>
<td>MgB$_2$</td>
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<td>2625</td>
<td>120-160</td>
<td>6761-7807</td>
<td>14–16</td>
</tr>
<tr>
<td>MgB$_2$</td>
<td>1.142</td>
<td>2625</td>
<td>86.1</td>
<td>5728</td>
<td>present work</td>
</tr>
</tbody>
</table>

*At low temperature (or 0 K). In the behavior of the elastic constants, we found no anomaly such as lattice instability as seen in the A15 superconductors. The polycrystalline elastic constants extrapolated to 0 K are $c_{11} = 196$, $c_{44} = 74.2$, $E = 177$, and $B = 96.6$ GPa. Our Young’s modulus is close to 167 GPa (uncorrected for porosity, impurities, and temperature) acoustically measured by Cordero et al., and the longitudinal and transverse sound velocities at 77 K (Ref. 18) are very consistent with our measurements. However, the bulk modulus appears to be very different from the isotothermal bulk modulus by x-ray measurements and the ab initio calculations. $B$ ranges from 120 to 160 GPa in the literature.

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The Debye temperature $\Theta_D$ is an important quantity to estimate the electron-phonon coupling constant $\lambda$, which is proportional to the mean sound velocity $v_m$,.

$$\Theta_D = \left( \frac{h}{kB} \right)^{1/2} \left( \frac{3}{4\pi\Omega} \right)^{1/2},$$

(4)

where $h/k_B$ has the usual meaning in quantum mechanics, $\Omega$ is the mean atomic volume, and $v_l$ and $v_s$ are the isotropic sound velocities given by $(B + 4G/3)/\rho$ and $(G/\rho)^{1/2}$, respectively. The present elastic constants at 0 K yield $\Theta_D = 819$ K. According to the McMillan formula, the critical temperature $T_c$ is expressed using $\Theta_D$, $\lambda$, and the Coulomb pseudopotential $\mu^*$ as

Table I. Lattice properties and bulk moduli of AlB$_2$-type compounds at room temperature. $\sqrt{B/\rho}$ is a quantity with a dimension of the sound velocity.
\[ T_c = \frac{\Theta_D}{1.45} \exp \left[ \frac{-1.04(1 + \lambda)}{\lambda - \mu^* (1 + 0.62\lambda)} \right]. \]  

The square of frequency corresponds to the elastic constant \(\mu^*\). The difference in the frequency ranges used in both measurements is probably attributed to the difference in the high-Debye temperature \(T_c\) between 70–120 K, being different from our measurement, which is probably attributed to the different temperature. However, the degree of softening is much less that the elastic softening occurs with the superconducting transition. The bulk modulus \(B\) and the internal friction of the mode, being shifted downward to avoid the overlap of the two curves. The background attenuation is \(\approx 2.5 \times 10^{-4}\) \(\mu\)s/MHz. The magnetic field of \(\sim 0.2\) T was applied for the EMAR excitation.

In conclusion, we have examined the elastic property and ultrasonic attenuation behavior of the high-\(T_c\) intermetallic superconductor MgB\(_2\). The salient results are summarized as follows:

(i) The elastic constants are determined to be \(B = 96.6\) and \(G = 74.2\) GPa at 0 K through the micromechanics calculation. The bulk modulus \(B\) is smaller than the previous reports. However, the presented value is more reliable and reasonable in terms of both the axial ratio of the AlB\(_2\) structure and its low condensation.

(ii) The elastic constants determined in this work yield the Debye temperature \(\Theta_D = 819\) K and the intermediate coupling constant \(\lambda = 0.76–0.89\) with the isotropic McMillan formula.

(iii) In the temperature behavior of ultrasonic attenuation, two anomalous peaks were observed.

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25 H. Inui, K. Tanaka, and S. Otani (private communication).